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Volterra Series Approximation for Multi-Degree of Freedom, Multi-Input, Multi-Output, Aircraft Dynamics

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VOLTERRA SERIES APPROXIMATION FOR MULTI-DEGREE OF FREEDOM, MULTI-INPUT, MULTI-OUTPUT, AIRCRAFT DYNAMICS

by

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ABSTRACT

VOLTERRA SERIES APPROXIMATION FOR MULTI-DEGREE OF FREEDOM, MULTI-INPUT, MULTI-OUTPUT, AIRCRAFT DYNAMICS

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Old Dominion University, 2018
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An analytical model of a second order system is extended from a single-axis framework, to a multi-axis, multi-degree of freedom framework for a multiple input, multiple output system. This mathematical model is built from the variational approach of the Volterra series representation of nonlinear systems. The new representation describes the second order, oscillatory natural modes of a system, and shows how to organize the Volterra terms in intuitive ways. The constructed mathematical model aims to establish an organization of the Volterra kernels to allow for analytical cause and effect type analysis on system behavior.

To demonstrate the accuracy of the developed Volterra model, the model is applied to atmospheric flight dynamics. A numerical simulation of an F-16 aircraft was developed based on the experimental data collected at NASA Langley and is compared to the Volterra model. Both longitudinal and latitudinal aircraft dynamics are analyzed, and the results show that the Volterra model effectively tracks the numerical simulations and has less error than a more conventional linearized system. The results show that weak nonlinearities of a system are predicted based on this new model. The construction of the model allows for a more effective analysis to the cause and effect of the response. Individual responses of each nonlinear component are separated for analysis, and each component’s effects on the total system response are observed.

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NOMENCLATURE

A	State matrix in state space
B	Input matrix in state space
C	Output matrix in state space
D	Direct input to output matrix in state space
H	Altitude
K	Volterra system gains related to the first state derivative
L	Volterra system gains related to the second state derivative
M	Mach number
T	Thrust
V_T	Total velocity
X	X-axis position in the inertial frame
Y	Y-axis position in the inertial frame
Z	Z-axis position in the inertial frame
h	Volterra kernel
ω_d	Damped natural frequency
ω_n	Undamped natural frequency
ζ	Damping ratio
σ	Damping factor
$\bar{s}$	Wing span area
$\bar{c}$	Wing chord, aerodynamic mean
m	Aircraft mass
u	Velocity in the body x-axis

v Velocity in the body y-axis

w Velocity in the body z-axis

α Angle of attack

β Sideslip angle

φ Roll Euler angle

θ Pitch Euler angle

ψ Yaw Euler angle

p Roll rate

q Pitch rate

r Yaw rate

δ_{hrzt} Horizontal tail deflection input

δ_{alrn} Aileron deflection input

δ_{spbr} Speed brake deflection input

δ_{lef} Leading edge flap deflection input

δ_{rdr} Rudder deflection input

δ_{th} Throttle percentage input

I_{xx} Moment of inertia about the body x-axis

I_{yy} Moment of inertia about the body y-axis

I_{zz} Moment of inertia about the body z-axis

I_{xy} Product of inertia with respect to the body x and y axes

I_{yz} Product of inertia with respect to the body y and z axes

I_{xz} Product of inertia with respect to the body x and z axes

H_e Engine angular momentum

x State in state space

u	Input in state space
y	Output in state space
t	Time
τ	Time integration/dummy variable
Φ	State transition matrix
$C_{x,t}$	Total body x-axis aerodynamic coefficient
$C_{y,t}$	Total body y-axis aerodynamic coefficient
$C_{z,t}$	Total body z-axis aerodynamic coefficient
$C_{l,t}$	Total roll moment aerodynamic coefficient
$C_{m,t}$	Total pitch moment aerodynamic coefficient
$C_{n,t}$	Total yaw moment aerodynamic coefficient
H_x^B	Angular momentum with respect to the body x-axis
H_y^B	Angular momentum with respect to the body y-axis
H_z^B	Angular momentum with respect to the body z-axis
$\bar{q}$	Dynamic pressure
$\bar{x}_{cm}$	Actual center of mass location
$\bar{x}_{cm_r}$	Reference center of mass location for aerodynamic data
φ_1	Phase corresponding to first element of state transition matrix
φ_2	Phase corresponding to fourth element of state transition matrix

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CHAPTER 1

INTRODUCTION

1.1 Overview

Engineers utilize a number of toolsets to conceptualize, develop, and optimize control systems. As stated by Etkin[1], these toolsets usually fall under three categories: experimental, computational, or analytical methods. Experimental methods consist of analyzing flight and wind tunnel test data and feeding back the test results to design the system. This category is used in tandem with computational methods to simulate dynamics, develop controllers, and perform optimization via numerical computation to drive the desired performance. Numerical simulation has proven to be an indispensable tool to drive design prior to developing a full-scale model. Finally, analytical methods consist of the mathematical description of the system and lead to an initial understanding and intuition of the system's dynamics. This method includes mathematical analysis, which allows for stability, robustness, and performance predictions. The mathematical toolset involves a combination of vector/matrix algebra, Laplace and Fourier transforms, system theory, state space methods, analytical linearization, and nonlinear analysis techniques. Analytical methods are important for the engineer to quickly provide feedback early in the design phase, as well as to provide insight about how the system behaves while it is being designed and iterated.

A proper understanding of all three categories leads to better design in a modern control system. For example, early on in the design process, the analytical toolset is reflected on to quickly describe the system under certain assumptions and simplifications. Analytical knowledge allows the early design of a control system and the ability to grasp essential concepts and predict performance. Later on, numerical simulations are used to describe a more detailed characterization of the system, whose results are fed back to the analytical model to test accuracy. Small-scale models and prototypes are then built to obtain experimental data, and this in turn is fed back to the results of the analytical model and simulations. This interplay between all three toolsets provides insight for iteration of the control system to meet performance requirements. The focus of this thesis is primarily to build upon the analytical toolset that engineers use to study and understand the nonlinear dynamics of a system.

A time-tested technique to develop an analytical description of the system is to approximate a dynamical system through the method of linearization; that is, simplifying the dynamics of the system based on linear time-invariant (LTI) system theory. This process involves linearization of the system about an equilibrium reference point and then applying small perturbations for analysis. This technique has proven sufficient to offer significant accuracy for predictions of system behavior across a broad range of performances. While effective, this approximation is just that, an approximation. A true linear system is never observed in the real world, as nonlinearities are always present. Therefore, the study of nonlinear system theory is an obvious continuation to linear system theory with the goal being to achieve greater accuracy and insights to real-world physics.

This thesis expands upon the work started by Omran and Newman, References 2 through 11, in which an analytical model of a nonlinear system was developed based on a Volterra series solution. Volterra theory has become one of the more popular methodologies for describing nonlinear dynamics, as it is seen as an extension of the already familiar LTI description of a system. The effort by Omran and Newman established an analytical Volterra model for a single degree of freedom, single-axis system for studying weak nonlinearities. Omran and Newman then applied this description to model aircraft flight dynamics. This work had real-world application with their utilization of F-16 experimental wind tunnel data captured by NASA Langley Research Center.[11]

Atmospheric flight dynamics is an important topic for studying nonlinear system theory. The aircraft equations of motion are a set of nonlinear ordinary differential equations covering six degrees of freedom. While linear system theory provides relatively accurate results across a large spectrum of the flight envelope, there are specific maneuvers and effects that cannot be predicted or studied using this method. This situation includes roll resonance, spin-yaw coupling, inertia coupling, wing rock, and limit cycles. This list is in addition to the weak nonlinearities inherent during normal maneuvers. These flight characteristics can often be traced directly back to certain nonlinear terms in the equations of motion. However, with linearization, underlying assumptions are made to neglect these nonlinearities.

There has been extensive effort to develop analytical techniques to describe these nonlinearities. Current methodologies include bifurcation, describing function, and power series descriptions. While these methods have provided insight, they do not have the predictive capability needed to fully understand the nonlinear phenomenon. In this respect, the Volterra method applied by Omran and Newman has an advantage, as this description allows a cause and effect analysis. This advantage is due to the analytical nature of the Volterra series that is very similar to the linear system theory solution. In both Volterra and linear system theories, the system responses from input to output are clearly represented in an analytical form, unlike the other methods. Volterra theory does have a drawback in that the system's Volterra kernels, which form the core analytical descriptions of the system, can be cumbersome and computationally time consuming to derive in complex systems. The work of Omran and Newman laid the foundation for continuing research in this area. Their work led to a second order, single degree of freedom, single-axis Volterra description for studying atmospheric flight dynamics. This thesis expands on that work and derives a Volterra system description for a second order, multi-axis, multi-degree of freedom system. This new description is then applied to atmospheric flight dynamics to continue research into the nonlinear phenomenon experienced in the study of flight.

1.2 Literature Review

The study of nonlinear dynamics in aircraft systems has been investigated extensively, with many methods published in the literature. This section is broken down into three subsections to outline the research important to this thesis: literature written on general nonlinear system theory, Volterra system theory, and nonlinear aircraft dynamics. These subsections provide just a sample of the research in each of these areas.

1.2.1 Linear and Nonlinear System Theory Background

Linear system theory has proven to be one of the most effective techniques to analyze dynamics and controls. There are a multitude of research articles and books on the subject. A general overview was written by Rugh, Reference 12, which covers the basic theory, proofs, and analytical expressions that sometimes are glossed over in other publications. Details of the convolution integral, state transition matrix (STM), perturbation linearization theory, and Taylor series expansion are key areas that are described in detail and are applicable to the nonlinear system description derived in this thesis. As shown later in this thesis, the properties of Volterra theory allow linear system descriptors to be used for a nonlinear description. For example, the state transition matrix is still one of the key system descriptors for the multi-input, multi-output (MIMO) Volterra model. Also, the convolution integral used to describe LTI systems is also used for the first order kernel solution for the Volterra series, with higher degree kernels using multi-dimensional convolution integrals. Utilizing Taylor series expansions and perturbating from an equilibrium solution are also techniques used to set up the nonlinear analysis of the aircraft model.

Note, Rugh provides a few examples for real-world application, but the majority of this reference covers general theory. Other books provide application of linear system theory to flight dynamics explicitly, such as the books by Etkin, McRuer, and Stevens and Lewis, References 1, 13, and 14. These books explicitly cover linearization of the aircraft equations of motion and its application to studying open and closed loop control. The works by McRuer and Etkin explore aircraft dynamics analytically, each expanding on stability and control using classical control theory. The work by Stevens and Lewis focuses on numerical methods and simulation-based analysis, using modern control theory and nonlinear analysis that full computer simulations have provided. These three works each provide reduced order derivations of the equations of motion. This reduction includes both longitudinal and latitudinal natural modes for the short period, phugoid, dutch roll, roll subsidence, and spiral modes. The short period and dutch roll oscillatory modes are applicable for the nonlinear analysis in this thesis.

The study of nonlinear system theory has also been covered extensively in the literature. A general overview of early work on the subject is the book written by Graham and McRuer, Reference 15. In particular, this book provides the methodology of the describing function used for nonlinear analysis. The concept of this theory is to replace nonlinear elements with linear describing functions, such as a sinusoidal function. McRuer and Graham show that this sinusoidal approximation for a nonlinear element can be used to describe a variety of weak nonlinear phenomenon, such as saturation and Coulomb friction. Another method outlined in the book is the phase plane method, a graphical representation and method of analysis for nonlinearities such as the limit cycle, a nonlinear oscillation of fixed frequency and amplitude. While the method outlined by Graham and McRuer is helpful for studying specific nonlinearities, they are not full, analytical descriptions. The first method approximates a specific set of nonlinearities with a linear function, while the other method can be considered a graphical approach to analysis.

Another publication on the subject is that of Vidyasagar, Reference 16, in which he outlines two methodologies that provide analytical solutions for nonlinear system analysis. One is the Krylov-Boguliubov averaging method and the other is the power series method. However, these two methods can only be applied for specific forms of second order differential equations and therefore are not desirable to the nonlinear equations involved with aircraft dynamics in this thesis.

One method of nonlinear system analysis that has proven useful for studying aircraft dynamics is bifurcation theory. Reference 17 shows the application of bifurcation theory to generalize the nonlinear behavior in high angle of attack maneuvers for an F-4 aircraft. Bifurcation theory involves creating equilibrium surface maps, or bifurcation diagrams, which capture the change of equilibrium points when certain system parameters and inputs are changed. Using these diagrams, it is possible to derive a limit point at which the solution curve folds back into itself, called a bifurcation point. Plotting these bifurcation points provides useful predictions such as stable and unstable regions of performance. Carroll and Mehra were able to confirm that bifurcation theory could be used to predict limit cycles and other high angle of attack phenomenon for aircraft.

Bifurcation theory is a promising area of study; however, it is still not a comprehensive cause and effect analytical solution. The bifurcation analysis requires the computation of numerous equilibrium solutions and mappings. While this form of analysis is a tool for predicting nonlinearities, a true analytical mathematical model is still sought.

1.2.2 Volterra Theory Background

Volterra theory has shown potential in providing additional, analytical insight to nonlinear dynamics. The Volterra series description was pioneered by Vito Volterra in the 1880s and was formally published in 1959, Reference 18. Volterra's integral function theory was applied by Norbert Wiener to study the nonlinear response of a resistor due to white noise in a resistor-inductor-capacitor (RLC) circuit in 1946. Wiener also contributed greatly to the advancement of nonlinear system theory, developing his own method of analyzing nonlinear systems. He published the combination of his prior theory with the Volterra series in 1958 in Reference 19.

Volterra theory can be seen as a generalization of the Taylor series expansion of a function, with a form similar to the first order convolution approach to linear, time-invariant systems. The Volterra series adds more terms beyond the first order approximation, to capture the nonlinearities. It shows that a nonlinear, time-invariant system can be represented by an infinite summation of increasing order, multi-dimensional convolution integrals. This form has a number of benefits including a direct analytical expression of the input-output, cause-effect relationship. Another benefit is the intuitional basis of convolution, which is well-developed and has been used in linear, time-invariant models.

The drawback to Volterra theory is the complexity arising from developing the integration terms, henceforth called the Volterra kernels. These kernels are the key system descriptors for the input to output relationship of the nonlinear system. If a system is either being described at a high order, or multivariate dynamics are included, the Volterra kernels become computationally expensive and unwieldy to formulate.

Various methods for developing the Volterra kernels have been researched and published. A general summary of some of these methods was published by Rugh in 1981, Reference 20, and provides a number of methods to derive the kernels. The Carleman linearization approach is one example. This method can be used on linear-analytic state equations to transform the state equations into a bilinearized form. The linear-analytic terms are expanded by power series representations, and the system is reformed in the bilinear state form to make derivation of the Volterra kernels straightforward. This method has been successful in analytically describing the nonlinearities of flight dynamics, as has been shown in Reference 21. However, this method leaves the kernels in triangular form, which will be explained later.

Another method outlined by Rugh is the growing exponential approach. This approach inserts an exponential series as the input, which gives a similar output in the form of another power series. After a few substitutions are made, the kernels are formed based on equating coefficients of like-exponentials. This approach gives kernels in relatively straightforward, simple terms; however, the length of the kernels becomes too large for more complex systems.

The variational expansion method is the final approach outlined by Rugh. This method involves describing a nonlinear system by expanding the nonlinear terms around the state vector and input vector as a power series. The state vector, itself, is then expanded in series in terms of an arbitrary parameter of increasing order to degree-n. An infinite set of subsystem differential equations are obtained based on equating like-ordered terms. Each subsystem is homogenous, and starting from the first order, each subsystem is fed into the next higher ordered subsystem.

The key to the approach is that the first order subsystem solution is the well-known linear, time-invariant solution, which is then fed into every subsequent expansion order. This process has the effect that the nonlinearities can be solved for in a linear way, as each subsystem has the first order terms. The Volterra kernels can thus be solved for analytically and will inherit certain attributes comparable to linear system theory.

This method provides the analytical framework sought for this research; however, it does carry some limitations. First, the kernel calculations are rigorous for all except the simplest of systems. As complexity of the system increases, the kernels become more and more computationally expensive. Also, the kernel solutions themselves become cumbersome due to length. It will take significant analysis to extract the kernels of interest from the solutions for a given system.

1.2.3 Aerospace Applications of Volterra Theory

The approach outlined by Rugh represents an approach for single-axis systems and has been used a number of times to study aircraft dynamics. Charles Suchomel dedicated a great amount to the subject in the late 1980s. Reference 22 describes some initial research in defining nonlinear metrics and another approach on how to form an analytical Volterra model of an aircraft. Specifically, Suchomel showed how a truncated three order variational expansion accurately modeled the nonlinear response of an F-8 aircraft pull-up maneuver. For these results, however, Suchomel numerically calculated the Volterra integrals to obtain the solution, and an analytical model was left to further research.

The released study from the 1988 Air Force Wright Aeronautical Laboratory, Reference 23, expounded on Suchomel's research and derived analytical kernels for a first order system with one-state, as well as the kernels for a two-state system. However, these analytical kernels were not developed for a general system; that is, the model left out some nonlinearities that didn't pertain to the specific flight mechanics under investigation. The authors in this paper studied two nonlinear aircraft characteristics of a T-2C aircraft, the limit cycle produced at high angle of attack, and wing rock. They showed that the Volterra series approximation is able to capture the nonlinear effect to much greater accuracy than that of a linear model and that the specific nonlinearities could be traced back through the analytical terms.

Volterra theory has also been used in research in other areas of aerospace, such as with the work undertaken by Silva, Reference 24. This research utilized Volterra theory to analyze transonic performance of a NACA 64A0110 rectangular wing. Silva investigated a time domain, unit impulse response method of identifying the Volterra kernels for a discrete, nonlinear system. This discrete formulation determines the kernel values at each individual time-step, and Silva provided a comparison to the true, analytic solution to prove its success. The discrete time representation of the kernels has continued on as a separate method to nonlinear aircraft research.

The Volterra variational approach to studying nonlinear flight mechanics was somewhat dormant for the next couple of decades. The approach was revisited by Omran and Newman in a series of published articles, References 2 through 10, in the late 2000s. Their research culminated in calculating full, analytical Volterra kernels for two general systems: the first order single-axis system and the second order single-axis system.[11] This work showed that a full cause and effect analytical approach could be formulated. Omran and Newman also developed a piecewise, parameter varying approach so that a truncated Volterra representation could be used to describe nonlinear responses in a global sense. That is, the parameters of a flight region were joined to the parameters of another flight region so that the model could handle a greater range. Due to the added complexity of the Volterra solution at higher orders, Omran and Newman developed this approach to include only the second order expansion. The parameter varying method then extended the model. Omran and Newman studied both longitudinal and latitudinal flight dynamics using these models. They utilized F-16 experimental data to both compare the models to numerical simulations and show nonlinearities inherent in that aircraft at various operating points.

As outlined in Omran and Newman's papers, there is a continuing need to expand the Volterra model to multiple degrees of freedom. Aircraft flight mechanics are described by equations of motion in six degrees of freedom. A nonlinear description of the inherent coupling between the states in the equations is an obvious need to further the research. This expansion would allow transparency into the nonlinear effects due to the cross coupling. The Volterra theory presented thus far is limited to only one degree of freedom, or a single-axis. The publication by Worden in 1996, Reference 25, extended the general Volterra theory to multiple inputs and multiple outputs. This extension identifies cross kernels from each input to each output. Worden investigated a harmonic probing algorithm to identify the direct and cross kernels involved and demonstrated its application to a two degree of freedom, spring-damper system.

Worden's multiple input, multiple output Volterra series has been applied to aerodynamics with the research produced by Balajewicz, Nitzsche, and Feszty, Reference 26, in 2010. They applied Worden's model to describe the pitch and heave unsteady motion of a two-dimensional NACA 0012 airfoil. Balajewicz was able to successfully demonstrate multi-input Volterra theory's accuracy to computational fluid dynamics (CFD) results. This research also showed the opportunities that the theory provides for studying the nonlinear cross coupling mechanisms. Balajewicz published a second paper in 2012, building off of the NACA 0012 work, to describe the airfoil's nonlinear flutter and limit cycle oscillations, again using multi-input Volterra theory.[27]

In each of these two cases, the Volterra kernels were identified using the previously mentioned discrete time approach. That is, the kernel values were computed numerically at each time-step. The purpose of this thesis research is to develop an analytical solution to the multi-input, multi-output system. This result will then be applied to aircraft flight mechanics to demonstrate its accuracy and investigate certain nonlinear phenomenon.

1.2.4 Experimental Data

The primary experimental data used for this research is wind tunnel data of an F-16 Fighting Falcon aircraft, manufactured by General Dynamics. This airframe is a highly maneuverable, single-engine, supersonic, multi-role, fighter aircraft. A three-view drawing of this aircraft is shown in Figure 1.1. The air vehicle was developed in the 1970s and introduced in 1978. At close to 40 years old, this aircraft is still in service in over 25 nations, including the U.S. Air Force. As a small, highly maneuverable aircraft with a large flight envelope, it is expected that the data will show nonlinearities inherent to large angles of attack, large sideslip angles, and high angular rates. This experimental data was published by Nguyen, Ogburn, Gilbert, Kibler, Brown, and Deal in a technical paper at NASA Langley Research Center in Hampton, Virginia, Reference 28. The data published covers aerodynamic coefficient values at angles of attack from -20 to +90 degrees, and side-slip angles from -30 to +30 degrees. This data also covers various control surface deflection points for the horizontal tail, ailerons, rudder, leading edge flap, speed brake, and thrust power level inputs.

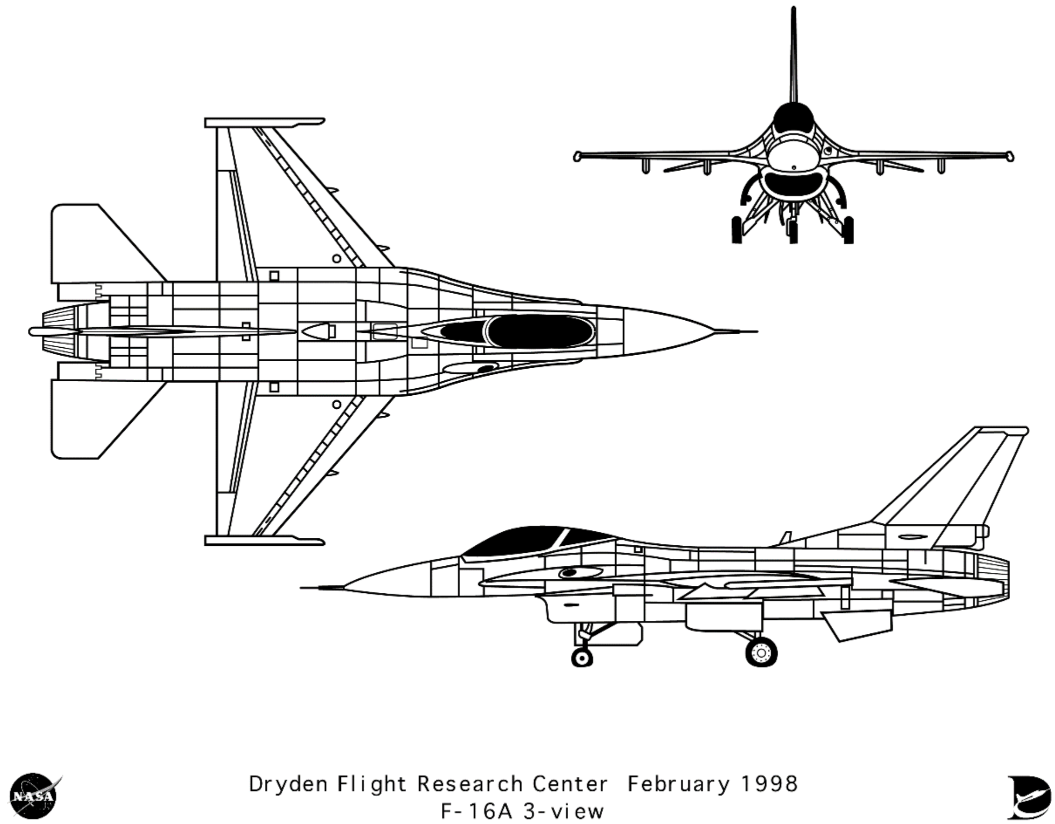

Figure 1.1 NASA Three-View of F-16A Aircraft

A simplified form of the F-16 data is published and investigated in the previously mentioned book by Lewis and Stevens. This book provides a summary of numerical simulation concepts and an initial analysis of longitudinal and latitudinal dynamics of the F-16 data. Most of this analysis uses linear system theory; therefore, investigation of the F-16 nonlinear dynamics based on Volterra theory could prove insightful. The research by Omran and Newman also utilized this F-16 data to investigate their single-axis solutions. Therefore, a continuing investigation of F-16 nonlinearities with a multi-axis solution is an obvious extension which may yield additional insights.

1.3 Problem Statement

The purpose of this thesis is to develop an analytical, mathematical model for a nonlinear, multiple degree of freedom (multi-axis) system. Based on previous literature, Volterra theory is seen to have the most potential to describe the nonlinear dynamics sought. This research is a natural extension to previous research performed by Omran and Newman. Due to the added complexity that is driven by the Volterra order and the number of degrees of freedom, only a second order system description with two states and two inputs is pursued. A direct application of this model is aircraft flight dynamics. The nonlinear dynamics, and their effects on the entire aircraft response, will be investigated using the F-16 experimental data provided by NASA Langley. The nonlinear Volterra model response will be compared to the numerical simulation to show the accuracy of the model. Finally, a cause and effect analysis will be performed on the discovered nonlinearities to show which system components contribute significantly to the nonlinear phenomenon.

1.4 Thesis Outline

This thesis is composed of six chapters. Chapter 2 is an introduction to Volterra theory and a review of the variational expansion method for generating Volterra kernels. The chapter also describes the single degree of freedom work performed thus far before reviewing the multiple input, multiple output extension developed by Worden. Chapter 3 describes the mathematical model, first explaining the organization of the Volterra equations derived, and the unique notation used in order to trace Volterra kernels to their state responses. The chapter then proceeds with the derivation of the Volterra equations for the MIMO system and discusses the overall kernel integrals. A normalized set of surface plots of the kernels are also examined. This chapter then proceeds to the calculation of the step response equations. Again, a normalized set of system responses is examined. Chapter 4 describes the derivation of the aircraft equations of motion utilized for this thesis, including the steps and assumptions taken to simplify both longitudinal and latitudinal equations in the form of two-state two-input approximations. Specifically, this chapter shows how the short period and dutch roll aircraft natural modes are approximated with two degree of freedom motion. Chapter 5 covers the state response results and analysis for each of the natural modes. Chapter 6 covers the conclusions that can be made from the work and an overview of future work that can be made in this area of dynamics.

CHAPTER 2

VOLTERRA SYSTEM THEORY

2.1 Overview

In this chapter, Volterra theory is reviewed. This review includes the general Volterra series description, comparison to the standard linear time-invariant convolution integral, its structure for single degree of freedom systems, and the extension to multiple input, multiple output systems. Finally, this chapter will discuss the variational expansion approach to form the input-output expressions of the system in the time domain.

2.2 Volterra Series

Physical systems are generally described by a set of differential equations which map the input signal to the system states and the output signals. The signal transmission through linear system in the time domain from time $\tau = 0$ to t, is described by the convolution integral

$$y(t) = \int_0^t h(t-\tau)u(\tau)\,d\tau \tag{2.1}$$

which is assumed to be continuous, time-invariant and casual. This function describes the input-output relationship of an LTI system, where $h(t)$ represents the impulse response function, or the system kernel, $u(t)$ represents the system input, and $y(t)$ is the output response. The Volterra series is a generalization of this input-output representation to a nonlinear system, where the system is also assumed to be continuous, time-invariant and causal. While many real-world systems fall under some or all of these assumptions, it should be noted that not all do (or can be simplified to satisfy these assumptions), and in those cases the Volterra series cannot be used to describe them. Continuity is the property that arbitrarily small inputs cause arbitrarily small outputs over the function domain, or there are no discontinuities present. Time-invariance is the property that the system output characteristics do not depend explicitly on time, or the system is only indirectly dependent on the time domain via inputs. Finally, causality is the property where the system output only depends on past and present inputs, not future ones.

The Volterra series is of the form

$$y(t) = h_0(t) + \int_{-\infty}^{\infty} h_1(\tau_1)u(t-\tau_1)\,d\tau_1 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau_1,\tau_2)u(t-\tau_1)u(t-\tau_2)\,d\tau_1 d\tau_2 + \cdots + \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} h_n(\tau_1,\cdots,\tau_n)u(t-\tau_1)\cdots u(t-\tau_n)\,d\tau_1\cdots d\tau_n \tag{2.2}$$

where again, $y(t)$ is the system output response, $u(t)$ is the system input, and $h_n(t)$ represents the Volterra kernels of order n. The form above shows that the first order Volterra kernel term is the same as the LTI convolution integral. This equivalence is important when discussing the variational expansion approach to computing the kernels. In essence, the Volterra series is an infinite sum of multi-dimensional convolution integrals of increasing order. In shorthand, the Volterra series can instead be written as

$$y(t) = h_0(t) + \sum_{n=1}^{\infty} \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} h_n(\tau_1,\cdots,\tau_n)\prod_{i=1}^{n} u(t-\tau_i)\,d\tau_i \tag{2.3}$$

As Rugh[20] showed, due to the one-sided assumptions of physical systems (causality) to $h(t)$ and $u(t)$, the upper limits can be lowered to t, the lower limits can be raised to 0, and a change of variables can be performed to show that the Volterra system can also be written in the form

$$y(t) = h_0(t) + \sum_{n=1}^{\infty} \int_0^t \cdots \int_0^t h_n(t-\tau_1, \cdots, t-\tau_n) \prod_{i=1}^{n} u(\tau_i)\, d\tau_i \qquad (2.4)$$

Note that the zeroth order kernel, $h_0(t)$, is the zero-input response. For the purpose of this research, it is a treated as a constant representing the initial equilibrium output of the system.

The system defined above suits only a single-axis system. The Volterra series was extended to a multiple input, multiple output system by Worden[25], with a general nth degree form specified by Chatterjee[29]. The MIMO Volterra series representation used for this thesis is tailored from the reference material to suit this research application. The representation is

$$\begin{aligned} y_j(t) &= h_0(t) \\ &+ \sum_{\eta_1 = u_1, u_2, \ldots} \int_{-\infty}^{\infty} h_{1_{(y_j, \eta_1)}}(\tau_1) \eta_1(t-\tau_1)\, d\tau_1 \\ &+ \sum_{\eta_1 = u_1, u_2, \ldots} \sum_{\eta_2 = u_1, u_2, \ldots} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_{2_{(y_j, \eta_1, \eta_2)}}(\tau_1, \tau_2) \eta_1(t-\tau_1) \eta_2(t-\tau_2)\, d\tau_1 d\tau_2 \\ &+ \cdots \\ &+ \sum_{\eta_1 = u_1, u_2, \ldots} \cdots \sum_{\eta_n = u_1, u_2, \ldots} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_{n_{(y_j, \eta_1, \cdots, \eta_n)}}(\tau_1, \cdots, \tau_n) \eta_1(t-\tau_1) \cdots \eta_n(t-\tau_n)\, d\tau_1 \cdots d\tau_n \\ &+ \cdots \end{aligned} \qquad (2.5)$$

where the output $y_j(t)$ represents each output response ($y_1(t)$, $y_2(t)$,…etc.), and η_n represents each input signal ($u_1(t)$, $u_2(t)$,…etc.). For the second degree kernels and above, it is seen that the summations result in permutations based on the inputs. For the second degree terms and higher, the kernels where $\eta_1 = \eta_2 = \cdots = \eta_n$ are called direct kernels. When this condition is not satisfied, the associated kernels are called cross kernels.

In shorthand, the general representation is

$$y_j(t) = h_0(t) + \sum_{n=1}^{\infty} \sum_{\eta_1=u_1,u_2,\dots} \cdots \sum_{\eta_n=u_1,u_2,\dots} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_{n_{(y_j,\eta_1,\cdots,\eta_n)}}(\tau_1,\cdots,\tau_n)$$

$$\times \eta_1(t-\tau_1)\cdots\eta_n(t-\tau_n)d\tau_1\cdots d\tau_n \tag{2.6}$$

The one-sidedness assumptions used on the single-axis representation can also be used for the multiple input case. This causality assumption results in

$$y_j(t) = h_0(t)$$

$$+ \sum_{\eta_1=u_1,u_2,\dots} \int_0^t h_{1_{(y_j,\eta_1)}}(t-\tau_1)\eta_1(\tau_1)\,d\tau_1$$

$$+ \sum_{\eta_1=u_1,u_2,\dots} \sum_{\eta_2=u_1,u_2,\dots} \int_0^t \int_0^t h_{2_{(y_j,\eta_1,\eta_2)}}(t-\tau_1,t-\tau_2)\eta_1(\tau_1)\eta_2(\tau_2)\,d\tau_1 d\tau_2$$

$$+\cdots$$

$$+ \sum_{\eta_1=u_1,u_2,\dots} \cdots \sum_{\eta_n=u_1,u_2,\dots} \int_0^t \cdots \int_0^t h_{n_{(y_j,\eta_1,\cdots,\eta_n)}}(t-\tau_1,\cdots,t-\tau_n)\eta_1(\tau_1)\cdots\eta_n(\tau_n)\,d\tau_1\cdots d\tau_n$$

$$+\cdots \tag{2.7}$$

or

$$y_j(t) = h_0(t) + \sum_{n=1}^{\infty} \sum_{\eta_1=u_1,u_2,\dots} \cdots \sum_{\eta_n=u_1,u_2,\dots} \int_0^t \cdots \int_0^t h_{n_{(y_j,\eta_1,\cdots,\eta_n)}}(t-\tau_1,\cdots,t-\tau_n)$$

$$\times \eta_1(\tau_1)\cdots\eta_n(\tau_n)d\tau_1\cdots d\tau_n \tag{2.8}$$

As an example, for a two degree of freedom system with two inputs, two outputs, and described to second order, the MIMO Volterra series with zero initial value is

$$y_1(t) = \int_{-\infty}^{\infty} h_{1_{(y_1,u_1)}}(t-\tau)u_1(\tau)\,d\tau + \int_{-\infty}^{\infty} h_{1_{(y_1,u_2)}}(t-\tau)u_2(\tau)\,d\tau$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_{2_{(y_1,u_1,u_1)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_1(\tau_2)\,d\tau_1 d\tau_2$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_{2_{(y_1,u_1,u_2)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_2(\tau_2)\,d\tau_1 d\tau_2$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_{2_{(y_1,u_2,u_1)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_1(\tau_2)\,d\tau_1 d\tau_2$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_{2_{(y_1,u_2,u_2)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_2(\tau_2)\,d\tau_1 d\tau_2 \quad (2.9)$$

$$y_2(t) = \int_{-\infty}^{\infty} h_{1_{(y_2,u_1)}}(t-\tau)u_1(\tau)\,d\tau + \int_{-\infty}^{\infty} h_{1_{(y_2,u_2)}}(t-\tau)u_2(\tau)\,d\tau$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_{2_{(y_2,u_1,u_1)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_1(\tau_2)\,d\tau_1 d\tau_2$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_{2_{(y_2,u_1,u_2)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_2(\tau_2)\,d\tau_1 d\tau_2$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_{2_{(y_2,u_2,u_1)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_1(\tau_2)\,d\tau_1 d\tau_2$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_{2_{(y_2,u_2,u_2)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_2(\tau_2)\,d\tau_1 d\tau_2 \quad (2.10)$$

As shown, the summation resulted in terms representing each permutation of the inputs. The kernels with respect to u_1 and u_2 together (e.g., $h_{2_{(y_j,u_1,u_2)}}$ and $h_{2_{(y_j,u_2,u_1)}}$), are the cross kernels. The remaining kernels are the direct kernels.

Interpretation of the MIMO Volterra series reveals that the overall response is a superposition of all direct and cross kernel components. These components are captured for every input-output permutation. For the second degree kernels, the components are actually dependent on two inputs, either two direct inputs or two cross inputs. The complexity rises with each subsequent order. Each subsystem response is homogeneous; that is, for an input $\alpha u(t)$ where α

is an arbitrary parameter, each subsystem response is scaled by α^j. Each kernel now represents a specific multiple input-output part of the dynamic response.

2.3 Variational Expansion Approach for Input - Output Representation

As discussed in the preceding chapter, Rugh[20] showed an approach to derive the input-output representation kernels by the variational expansion method. The variational expansion method replaces the state derivative function terms by an infinite power series around an equilibrium point, much like a Taylor series. The general state equation for one input and multiple states is

$$\dot{x}(t) = f(x(t), u(t), t), \quad x(0) = x_0 \tag{2.11}$$

The nonlinear function with a representation of the state in terms of a perturbation from an equilibrium point can be described by

$$f(x, u, t) = f(x_e + \delta x, u_e + \delta u, t) \tag{2.12}$$

where x_e and u_e are the initial equilibrium state and input to be perturbed from, and δx and δu are the perturbations. The expansion of the nonlinear function terms of the perturbed state and input is represented by

$$f(\delta x, \delta u, t) = K_{00} + \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} K_{ij}\delta x^{(i)}\delta u^j \tag{2.13}$$

which, when expanded is

$$f(\delta x, \delta u, t) = K_{00} + K_{10}\delta x + K_{01}u + K_{20}\delta x \otimes \delta x + K_{11}\delta x u + K_{02}u^2 + K_{30}\delta x \otimes \delta x \otimes \delta x$$

$$+K_{21}\delta x \otimes \delta x \times u + K_{12}\delta x \times u \times u + K_{03}u^3 + \cdots \tag{2.14}$$

where $\otimes$ is the Kronecker product, K_{ij} is a coefficient matrix, and $\delta x^{(i)} = \delta x \otimes \cdots \otimes \delta x$ for i terms. The initial condition here is $\delta x(0) = 0$; therefore, the K_{00} term is zero. The perturbed state $\delta x(t)$ is then expanded by considering inputs of the form $\alpha\delta u(t)$, where α is an arbitrary parameter. The state $\delta x(t)$ is expanded in the parameter α as

$$\delta x(t) = \sum_{n=1}^{\infty} \alpha^n \delta x_n(t) = \alpha\delta x_1(t) + \alpha^2\delta x_2(t) + \alpha^3\delta x_3(t) + \cdots \tag{2.15}$$

By substituting into Equation (2.14), the system expansion is formed as

$$\begin{aligned}
&\alpha\delta\dot{x}_1(t) + \alpha^2\delta\dot{x}_2(t) + \alpha^3\delta\dot{x}_3(t) + \cdots = \\
&K_{10}(\alpha\delta x_1(t) + \alpha^2\delta x_2(t) + \alpha^3\delta x_3(t) + \cdots) + K_{01}\big(\alpha\delta u(t)\big) \\
&+K_{11}(\alpha\delta x_1(t) + \alpha^2\delta x_2(t) + \alpha^3\delta x_3(t) + \cdots)\big(\alpha\delta u(t)\big) \\
&+K_{20}(\alpha\delta x_1(t) + \alpha^2\delta x_2(t) + \alpha^3\delta x_3(t) + \cdots) \otimes (\alpha\delta x_1(t) + \alpha^2\delta x_2(t) + \alpha^3\delta x_3(t) + \cdots) \\
&+K_{30}(\alpha\delta x_1(t) + \alpha^2\delta x_2(t) + \alpha^3\delta x_3(t) + \cdots) \otimes (\alpha\delta x_1(t) + \alpha^2\delta x_2(t) + \alpha^3\delta x_3(t) + \cdots) \\
&\otimes (\alpha\delta x_1(t) + \alpha^2\delta x_2(t) + \alpha^3\delta x_3(t) + \cdots) + K_{21}(\alpha\delta x_1(t) + \alpha^2\delta x_2(t) + \alpha^3\delta x_3(t) + \cdots) \\
&\otimes (\alpha\delta x_1(t) + \alpha^2\delta x_2(t) + \alpha^3\delta x_3(t) + \cdots)\big(\alpha\delta u(t)\big) \\
&+K_{12}(\alpha\delta x_1(t) + \alpha^2\delta x_2(t) + \alpha^3\delta x_3(t) + \cdots)\big(\alpha\delta u(t)\big)\big(\alpha\delta u(t)\big) \\
&+K_{02}\big(\alpha\delta u(t)\big)\big(\alpha\delta u(t)\big) + K_{03}\big(\alpha\delta u(t)\big)\big(\alpha\delta u(t)\big)\big(\alpha\delta u(t)\big) + \cdots
\end{aligned} \tag{2.16}$$

The variational expansion is completed by equating like-ordered parameters of α, in which a set of differential equations can be formed

$$\delta \dot{x}_1 = K_{10}\delta x_1 + K_{01}\delta u$$

$$\delta \dot{x}_2 = K_{10}\delta x_2 + K_{20}\delta x_1^{(2)} + K_{11}\delta x_1 \delta u + K_{02}\delta u^2$$

$$\delta \dot{x}_3 = K_{10}\delta x_3 + K_{30}\delta x_1^{(3)} + 2K_{20}\delta x_1 \otimes \delta x_2 + K_{11}\delta x_2 \delta u + K_{21}\delta x_1^{(2)}\delta u + K_{12}\delta x_1 \delta u^2 + K_{03}\delta u^3$$

$$\vdots \tag{2.17}$$

with $\delta x_1(0) = 0$, $\delta x_2(0) = 0$ and $\delta x_3(0) = 0$. Note, to clean up the notation, from this point forward the perturbation notation δ is dropped here and in the following chapters. The perturbation is assumed in this context.

The equations are then solved starting from the simplest first expansion subsystem, x_1, and then substituted into the next subsystem, and so on. The first equation in Equation (2.17) has the linear solution

$$x_1(t) = \int_0^t h_1(t,\tau)u(\tau)\,d\tau \tag{2.18}$$

where h_1 is defined by the state transition matrix, $\Phi(t,\tau)$.

$$h_1(t,\tau) = \Phi(t,\tau)K_{01} \tag{2.19}$$

Note, $\Phi(t,\tau)$ will depend on matrix K_{10}, which is common to all the subsystems in Equation (2.17). The state transition matrix can be computed a number of ways. Most notably is through the Peano-Baker series, or using the matrix exponential, e^{At}. Proceeding to the second subsystem, the solution to x_1 is substituted into the second equation in Equation (2.17) to solve for x_2.

$$x_2(t) = \int_0^t \Phi(t,\tau)\left[K_{20}\int_0^\tau\int_0^\tau h_1(\tau,\tau_1)h_1(\tau,\tau_2)u(\tau_1)u(\tau_2)d\tau_1 d\tau_2\right.$$

$$\left.+K_{11}\int_0^\tau h_1(\tau,\tau_1)u(\tau_1)d\tau_1 u(\tau) + K_{02}u^2(\tau)\right]d\tau \quad (2.20)$$

While the terms higher than order two will not be shown, this solution for x_2 would then be substituted into the third order equation, and this process would continue on for the higher orders sought. The final response is now a summation of the components and the initial equilibrium state.

$$x(t) = x_e(t) + x_1(t) + x_2(t) + x_3(t) + \cdots \quad (2.21)$$

2.4 MIMO Variational Expansion Approach

The previous subsection described the variational expansion method for a single-axis system. The extension to a multiple input, multiple output system is not far out of reach. The general system description is extended to

$$\dot{x}(t) = f(x(t), u(t), t), \quad x(0) = x_0 \quad (2.22)$$

where $x(t)$ is again a vector of states, $u(t)$ is a vector of inputs, and $\dot{x}(t)$ is the corresponding time derivative vector of the states described by the function f. Note, again that the perturbation notation of δ is dropped to simplify the expressions. Shown another way, the system is now

$$\dot{x}_1 = f_1(x(t), u(t), t)$$

$$\dot{x}_2 = f_2(x(t), u(t), t)$$

$$\dot{x}_3 = f_3(x(t), u(t), t)$$

$$\vdots$$

$$\dot{x}_n = f_n(x(t), u(t), t) \quad (2.23)$$

where $\dot{x}_i(t)$ is the i[th] state derivative within vector $\dot{x}(t)$, not to be mistaken as the expanded state derivative $\dot{x}_i(t)$ in Equation (2.17). Each function can now be expanded separately via the power series as before.

$$f_1(x,u,t) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} K_{1_{ij}}\, x^{(i)} u^{(j)}$$

$$f_2(x,u,t) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} K_{2_{ij}}\, x^{(i)} u^{(j)}$$

$$\vdots$$

$$f_n(x,u,t) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} K_{n_{ij}}\, x^{(i)} u^{(j)}$$

(2.24)

The multi-axis variational expansion proceeds the same way as the single-axis case. All states $x_i(t)$ are expanded by considering inputs of the form $\alpha u(t)$, where α is an arbitrary parameter.

$$x_1(t) = \sum_{i=1}^{\infty} \alpha^i x_{1,i}(t) = \alpha x_{1,1}(t) + \alpha^2 x_{1,2}(t) + \alpha^3 x_{1,3}(t) + \cdots$$

$$x_2(t) = \sum_{i=1}^{\infty} \alpha^i x_{2,i}(t) = \alpha x_{2,1}(t) + \alpha^2 x_{2,2}(t) + \alpha^3 x_{2,3}(t) + \cdots$$

$$\vdots$$

$$x_n(t) = \sum_{i=1}^{\infty} \alpha^i x_{n,i}(t) = \alpha x_{n,1}(t) + \alpha^2 x_{n,2}(t) + \alpha^3 x_{n,3}(t) + \cdots \quad (2.25)$$

Now each expanded state is substituted into the system equations in Equation (2.24). Terms of like-ordered coefficients of α are grouped together. The single state variable case produced one differential equation for each subsystem in the expansion. In the multi-variable state case, each subsystem of differential equations contains n relations corresponding to the number of state equations. Also, each of these subsystem equation sets have an initial condition of $x_{1,i}(0) = 0$, $x_{2,i}(0) = 0, \cdots$. The Volterra kernels are computed for each set just as in the single state case. The final solution for each state is

$$
\begin{aligned}
x_1(t) &= x_{1_e}(t) + x_{1,1}(t) + x_{1,2}(t) + x_{1,3}(t) + \cdots \\
x_2(t) &= x_{2_e}(t) + x_{2,1}(t) + x_{2,2}(t) + x_{2,3}(t) + \cdots \\
&\vdots \\
x_n(t) &= x_{n_e}(t) + x_{n,1}(t) + x_{n,2}(t) + x_{n,3}(t) + \cdots
\end{aligned}
\tag{2.26}
$$

where $x_{1_e}, x_{2_e}, \cdots$ are the initial equilibrium conditions.

2.5 Volterra Kernel Types

A property of the variational expansion methodology is that the resulting kernels are not of a defined type. There are three types of Volterra kernels: symmetric, regular, and triangular. When computing the Volterra kernels, certain mathematical transformations may need to be taken in order for the kernels to be in a desired usable form.

The symmetric kernel is defined by

$$h_{sym}(\tau_1, \cdots, \tau_n) = h_{sym}\left(\tau_{\pi(1)}, \cdots, \tau_{\pi(n)}\right) \tag{2.27}$$

for the stationary case, and

$$h_{sym}(t, \tau_1, \cdots, \tau_n) = h_{sym}\left(t, \tau_{\pi(1)}, \cdots, \tau_{\pi(n)}\right) \tag{2.28}$$

for the nonstationary case. Here, $\pi(.)$ denotes any permutation of the integers 1 to n. The symmetric kernel is important for this research since it allows the dummy time variables $\tau_1, \tau_2 \cdots, \tau_n$ to be arbitrary. As will be shown, this allows an easy change of variables and change in the order of integration when the kernels are being set up for solution. As Rugh[20] has stated, a homogeneous system can be assumed to be symmetric without any loss of generality.

The next kernel type, the triangular kernel, is defined by $h(t, \tau_1, \cdots, \tau_n) = 0$ for $\tau_{i+j} > \tau_j$ for i and j integers. As will be seen, mathematical manipulation will sometimes produce a triangular kernel during kernel formulation. This trait is evident from step functions that appear in the kernel. Step functions $\Delta(\tau_i - \tau_j)$ can be inserted to emphasize triangularity where it exists.

$$h_{tri}(t, \tau_1, \tau_2, \cdots, \tau_n) = h_{tri}(t, \tau_1, \tau_2, \cdots, \tau_n)\Delta(\tau_1 - \tau_2) \cdots \Delta(\tau_{n-1} - \tau_n) \qquad (2.29)$$

The final type of kernel is the regular kernel. The regular kernel is similar to the triangular kernel and is defined by

$$h_{reg}(\tau_1, \tau_2, \cdots, \tau_n) = h_{tri}(\tau_1 + \cdots + \tau_n, \tau_2 \cdots + \tau_n, \tau_{n-1} + \tau_n) \qquad (2.30)$$

For these three types of kernel forms, each kernel form may be natural for a particular system structure. The kernel structure needed for this thesis research is the symmetric kernel due to its generality. The other kernel types will be encountered during the formulation of the kernels; however, any triangular or regular kernel can be symmetrized by the formula

$$h_{sym}(\tau_1, \cdots, \tau_n) = \frac{1}{n!} \sum_{\pi(.)} h(\tau_{\pi(1)}, \cdots, \tau_{\pi(n)}) \qquad (2.31)$$

2.6 Truncated Volterra Series

The work performed by Omran and Newman[2-11] outlined the benefits and drawbacks of truncating the Volterra series. They outline that the order of the system needed depends on two things, the strength of the nonlinearity of the system and the operating space of the model. If at a certain operating point a high order series is required, computing the convolution integrals of the Volterra series may quickly become infeasible. Omran and Newman devised a global approach that would require only a Volterra series extended to order two. This alone would only describe weak nonlinearities. However, Omran and Newman devised an approach to calculate the kernels at different operating points for different sub-regions of the system. A piecewise interpolation scheme is then used to move from one region to the next.

This method allows for an analysis of global nonlinearities, while also maintaining a low complexity in regards to the Volterra series formulation. For the multiple input, multiple output focus of this research, the Volterra series will naturally be more lengthy and complex than the single-axis solution from previous research. Therefore, reducing the formulation complexity as much as possible is both important and necessary. The focus of this research is to formulate the MIMO Volterra series for two inputs and two outputs to the second order. By itself, this will offer a description of weak nonlinearities. However, the methodology devised by Omran and Newman can be implemented to extend the second order MIMO Volterra formulation to describe global nonlinearities. While this use of the global approach is out of scope for this thesis, it is an area for future work.

CHAPTER 3

MIMO VOLTERRA ANALYTICAL MODEL

3.1 Overview

In this chapter, the Volterra model is derived for a generic, two degree of freedom system. This process relies on the variational expansion technique described in the preceding chapter. First, the solution to the single-axis first order system is described from previous research, followed by the single-axis second order system. Finally, the formulation of the new MIMO Volterra model is developed. Note that the single-axis systems and solutions are given in a rapid manner by skipping over steps of the variational expansion methodology. These steps are covered in more detail for the new MIMO Volterra model.

3.2 Single-Axis First Order System Analytical Volterra Kernels

The solution of the Volterra kernels to a single-axis, first order system has been computed and published a number of times. The first known derivation was by Baumann, Herdman, Stalford, and Garrett[23] in their technical report for the Air Force Research Laboratory in 1988. This solution was reinvestigated and expanded by Omran[11]. For the general equation

$$\dot{x} = f(x, u) \tag{3.1}$$

where $x \in \mathbb{R}^1$ is a single state variable, $u \in \mathbb{R}^1$ is a single input variable, and $f \in \mathbb{R}^1$ is the state derivative function. Taking the variational expansion approach described by Gilbert[30] and outlined in Rugh[20], the system is defined as

$$\dot{x} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} K_{ij}x^i u^j = K_{10}x + K_{01}u + K_{20}x^2 + K_{11}xu + K_{02}u^2 + \cdots, \qquad K_{00} = 0 \tag{3.2}$$

where, through variational expansion, the state is described by a sum of infinite terms

$$x = \alpha x_1 + \alpha^2 x_2 + \alpha^3 x_3 + \cdots \tag{3.3}$$

for which the homogeneous subsystem state equations are generated to second order as

$$\begin{aligned} \dot{x}_1 &= K_{10}x_1 + K_{01}u \\ \dot{x}_2 &= K_{10}x_2 + K_{20}x_1^2 + K_{11}x_1 u + K_{02}u^2 \end{aligned} \tag{3.4}$$

The solution to this system in time-invariant form is

$$x \approx \int_0^t h_1(t-\tau)u(\tau)\,d\tau + \int_0^t\int_0^t h_2(t-\tau_1, t-\tau_2)u(\tau_1)u(\tau_2)\,d\tau_1 d\tau_2 \tag{3.5}$$

where

$$h_1(t-\tau) = K_{01}e^{K_{10}(t-\tau)} \tag{3.6}$$

and

$$h_2(t-\tau_1, t-\tau_2) = h_2^{qs}(t-\tau_1, t-\tau_2) + h_2^{bsi}(t-\tau_1, t-\tau_2) + h_2^{qi}(t-\tau_1, t-\tau_2) \tag{3.7}$$

where the superscript notation qs, bsi, and qi indicate quadratic state, bilinear state input, and quadratic input components defined as

$$h_2^{qs}(t-\tau_1, t-\tau_2) = \frac{K_{20}K_{01}^2}{K_{10}} e^{K_{10}((t-\tau_1)+(t-\tau_2))}\left(1 - e^{-K_{10}\min(t-\tau_1, t-\tau_2)}\right) \tag{3.8}$$

$$h_2^{bsi}(t-\tau_1,t-\tau_2)=\frac{K_{11}K_{01}}{2}e^{K_{10}\max(t-\tau_1 t-\tau_2)} \tag{3.9}$$

$$h_2^{qi}(t-\tau_1,t-\tau_2)=K_{02}e^{K_{10}(t-\tau_1)}\delta\big(-(t-\tau_1)+(t-\tau_2)\big) \tag{3.10}$$

Equation (3.10) makes use of the impulse, Dirac Delta function $\delta(.)$.

3.3 Single-Axis Second Order System Analytical Volterra Kernels

The solution of the Volterra kernels to a single-axis, second order system was investigated by Omran and Newman[6]. The following discussion is the system setup and Volterra kernel solutions derived. These results will be useful as a comparison to the MIMO Volterra model.

The system is defined as

$$\ddot{x}=f(x,\dot{x},u) \tag{3.11}$$

where $x\in\mathbb{R}^1$ is a single state variable, $\dot{x}\in\mathbb{R}^1$ is the single state derivative variable, $u\in\mathbb{R}^1$ is a single input variable, and $f\in\mathbb{R}^1$ is the state second derivative function. Taking the variational expansion approach described by Gilbert[30] and outlined in Rugh[20], the system is defined as

$$\begin{aligned}\ddot{x}&=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}K_{ijk}x^i\dot{x}^ju^k\\ &=K_{100}x+K_{010}\dot{x}+K_{001}u+K_{200}x^2+K_{110}x\dot{x}\\ &\quad+K_{020}\dot{x}^2+K_{101}xu+K_{011}\dot{x}u+K_{002}u^2+\cdots,\qquad K_{00}=0\end{aligned} \tag{3.12}$$

where, if $\dot{x}$ is expressed as the rate of the state v, or $\dot{x}=v$, then the system can be written in a state space-like form as

$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ K_{100} & K_{010} \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ K_{001} \end{bmatrix} u + \begin{bmatrix} 0 \\ K_{200} \end{bmatrix} x^2 + \begin{bmatrix} 0 \\ K_{110} \end{bmatrix} xv$$
$$+ \begin{bmatrix} 0 \\ K_{020} \end{bmatrix} v^2 + \begin{bmatrix} 0 \\ K_{101} \end{bmatrix} xu + \begin{bmatrix} 0 \\ K_{011} \end{bmatrix} vu + \begin{bmatrix} 0 \\ K_{002} \end{bmatrix} u^2 \quad (3.13)$$

As a second order system in companion canonical form, the terms K_{100} can be expressed as $-\omega_n^2$, and K_{010} can be expressed as $-2\zeta\omega_n$ for purposes of natural frequency, ω_n, and damping ratio, ζ, discussions. Through variational expansion, the state and rate are described by a sum of infinite terms.

$$x = \alpha x_1 + \alpha^2 x_2 + \alpha^3 x_3 + \cdots$$
$$v = \alpha v_1 + \alpha^2 v_2 + \alpha^3 v_3 + \cdots \quad (3.14)$$

for which the homogeneous subsystem state equations are generated to second order as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \end{bmatrix} + \begin{bmatrix} 0 \\ K_{001} \end{bmatrix} u$$

$$\begin{bmatrix} \dot{x}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K_{200} \end{bmatrix} x_1^2 + \begin{bmatrix} 0 \\ K_{110} \end{bmatrix} x_1 v_1 + \begin{bmatrix} 0 \\ K_{020} \end{bmatrix} v_1^2$$
$$+ \begin{bmatrix} 0 \\ K_{101} \end{bmatrix} x_1 u + \begin{bmatrix} 0 \\ K_{011} \end{bmatrix} v_1 u + \begin{bmatrix} 0 \\ K_{002} \end{bmatrix} u^2 \quad (3.15)$$

The solution to this system for x in time-invariant form is

$$x \approx \int_0^t h_{1_x}(t-\tau)u(\tau)\,d\tau + \int_0^t \int_0^t h_{2_x}(t-\tau_1, t-\tau_2)u(\tau_1)u(\tau_2)\,d\tau_1 d\tau_2 \quad (3.16)$$

where

$$h_{1_x}(t-\tau) = \frac{K_{001}}{\omega_d} e^{-\sigma(t-\tau)} \sin\big(\omega_d(t-\tau)\big) \quad (3.17)$$

In Equation (3.17), ω_d is the damped natural frequency and σ is the damping factor. To calculate h_{2_x}, the first expansion kernel for v_1 is needed from the Volterra solution

$$v \approx \int_0^t h_{1_v}(t-\tau)u(\tau)\,d\tau + \int_0^t \int_0^t h_{2_v}(t-\tau_1, t-\tau_2)u(\tau_1)u(\tau_2)\,d\tau_1 d\tau_2 \qquad (3.18)$$

The second kernel for v, however, will not be needed to compute x to second order. Therefore, for v in Equation (3.18)

$$h_{1_v}(t-\tau) = \frac{-K_{001}}{\sqrt{1-\zeta^2}} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau) - \varphi) \qquad (3.19)$$

Going back to h_{2_x} from Equation (3.16) for x,

$$h_{2_x} = h_{2_x}^{qs} + h_{2_x}^{qr} + h_{2_x}^{bsr} + h_{2_x}^{bsi} + h_{2_x}^{bri} + h_{2_x}^{qi} \qquad (3.20)$$

where the superscript notation qs, qr, bsr, bsi, bri, and qi indicate quadratic state, quadratic rate, bilinear state rate, bilinear state input, bilinear rate input, and quadratic input components defined as

$$h_{2_x}^{qs}(t-\tau_1, t-\tau_2) =$$

$$\frac{A_2^3 K_{001}^2 K_{200}}{4} e^{-\sigma((t-\tau_1)+(t-\tau_2))} \Bigg\{ \frac{8\omega_d^2 \sigma}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)} \sin\Big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\Big)$$

$$-\frac{2\omega_d(\sigma^2-3\omega_d^2)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)} \cos\Big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\Big)$$

$$+\frac{2\omega_d}{\sigma^2+\omega_d^2} \cos\Big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)\Big)$$

$$+\frac{\sigma}{\sigma^2+\omega_d^2} e^{\sigma(t+\min(-\tau_1,-\tau_2))} \Big[\sin\Big(\omega_d\Big((t-\tau_1)-(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\Big)\Big)$$
$$+\sin\Big(\omega_d\Big(-(t-\tau_1)+(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\Big)\Big)$$
$$-\sin\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)\Big)\Big]$$

$$-\frac{\sigma}{\sigma^2+9\omega_d^2} e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d\Big(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\Big)\Big)$$

$$-\frac{\omega_d}{\sigma^2+\omega_d^2} e^{\sigma(t+\min(-\tau_1,-\tau_2))} \Big[\cos\Big(\omega_d\Big((t-\tau_1)-(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\Big)\Big)$$
$$+\cos\Big(\omega_d\Big(-(t-\tau_1)+(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\Big)\Big)$$
$$+\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)\Big)\Big]$$

$$+\frac{3\omega_d}{\sigma^2+9\omega_d^2} e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\Big(\omega_d\Big(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\Big)\Big)\Bigg\} \qquad (3.21)$$

$$h_{2_x}^{bsr}(t-\tau_1,t-\tau_2) =$$

$$\frac{A_2^2 A_4 K_{001}^2 K_{110}}{4} e^{-\sigma((t-\tau_1)+(t-\tau_2))} \left\{ \frac{\sigma}{\sigma^2+\omega_d^2} \sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big) \right.$$

$$-\frac{\sigma}{\sigma^2+9\omega_d^2} \sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{2\omega_d\left(-\sigma^2+3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)} \cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{2\omega_d}{\sigma^2+\omega_d^2} \cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_2\big)$$

$$\begin{aligned} +\frac{\sigma}{\sigma^2+\omega_d^2} e^{\sigma(t+\min(-\tau_1,-\tau_2))} \Big[&\sin\Big(-\omega_d\Big((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_2\Big) \\ &-\sin\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_2\Big) \\ &-\sin\Big(-\omega_d\Big((t-\tau_1)-(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_2\Big)\Big] \end{aligned}$$

$$+\frac{\sigma}{\sigma^2+9\omega_d^2} e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_2\Big)$$

$$\begin{aligned} -\frac{\omega_d}{\sigma^2+\omega_d^2} e^{\sigma(t+\min(-\tau_1,-\tau_2))} \Big[&\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_2\Big) \\ &+\cos\Big(-\omega_d\Big((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_2\Big) \\ &+\cos\Big(-\omega_d\Big((t-\tau_1)-(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_2\Big)\Big] \end{aligned}$$

$$\left. +\frac{3\omega_d}{\sigma^2+9\omega_d^2} e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_2\Big)\right\} \quad (3.22)$$

$$h_{2_x}^{qr}(t-\tau_1,t-\tau_2)=$$

$$\frac{A_2A_4^2K_{001}^2K_{020}}{4}e^{-\sigma((t-\tau_1)+(t-\tau_2))}\left\{\frac{8\omega_d^2\sigma}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)\right.$$

$$+\frac{2\omega_d}{\sigma^2+\omega_d^2}\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$-\frac{2\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{\sigma}{\sigma^2+\omega_d^2}e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\left[\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)\right.$$
$$+\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$
$$\left.-\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)\right]$$

$$+\frac{\sigma}{\sigma^2+9\omega_d^2}e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{\omega_d}{\sigma^2+\omega_d^2}e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\left[\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)\right.$$
$$+\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$
$$\left.+\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)\right]$$

$$\left.+\frac{3\omega_d}{\sigma^2+9\omega_d^2}e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)\right\}\quad(3.23)$$

$$h_{2_x}^{bsi}(t-\tau_1,t-\tau_2)=\frac{A_2^2K_{001}K_{101}}{2}e^{-\sigma\left(\max\left(-(t-\tau_1)+(t-\tau_2),(t-\tau_1)-(t-\tau_2)\right)+\min(t-\tau_1,t-\tau_2)\right)}$$

$$\times\sin\left(\omega_d\max\left(-(t-\tau_1)+(t-\tau_2),(t-\tau_1)-(t-\tau_2)\right)\right)\times\sin\left(\omega_d\min(t-\tau_1,t-\tau_2)\right)\quad(3.24)$$

$$h_{2_x}^{bri}(t-\tau_1,t-\tau_2)=\frac{A_2A_4K_{001}K_{011}}{2}e^{-\sigma\left(\max\left(-(t-\tau_1)+(t-\tau_2),(t-\tau_1)-(t-\tau_2)\right)+\min(t-\tau_1,t-\tau_2)\right)}$$

$$\times\sin\left(\omega_d\max\left(-(t-\tau_1)+(t-\tau_2),(t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)\times\sin\left(\omega_d\min(t-\tau_1,t-\tau_2)\right)\quad(3.25)$$

$$h_{2_x}^{qi}(t-\tau_1,t-\tau_2)=A_2K_{002}e^{-\sigma(t-\tau_1)}\sin\left(\omega_d(t-\tau_1)\right)\delta\left((t-\tau_2)-(t-\tau_1)\right)\quad(3.26)$$

where in the parameters A_2, A_4, σ, and ω_d are constants derived from the state transition matrix

$$\Phi(t-\tau)=\begin{bmatrix}A_1e^{-\sigma(t-\tau)}\sin(\omega_d(t-\tau)+\varphi_1) & A_2e^{-\sigma(t-\tau)}\sin\left(\omega_d(t-\tau)\right)\\ A_3e^{-\sigma(t-\tau)}\sin\left(\omega_d(t-\tau)\right) & A_4e^{-\sigma(t-\tau)}\sin(\omega_d(t-\tau)+\varphi_2)\end{bmatrix}$$

where

$${\omega_n}^2=-K_{100}$$

$$2\zeta\omega_n=-K_{010}$$

$$\sigma=\zeta\omega_n$$

$$\omega_d=\omega_n\sqrt{1-\zeta^2}$$

$$\varphi_1=\cos^{-1}\left(\frac{-(K_{010}+\sigma)}{\omega_d\sqrt{1+\left(\frac{K_{010}+\sigma}{\omega_d}\right)^2}}\right)$$

$$\varphi_2=\cos^{-1}\left(\frac{-\sigma}{\omega_d\sqrt{1+\left(\frac{\sigma}{\omega_d}\right)^2}}\right)$$

and where A_1, A_2, A_3, and A_4 are simplified constants defined as

$$A_1 = \frac{1}{\sqrt{1-\zeta^2}}$$

$$A_2 = \frac{1}{\omega_d}$$

$$A_3 = \frac{-{\omega_n}^2}{\omega_d}$$

$$A_4 = \frac{1}{\sqrt{1-\zeta^2}}$$

3.4 Two Degree Of Freedom Second Order System Analytical Volterra Kernels

The remainder of this chapter focuses on extending the Volterra model to two degrees of freedom, with two states and two inputs modeled. The general nonlinear system is

$$\dot{x} = f(x(t), u(t), t) \tag{3.27}$$

where $x \in \mathbb{R}^n$ denotes a vector of $n \times 1$ states, $u \in \mathbb{R}^n$ denotes a vector of $m \times 1$ inputs, t denotes time, and f denotes the state derivative vector function corresponding to $\dot{x}$. For the two degree of freedom case, the number of states in X will be limited to two, denoted as x_1 and x_2. Likewise, the number of inputs will be limited to two, denoted as u_1 and u_2. Therefore, the system representation is

$$\dot{x}_1 = f_1(x_1, x_2, u_1, u_2, t)$$
$$\dot{x}_2 = f_2(x_1, x_2, u_1, u_2, t) \tag{3.28}$$

A Taylor series expansion is then performed on $f_1(x_1, x_2, u_1, u_2, t)$ and $f_2(x_1, x_2, u_1, u_2, t)$ at initial points ${x_1}_0 = 0$, ${x_2}_0 = 0$, ${u_1}_0 = 0$ and ${u_2}_0 = 0$. This is represented the following way.

$$\dot{x}_1 = \sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\sum_{c=0}^{\infty}\sum_{d=0}^{\infty} K_{abcd} x_1^a x_2^b u_1^c u_2^d , \qquad K_{0000} = 0$$

$$\dot{x}_2 = \sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\sum_{c=0}^{\infty}\sum_{d=0}^{\infty} L_{abcd} x_1^a x_2^b u_1^c u_2^d , \qquad L_{0000} = 0 \tag{3.29}$$

For $\dot{x}_1$, K_{abcd} are the coefficients to the expanded terms, and a, b, c, d are integers from 0, 1, 2, 3, … and $K_{0000} = 0$ represents the equilibrium point. Likewise, for $\dot{x}_2$, L_{abcd} are the coefficients to the expanded terms, and a, b, c, d are integers from 0, 1, 2, 3, … and $L_{0000} = 0$ represents the equilibrium point.

Expanding the terms for both $\dot{x}_1$ and $\dot{x}_2$, the nonlinear terms are seen.

$$\begin{aligned}\dot{x}_1 = {} & K_{0001}u_2 + K_{0002}u_2^2 + K_{0003}u_2^3 + \cdots + K_{0010}u_1 + K_{0020}u_1^2 + K_{0030}u_1^3 + \cdots + K_{0011}u_1u_2 \\ & + K_{0012}u_1u_2^2 + K_{0021}u_1^2u_2 + \cdots + K_{0100}x_2 + K_{0200}x_2^2 + K_{0300}x_2^3 + \cdots + K_{1000}x_1 \\ & + K_{2000}x_1^2 + K_{3000}x_1^3 + \cdots + K_{1100}x_1x_2 + K_{1200}x_1x_2^2 + K_{2100}x_1^2x_2 + \cdots \\ & + K_{0101}x_2u_2 + K_{0102}x_2u_2^2 + K_{0201}x_2^2u_2 + \cdots + K_{0110}x_2u_1 + K_{0120}x_2u_1^2 \\ & + K_{0210}x_2^2u_1 + \cdots + K_{1001}x_1u_2 + K_{1002}x_1u_2^2 + K_{2001}x_1^2u_2 + \cdots + K_{1010}x_1u_1 \\ & + K_{1020}x_1u_1^2 + K_{2010}x_1^2u_1 + \cdots \end{aligned} \tag{3.30}$$

$$\begin{aligned}\dot{x}_2 = {} & L_{0001}u_2 + L_{0002}u_2^2 + L_{0003}u_2^3 + \cdots + L_{0010}u_1 + L_{0020}u_1^2 + L_{0030}u_1^3 + \cdots + L_{0011}u_1u_2 \\ & + L_{0012}u_1u_2^2 + L_{0021}u_1^2u_2 + \cdots + L_{0100}x_2 + L_{0200}x_2^2 + L_{0300}x_2^3 + \cdots + L_{1000}x_1 \\ & + L_{2000}x_1^2 + L_{3000}x_1^3 + \cdots + L_{1100}x_1x_2 + L_{1200}x_1x_2^2 + L_{2100}x_1^2x_2 + \cdots \\ & + L_{0101}x_2u_2 + L_{0102}x_2u_2^2 + L_{0201}x_2^2u_2 + \cdots + L_{0110}x_2u_1 + L_{0120}x_2u_1^2 \\ & + L_{0210}x_2^2u_1 + \cdots + L_{1001}x_1u_2 + L_{1002}x_1u_2^2 + L_{2001}x_1^2u_2 + \cdots + L_{1010}x_1u_1 \\ & + L_{1020}x_1u_1^2 + L_{2010}x_1^2u_1 + \cdots \end{aligned} \tag{3.31}$$

As described in Chapter 2, this model is expanded to the second order; therefore, terms of order three and higher are neglected. The reduced equations are thus

$$\dot{x}_1 = K_{0001}u_2 + K_{0002}u_2^2 + K_{0010}u_1 + K_{0011}u_1u_2 + K_{0020}u_1^2 + K_{0100}x_2 + K_{0101}x_2u_2 + K_{0110}x_2u_1 + K_{0200}x_2^2 + K_{1000}x_1 + K_{1001}x_1u_2 + K_{1010}x_1u_1 + K_{1100}x_1x_2 + K_{2000}x_1^2 \tag{3.32}$$

$$\dot{x}_2 = L_{0001}u_2 + L_{0002}u_2^2 + L_{0010}u_1 + L_{0011}u_1u_2 + L_{0020}u_1^2 + L_{0100}x_2 + L_{0101}x_2u_2 + L_{0110}x_2u_1 + L_{0200}x_2^2 + L_{1000}x_1 + L_{1001}x_1u_2 + L_{1010}x_1u_1 + L_{1100}x_1x_2 + L_{2000}x_1^2 \tag{3.33}$$

From this set of terms, the system's state space representation can now be formed.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}K_{1000} & K_{0100}\\ L_{1000} & L_{0100}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}K_{0010} & K_{0001}\\ L_{0010} & L_{0001}\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} + \begin{bmatrix}K_{2000} & K_{0200}\\ L_{2000} & L_{0200}\end{bmatrix}\begin{bmatrix}x_1^2\\ x_2^2\end{bmatrix} + \begin{bmatrix}K_{1100} & K_{0011}\\ L_{1100} & L_{0011}\end{bmatrix}\begin{bmatrix}x_1x_2\\ u_1u_2\end{bmatrix} + \begin{bmatrix}K_{1010} & K_{0110}\\ L_{1010} & L_{0110}\end{bmatrix}\begin{bmatrix}x_1u_1\\ x_2u_1\end{bmatrix} + \begin{bmatrix}K_{1001} & K_{0101}\\ L_{1001} & L_{0101}\end{bmatrix}\begin{bmatrix}x_1u_2\\ x_2u_2\end{bmatrix} + \begin{bmatrix}K_{0020} & K_{0002}\\ L_{0020} & L_{0002}\end{bmatrix}\begin{bmatrix}u_1^2\\ u_2^2\end{bmatrix} \tag{3.34}$$

The state and input matrices are organized to facilitate solving for the Volterra kernels. Note, they have been grouped based on the order of the terms and like-terms. Terminology-wise, x_1 and x_2 are known as the linear state terms, while u_1 and u_2 are known as the linear input terms. There are also the quadratic state terms, x_1^2 and x_2^2, and the quadratic input terms, u_1^2 and u_2^2. A combination of state and input terms are known as bilinear terms. That is, the term x_1x_2 is the bilinear state term, while u_1u_2 is the bilinear input term. The rest of the bilinear terms are comprised of the bilinear state-input 1 terms, that is x_1u_1 and x_2u_1, and the bilinear state-input 2 terms, that is x_1u_2 and x_2u_2.

With the system set up, the variational expansion method is now used to generate the Volterra kernels. As a side note on notation, due to the addition of a second state and input from the single-axis system, additional notation is needed to distinguish between state number and order that is referenced. The following notation is used from this point on to reference the state number and order number of the term of the variational expansion.

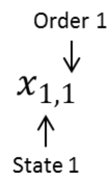

Figure 3.1 State Notation

As a sum of infinite terms, state 1 and state 2 are expressed as

$$x_1 = \alpha x_{1,1} + \alpha^2 x_{1,2} + \alpha^3 x_{1,3} + \cdots$$

$$x_2 = \alpha x_{2,1} + \alpha^2 x_{2,2} + \alpha^3 x_{2,3} + \cdots \tag{3.35}$$

The coefficients α, α^2, α^3, etc. are used as arbitrary constants to group like-ordered terms. The input term analogue to this is

$$u_1 = \alpha u_1$$

$$u_1^2 = \alpha^2 u_1^2$$

$$\vdots \tag{3.36}$$

and

$$u_2 = \alpha u_2$$

$$u_2^2 = \alpha^2 u_2^2$$

$$\vdots \tag{3.37}$$

After equating sets of terms with similar coefficients α^i, the variational expansion produces a set of homogeneous subsystem state equations that can be ordered as

$$\begin{bmatrix} \dot{x}_{1,1} \\ \dot{x}_{2,1} \end{bmatrix} = \begin{bmatrix} K_{1000} & K_{0100} \\ L_{1000} & L_{0100} \end{bmatrix} \begin{bmatrix} x_{1,1} \\ x_{2,1} \end{bmatrix} + \begin{bmatrix} K_{0010} & K_{0001} \\ L_{0010} & L_{0001} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_{1,2} \\ \dot{x}_{2,2} \end{bmatrix} = \begin{bmatrix} K_{1000} & K_{0100} \\ L_{1000} & L_{0100} \end{bmatrix} \begin{bmatrix} x_{1,2} \\ x_{2,2} \end{bmatrix} + \begin{bmatrix} K_{2000} & K_{0200} \\ L_{2000} & L_{0200} \end{bmatrix} \begin{bmatrix} x_{1,1}^2 \\ x_{2,1}^2 \end{bmatrix} + \begin{bmatrix} K_{1100} & K_{0011} \\ L_{1100} & L_{0011} \end{bmatrix} \begin{bmatrix} x_{1,1}x_{2,1} \\ u_1 u_2 \end{bmatrix}$$

$$+ \begin{bmatrix} K_{1010} & K_{0110} \\ L_{1010} & L_{0110} \end{bmatrix} \begin{bmatrix} x_{1,1}u_1 \\ x_{2,1}u_1 \end{bmatrix} + \begin{bmatrix} K_{1001} & K_{0101} \\ L_{1001} & L_{0101} \end{bmatrix} \begin{bmatrix} x_{1,1}u_2 \\ x_{2,1}u_2 \end{bmatrix} + \begin{bmatrix} K_{0020} & K_{0002} \\ L_{0020} & L_{0002} \end{bmatrix} \begin{bmatrix} u_1^2 \\ u_2^2 \end{bmatrix}$$

$$\vdots \tag{3.39}$$

Note, the variational expansion can be increased to expansion orders higher than 2; however, since the Volterra model is setup to the second order only, additional expansions are not needed. From the above expanded state equations, the following nomenclature is defined for state and input coefficient matrices.

$$A = \begin{bmatrix} K_{1000} & K_{0100} \\ L_{1000} & L_{0100} \end{bmatrix}$$

$$B = \begin{bmatrix} K_{0010} & K_{0001} \\ L_{0010} & L_{0001} \end{bmatrix}$$

$$B_{qs} = \begin{bmatrix} K_{2000} & K_{0200} \\ L_{2000} & L_{0200} \end{bmatrix}$$

$$B_{bsi1} = \begin{bmatrix} K_{1010} & K_{0110} \\ L_{1010} & L_{0110} \end{bmatrix}$$

$$B_{bsi2} = \begin{bmatrix} K_{1001} & K_{0101} \\ L_{1001} & L_{0101} \end{bmatrix}$$

$$B_{bsbi} = \begin{bmatrix} K_{1100} & K_{0011} \\ L_{1100} & L_{0011} \end{bmatrix}$$

$$B_{qi} = \begin{bmatrix} K_{0020} & K_{0002} \\ L_{0020} & L_{0002} \end{bmatrix} \tag{3.40}$$

where A and B are the linear state space coefficient matrices, B_{qs} indicates the B matrix for the quadratic states, B_{bsi1} indicates the B matrix for the bilinear states with input 1, B_{bsi2} indicates the B matrix for the bilinear states with input 2, B_{bsbi} indicates the B matrix for the bilinear states and bilinear inputs terms, and B_{qi} indicates the B matrix for the quadratic inputs.

In order to solve this set of subsystem state equations, the first order system is solved first. This is simply the linear state response solution to a forced input, a convolution integral, as shown below.

$$\begin{bmatrix} x_{1,1} \\ x_{2,1} \end{bmatrix} = \int_0^t C\Phi(t-\tau)B \begin{bmatrix} u_1(\tau) \\ u_2(\tau) \end{bmatrix} d\tau \tag{3.41}$$

In this instance and all those following, the C matrix is simply an identity matrix

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3.42}$$

since all that is sought for analysis is the state variable responses as the output. The next element in the convolution integral is the state transition matrix (STM), commonly denoted as Φ, or the exponential matrix e^{At}, which has multiple methods for derivation. The state transition matrix describes how the state $x(t)$ at time t evolves into, or from, the state $x(\tau)$ at some other time τ. Note that the computation of the state transition matrix equals the exponential matrix because the system is time-invariant, which is still the case for Volterra theory.

When calculating the state transition matrix, if the resolvent, $(sI-A)^{-1}$, is investigated in the complex frequency domain, frequency response terms such as the damping ratio, ζ, and natural frequency, ω_n, can be applied to the system. This will ultimately reduce the complexity of the analytical equations and give the equations some intuitive sense. Engineers can more effectively communicate analysis using the model. Note, a significant assumption is that the dynamics this model will describe are systems with two complex conjugate poles, or oscillatory motion. Therefore, the model will not describe exponential-like motion or a system with poles on the real axis. This assumption is justified, as the flight dynamics under analysis are oscillatory convergent in nature. Therefore, computation of Φ for the system results in

$$\Phi(t-\tau) = \begin{bmatrix} A_1 e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau)+\varphi_1) & A_2 e^{-\sigma(t-\tau)} \sin\left(\omega_d(t-\tau)\right) \\ A_3 e^{-\sigma(t-\tau)} \sin\left(\omega_d(t-\tau)\right) & A_4 e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau)+\varphi_2) \end{bmatrix} \tag{3.43}$$

where

$$\omega_n{}^2 = (L_{0100}K_{1000} - L_{1000}K_{0100})$$

$$2\zeta\omega_n = -(K_{1000} + L_{0100})$$

$$\sigma = \zeta\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\varphi_1 = \cos^{-1}\left(\frac{-(L_{0100}+\sigma)}{\omega_d\sqrt{1+\left(\frac{L_{0100}+\sigma}{\omega_d}\right)^2}}\right)$$

$$\varphi_2 = \cos^{-1}\left(\frac{-(K_{1000}+\sigma)}{\omega_d\sqrt{1+\left(\frac{K_{1000}+\sigma}{\omega_d}\right)^2}}\right)$$

and where A_1, A_2, A_3, and A_4 are simplified constants defined as

$$A_1 = \sqrt{\frac{L_{0100}^2 + 2\sigma L_{0100} + {\omega_n}^2}{{\omega_n}^2 - \sigma^2}}$$

$$A_2 = \frac{K_{0100}}{\omega_d}$$

$$A_3 = \frac{L_{1000}}{\omega_d}$$

$$A_4 = \sqrt{\frac{K_{1000}^2 + 2\sigma K_{1000} + {\omega_n}^2}{{\omega_n}^2 - \sigma^2}}$$

Equation (3.41) can now be computed as

$$x_{1,1} = \int_0^t \left(A_1 K_{0010} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau) + \varphi_1) + A_2 L_{0010} e^{-\sigma(t-\tau)} \sin\big(\omega_d(t-\tau)\big)\right) u_1(\tau) d\tau + \int_0^t \left(A_1 K_{0001} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau) + \varphi_1) + A_2 L_{0001} e^{-\sigma(t-\tau)} \sin\big(\omega_d(t-\tau)\big)\right) u_2(\tau) d\tau \tag{3.44}$$

$$x_{2,1} = \int_0^t \left(A_3 K_{0010} e^{-\sigma(t-\tau)} \sin\big(\omega_d(t-\tau)\big) + A_4 L_{0010} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau) + \varphi_2)\right) u_1(\tau) d\tau + \int_0^t \left(A_3 K_{0001} e^{-\sigma(t-\tau)} \sin\big(\omega_d(t-\tau)\big) + A_4 L_{0001} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau) + \varphi_2)\right) u_2(\tau) d\tau \tag{3.45}$$

A digression on notation, if the above equations are to be described by Volterra kernels, namely h_1, then more notation is needed to differentiate the specific kernel to the state output. The following notation is used for the first order kernels to reference both the state and input association to Equations (3.44) and (3.45) above.

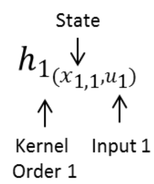

Figure 3.2 1st Order Kernel Notation

Therefore, from Equations (2.9) and (2.10) in Chapter 2, the first degree response is defined as

$$x_{1,1} = \int_0^t h_{1_{(x_{1,1},u_1)}}(t-\tau)u_1(\tau)\,d\tau + \int_0^t h_{1_{(x_{1,1},u_2)}}(t-\tau)u_2(\tau)\,d\tau \tag{3.46}$$

and

$$x_{2,1} = \int_0^t h_{1_{(x_{2,1},u_1)}}(t-\tau)u_1(\tau)\,d\tau + \int_0^t h_{1_{(x_{2,1},u_2)}}(t-\tau)u_2(\tau)\,d\tau \tag{3.47}$$

The first order kernels were computed to be

$$\begin{aligned} h_{1_{(x_{1,1},u_1)}}(t-\tau) &= A_1 K_{0010} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau)+\varphi_1) \\ &+ A_2 L_{0010} e^{-\sigma(t-\tau)} \sin\big(\omega_d(t-\tau)\big) \end{aligned} \tag{3.48}$$

$$\begin{aligned} h_{1_{(x_{1,1},u_2)}}(t-\tau) &= A_1 K_{0001} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau)+\varphi_1) \\ &+ A_2 L_{0001} e^{-\sigma(t-\tau)} \sin\big(\omega_d(t-\tau)\big) \end{aligned} \tag{3.49}$$

$$\begin{aligned} h_{1_{(x_{2,1},u_1)}}(t-\tau) &= A_3 K_{0010} e^{-\sigma(t-\tau)} \sin\big(\omega_d(t-\tau)\big) \\ &+ A_4 L_{0010} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau)+\varphi_2) \end{aligned} \tag{3.50}$$

$$h_{1_{(x_{2,1},u_2)}}(t-\tau) = A_3 K_{0001} e^{-\sigma(t-\tau)} \sin\big(\omega_d(t-\tau)\big)$$

$$+A_4 L_{0001} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau)+\varphi_2) \tag{3.51}$$

For the second expansion subsystem, the kernels for $x_{1,2}$ and $x_{2,2}$are much more complex to derive. As seen in Equation (3.39), the quadratic and bilinear terms total ten components. Realizing this additional complexity, additional notation is needed to distinguish the different components. The following table shows the nomenclature for each state and input combination.

Table 3.1 2nd Order State Component Nomenclature

Component	**Description**
$x_{1,2}^{qs1}$	State 1 response of degree 2 - Quadratic State 1 component
$x_{1,2}^{bs1s2}$	State 1 response of degree 2 - Bilinear State 1 State 2 component
$x_{1,2}^{qs2}$	State 1 response of degree 2 - Quadratic State 2 component
$x_{1,2}^{bs1i1}$	State 1 response of degree 2 - Bilinear State 1 Input 1 component
$x_{1,2}^{bs2i1}$	State 1 response of degree 2 - Bilinear State 2 Input 1 component
$x_{1,2}^{bs1i2}$	State 1 response of degree 2 - Bilinear State 1 Input 2 component
$x_{1,2}^{bs2i2}$	State 1 response of degree 2 - Bilinear State 2 Input 2 component
$x_{1,2}^{qi1}$	State 1 response of degree 2 - Quadratic Input 1 component
$x_{1,2}^{bi1i2}$	State 1 response of degree 2 - Bilinear Input 1 Input 2 component
$x_{1,2}^{qi2}$	State 1 response of degree 2 - Quadratic Input 2 component
$x_{2,2}^{qs1}$	State 2 response of degree 2 - Quadratic State 1 component
$x_{2,2}^{bs1s2}$	State 2 response of degree 2 - Bilinear State 1 State 2 component
$x_{2,2}^{qs2}$	State 2 response of degree 2 - Quadratic State 2 component
$x_{2,2}^{bs1i1}$	State 2 response of degree 2 - Bilinear State 1 Input 1 component
$x_{2,2}^{bs2i1}$	State 2 response of degree 2 - Bilinear State 2 Input 1 component
$x_{2,2}^{bs1i2}$	State 2 response of degree 2 - Bilinear State 1 Input 2 component
$x_{2,2}^{bs2i2}$	State 2 response of degree 2 - Bilinear State 2 Input 2 component
$x_{2,2}^{qi1}$	State 2 response of degree 2 - Quadratic Input 1 component
$x_{2,2}^{bi1i2}$	State 2 response of degree 2 - Bilinear Input 1 Input 2 component
$x_{2,2}^{qi2}$	State 2 response of degree 2 - Quadratic Input 2 component

Also, expanding upon the kernel notation for the first order kernels, the second order kernel notation is

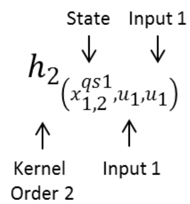

Figure 3.3 2nd Order Kernel Notation

The overall solution of $x_{1,2}$ and $x_{2,2}$ is

$$x_{1,2} = x_{1,2}^{qs1} + x_{1,2}^{bs1s2} + x_{1,2}^{qs2} + x_{1,2}^{bs1i1} + x_{1,2}^{bs2i1} + x_{1,2}^{bs1i2} + x_{1,2}^{bs2i2} + x_{1,2}^{qi1} + x_{1,2}^{bi1i2} + x_{1,2}^{qi2} \quad (3.52)$$

and

$$x_{2,2} = x_{2,2}^{qs1} + x_{2,2}^{bs1s2} + x_{2,2}^{qs2} + x_{2,2}^{bs1i1} + x_{2,2}^{bs2i1} + x_{2,2}^{bs1i2} + x_{2,2}^{bs2i2} + x_{2,2}^{qi1} + x_{2,2}^{bi1i2} + x_{2,2}^{qi2} \quad (3.53)$$

Each component is solved separately, and each component follows the MIMO Volterra form and structure seen in Equations (2.9) and (2.10) from Chapter 2. For example, the first component is $x_{1,2}^{qs1}$ and has the structure

$$\begin{aligned}
x_{1,2}^{qs1} = &\int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs1},u_1,u_1\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_1(\tau_2)\, d\tau_1 d\tau_2 \\
&+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs1},u_1,u_2\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 \\
&+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs1},u_2,u_1\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_1(\tau_2)\, d\tau_1 d\tau_2 \\
&+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs1},u_2,u_2\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 \qquad (3.54)
\end{aligned}$$

Each kernel needs to be calculated, and this work continues for each of the remaining components for $x_{1,2}$ and $x_{2,2}$. From Equations (3.52) through (3.54), the sheer number of kernels to calculate is daunting. Software tools were used whenever possible for consistency and to double and triple check work as progress was made. While computationally time consuming, the steps are not much different from the derivations for the single-axis second order system. There are now just more terms and components needed to capture the coupling between states and inputs.

Due to the amount of work needed for deriving the kernels, covering every kernel and integral in this thesis would be impractical. Therefore, the following sections include the steps necessary to set up each of the main kernels to integrate to a solution. One example for each of the kernel types will be given.

3.4.1 Volterra Kernels of the Quadratic State Type

Focusing on the first component listed for $x_{1,2}$, that is $x_{1,2}^{qs1}$, the subsystem is set up as

$$\begin{bmatrix} x_{1,2}^{qs} \\ x_{2,2}^{qs} \end{bmatrix} = \int_0^t C\Phi(t-\tau)B_{qs} \begin{bmatrix} x_{1,1}^2 \\ x_{2,1}^2 \end{bmatrix} d\tau \tag{3.55}$$

where the linear state solutions of $x_{1,1}$ and $x_{2,1}$, Equations (3.46) and (3.47), are substituted into this equation.

There are a few things to note here. First, note that this is a linear differential equation being fed into an equation to describe nonlinearities, which is the essential feature of the variational expansion method. Also note that the state transition matrix is also utilized. Finally, notice that the $x_{1,1}$ component is fed in as $x_{1,1}^2$. Since $x_{1,1}$ is the linear state response with an input, there are actually two inputs included in the quadratic state components.

The subsystem equation above is setup with both quadratic state components, $x_{1,2}^{qs}$ and $x_{2,2}^{qs}$, and the corresponding B_{qs} matrix. Combining like-kernels together allows mathematical manipulations to be performed simultaneously, saving computation time. However, due to space constraints, only the terms involved with the $h_{2_{\left(x_{1,2}^{qs1},u_1,u_1\right)}}$ kernel for the $x_{1,2}^{qs1}$component response will be shown in the example. This is accomplished by keeping only those terms with K_{2000} and L_{2000} gains (for quadratic state 1) and terms with only $u_1(\tau) \times u_1(\tau)$. Considering this, the above equation for $x_{1,2}^{qs}$ has the $x_{1,2}^{qs1}$ terms

$$
\begin{aligned}
x_{1,2}^{qs} = \int_0^t &\left(A_1 K_{2000} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau)+\varphi_1) + A_2 L_{2000} e^{-\sigma(t-\tau)} \sin\big(\omega_d(t-\tau)\big)\right) \\
&\times \int_0^\tau \Big(\big(A_1 K_{0010} e^{-\sigma(\tau-\tau_1)} \sin(\omega_d(\tau-\tau_1)+\varphi_1) \\
&+ A_2 L_{0010} e^{-\sigma(\tau-\tau_1)} \sin\big(\omega_d(\tau-\tau_1)\big)\big) u_1(\tau_1)\Big)\, d\tau_1 \\
&\times \int_0^\tau \Big(\big(A_1 K_{0010} e^{-\sigma(\tau-\tau_2)} \sin(\omega_d(\tau-\tau_2)+\varphi_1) \\
&+ A_2 L_{0010} e^{-\sigma(\tau-\tau_2)} \sin\big(\omega_d(\tau-\tau_2)\big)\big) u_1(\tau_2)\Big)\, d\tau_2\, d\tau \\
&+ \cdots \qquad (3.56)
\end{aligned}
$$

A mathematical manipulation can be performed to bring the two upper limits, τ, to t. This involves inserting step functions, $\Delta(\tau-\tau_1)$ and $\Delta(\tau-\tau_2)$ into the integral term for each. Since the limits of all three integrals in the equation match, from 0 to t, the order of integration can now be rearranged as so

$$x_{1,2}^{qs} = \int_0^t \int_0^t \int_0^t \left(A_1 K_{2000} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau)+\varphi_1) + A_2 L_{2000} e^{-\sigma(t-\tau)} \sin\big(\omega_d(t-\tau)\big)\right)$$

$$\times \Big(\big(A_1 K_{0010} e^{-\sigma(\tau-\tau_1)} \sin(\omega_d(\tau-\tau_1)+\varphi_1)$$

$$+ A_2 L_{0010} e^{-\sigma(\tau-\tau_1)} \sin\big(\omega_d(\tau-\tau_1)\big)\big) u_1(\tau_1)\Big) \times \Delta(\tau-\tau_1)$$

$$\times \Big(\big(A_1 K_{0010} e^{-\sigma(\tau-\tau_2)} \sin(\omega_d(\tau-\tau_2)+\varphi_1)$$

$$+ A_2 L_{0010} e^{-\sigma(\tau-\tau_2)} \sin\big(\omega_d(\tau-\tau_2)\big)\big) u_1(\tau_2)\Big) \times \Delta(\tau-\tau_2)\, d\tau\, d\tau_1\, d\tau_2$$

$$+ \cdots \tag{3.57}$$

Now that the order of integration has changed, a second mathematical manipulation can be performed to remove the step functions. The multiplication of the two step functions $\Delta(\tau-\tau_1) \times \Delta(\tau-\tau_2)$ is equivalent to setting the lower limit on the inside integral to $-\min(-\tau_1, -\tau_2)$. After this manipulation, the equation becomes

$$x_{1,2}^{qs} = \int_0^t \int_0^t \int_{-\min(-\tau_1,-\tau_2)}^t \big(A_1 K_{2000} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau)+\varphi_1)$$

$$+ A_2 L_{2000} e^{-\sigma(t-\tau)} \sin\big(\omega_d(t-\tau)\big)\big)$$

$$\times \big(A_1 K_{0010} e^{-\sigma(\tau-\tau_1)} \sin(\omega_d(\tau-\tau_1)+\varphi_1)$$

$$+ A_2 L_{0010} e^{-\sigma(\tau-\tau_1)} \sin\big(\omega_d(\tau-\tau_1)\big)\big)$$

$$\times \big(A_1 K_{0010} e^{-\sigma(\tau-\tau_2)} \sin(\omega_d(\tau-\tau_2)+\varphi_1)$$

$$+ A_2 L_{0010} e^{-\sigma(\tau-\tau_2)} \sin\big(\omega_d(\tau-\tau_2)\big)\big) u_1(\tau_1) u_1(\tau_2)\, d\tau\, d\tau_1\, d\tau_2$$

$$+ \cdots \tag{3.58}$$

To calculate the $h_{2_{\left(x_{1,2}^{qs1}, u_1, u_1\right)}}$ kernels, the inside integral can now be integrated from $-\min(-\tau_1, -\tau_2)$ to t, with respect to τ. Integration software was utilized to perform the operation.

After integration, additional mathematical rearrangements were performed to simplify and organize the kernel. The next three kernels for $x_{1,2}^{qs1}$ can be solved using the same methodology for $h_{2_{\left(x_{1,2}^{qs1},u_1,u_2\right)}}$, $h_{2_{\left(x_{1,2}^{qs1},u_2,u_1\right)}}$, and $h_{2_{\left(x_{1,2}^{qs1},u_2,u_2\right)}}$. Due to the extensive length of these solutions, these kernels are provided in Appendix A.

Due to the similarity between the $x_{1,2}^{qs1}$, $x_{2,2}^{qs1}$, $x_{1,2}^{qs2}$, $x_{2,2}^{qs2}$, $x_{1,2}^{bs1s2}$, and $x_{2,2}^{bs1s2}$ components, the steps explained above can also be used to setup and calculate the Volterra kernels for each of these components. They are all similar due to the substitution of two state solutions into the second degree expansion. The calculated kernels for all of these components are provided in Appendix A.

3.4.2 Volterra Kernels of the Bilinear State-Input Type

The next kernel type would be an example of a bilinear state and input case. This includes the following components: $x_{1,2}^{bs1i1}$, $x_{1,2}^{bs2i1}$, $x_{1,2}^{bs1i2}$, and $x_{1,2}^{bs2i2}$, along with the corresponding $x_{2,2}$ components. Each of these second degree subsystems can be setup via the following equations.

$$\begin{bmatrix} x_{1,2}^{bsi1} \\ x_{2,2}^{bsi1} \end{bmatrix} = \int_0^t C\Phi(t-\tau)B_{bsi1}\begin{bmatrix} x_{1,1}u_1 \\ x_{2,1}u_1 \end{bmatrix} d\tau \tag{3.59}$$

and

$$\begin{bmatrix} x_{1,2}^{bsi2} \\ x_{2,2}^{bsi2} \end{bmatrix} = \int_0^t C\Phi(t-\tau)B_{bsi2}\begin{bmatrix} x_{1,1}u_2 \\ x_{2,1}u_2 \end{bmatrix} d\tau \tag{3.60}$$

Here again, the linear state responses in Equations (3.46) and (3.47) are fed back into the second degree subsystem as seen in Equation (3.39). In these equations, there will be one input contribution from the substitution of the linear state response term and another input contribution due to the bilinear input term for the nonlinearity. Once again, there are two inputs. The focus for the remainder of this example type will be on $x_{1,2}^{bsi1}$ from the above. Due to space constraints, only the terms involved with the $h_{2_{\left(x_{1,2}^{bs1i1},u_1,u_1\right)}}$ kernel for the $x_{1,2}^{bs1i1}$component response will be carried forward in the example. This is accomplished by keeping only those terms with K_{1010} and L_{1010} gains (for quadratic state 1) and terms with only $u_1(\tau) \times u_1(\tau)$.When this component is expanded, the equation becomes

$$x_{1,2}^{bsi1} = \int_0^t \left(A_1 K_{1010} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau)+\varphi_1) + A_2 L_{1010} e^{-\sigma(t-\tau)} \sin\big(\omega_d(t-\tau)\big)\right)$$
$$\times \int_0^\tau \Big(\big(A_1 K_{0010} e^{-\sigma(\tau-\tau_1)} \sin(\omega_d(\tau-\tau_1)+\varphi_1)$$
$$+ A_2 L_{0010} e^{-\sigma(\tau-\tau_1)} \sin\big(\omega_d(\tau-\tau_1)\big)\big) u_1(\tau_1)\Big)\, d\tau_1 \times u_1(\tau)\, d\tau$$
$$+ \cdots \tag{3.61}$$

Here, τ can arbitrarily be changed to τ_1, and τ_1 can be changed to τ_2, in order to label the dummy integration variables to be in line with the second order Volterra form with $d\tau_1\, d\tau_2$.

$$x_{1,2}^{bsi1} = \int_0^t \left(A_1 K_{1010} e^{-\sigma(t-\tau_1)} \sin(\omega_d(t-\tau_1)+\varphi_1) + A_2 L_{1010} e^{-\sigma(t-\tau_1)} \sin\big(\omega_d(t-\tau_1)\big)\right)$$
$$\times \int_0^\tau \Big(\big(A_1 K_{0010} e^{-\sigma(\tau_1-\tau_2)} \sin(\omega_d(\tau_1-\tau_2)+\varphi_1)$$
$$+ A_2 L_{0010} e^{-\sigma(\tau_1-\tau_2)} \sin\big(\omega_d(\tau_1-\tau_2)\big)\big) u_1(\tau_2)\Big)\, d\tau_2 \times u_1(\tau_1)\, d\tau_1$$
$$+ \cdots \tag{3.62}$$

In order to set the upper limit of the inner integral to t, a step function $\Delta(\tau_1-\tau_2)$ is inserted. The order of integration can now be arbitrarily changed so that the equation is now

$$x_{1,2}^{bsi1} = \int_0^t \int_0^t \left(A_1 K_{1010} e^{-\sigma(t-\tau_1)} \sin(\omega_d(t-\tau_1)+\varphi_1) + A_2 L_{1010} e^{-\sigma(t-\tau_1)} \sin\big(\omega_d(t-\tau_1)\big)\right)$$

$$\times \Big(\big(A_1 K_{0010} e^{-\sigma(\tau_1-\tau_2)} \sin(\omega_d(\tau_1-\tau_2)+\varphi_1)$$

$$+ A_2 L_{0010} e^{-\sigma(\tau_1-\tau_2)} \sin\big(\omega_d(\tau_1-\tau_2)\big)\big) u_1(\tau_2)\Big) \Delta(\tau_1-\tau_2) \times u_1(\tau_1)\, d\tau_1\, d\tau_2$$

$$+ \cdots \tag{3.63}$$

Due to the step function inserted, the kernel is in a triangular form, with the kernel defined for only half the domain. As Rugh[20] states, the kernel is in triangular form if it satisfies the property that $h(t, \tau_1, \cdots, \tau_n) = 0$ when $\tau_{i+j} > \tau_j$ for i, j positive integers. The step function inserted renders the kernel zero when $\tau_2 > \tau_1$. Therefore, the equation must now be symmetrized. Rugh[20] outlined a method to symmetrize a triangular kernel by setting

$$h_{sym}(t_1, \cdots, t_n) = \frac{1}{n!} \sum_{\pi(.)} h_{tri}(t_{\pi(1)}, \cdots, t_{\pi(n)}) \tag{3.64}$$

where $\pi(.)$ denotes any permutation of the integers 1,…,n. Performing the symmetrization results in the equation

$$x_{1,2}^{bsi1} = \int_0^t \int_0^t \frac{1}{2}\Bigg(\bigg(A_1 K_{1010} e^{-\sigma(t-\tau_1)} \sin(\omega_d(t-\tau_1)+\varphi_1)$$

$$+ A_2 L_{1010} e^{-\sigma(t-\tau_1)} \sin\big(\omega_d(t-\tau_1)\big)$$

$$\times \Big(\big(A_1 K_{0010} e^{-\sigma(\tau_1-\tau_2)} \sin(\omega_d(\tau_1-\tau_2)+\varphi_1)$$

$$+ A_2 L_{0010} e^{-\sigma(\tau_1-\tau_2)} \sin\big(\omega_d(\tau_1-\tau_2)\big)\big)u_1(\tau_2)\Big)\bigg)\Delta(\tau_1-\tau_2)u_1(\tau_1)$$

$$+ \bigg(A_1 K_{1010} e^{-\sigma(t-\tau_2)} \sin(\omega_d(t-\tau_2)+\varphi_1) + A_2 L_{1010} e^{-\sigma(t-\tau_2)} \sin\big(\omega_d(t-\tau_2)\big)$$

$$\times \Big(\big(A_1 K_{0010} e^{-\sigma(\tau_2-\tau_1)} \sin(\omega_d(\tau_2-\tau_1)+\varphi_1)$$

$$+ A_2 L_{0010} e^{-\sigma(\tau_2-\tau_1)} \sin\big(\omega_d(\tau_2-\tau_1)\big)\big)u_1(\tau_1)\Big)\bigg)\Delta(\tau_2-\tau_1)u_1(\tau_2)\Bigg)\, d\tau_1\, d\tau_2$$

$$+ \cdots \tag{3.65}$$

Due to the two step functions, $\Delta(\tau_1-\tau_2)$ and $\Delta(\tau_2-\tau_1)$, an equivalent equation to combine and simplify terms is

$$x_{1,2}^{bsi1} = \int_0^t \int_0^t \frac{1}{2}\Big(\big(A_1 K_{1010} e^{-\sigma \min(t-\tau_1, t-\tau_2)} \sin(\omega_d \min(t-\tau_1, t-\tau_2)+\varphi_1)$$

$$+ A_2 L_{1010} e^{-\sigma \min(t-\tau_1, t-\tau_2)} \sin\big(\omega_d \min(t-\tau_1, t-\tau_2)\big)\big)$$

$$\times \big(A_1 K_{0010} e^{-\sigma \max(\tau_1-\tau_2, \tau_2-\tau_1)} \sin(\omega_d \max(\tau_1-\tau_2, \tau_2-\tau_1)+\varphi_1)$$

$$+ A_2 L_{0010} e^{-\sigma \max(\tau_1-\tau_2, \tau_2-\tau_1)} \sin\big(\omega_d \max(\tau_1-\tau_2, \tau_2$$

$$- \tau_1)\big)\big)\Big)\, u_1(\tau_1) u_1(\tau_2)\, d\tau_1\, d\tau_2$$

$$+ \cdots \tag{3.66}$$

where the $\max(\tau_1-\tau_2, \tau_2-\tau_1)$ operation is the maximum value between $\tau_1-\tau_2$ and $\tau_2-\tau_1$. The kernel is now complete and the $h_{2_{\left(x_{1,2}^{bs1i1}, u_1, u_1\right)}}$ component is defined as

$$h_{2_{\left(x_{1,2}^{bs1i1},u_1,u_1\right)}} =$$

$$\frac{1}{2}\Big(\big(A_1 K_{1010} e^{-\sigma \min(t-\tau_1, t-\tau_2)} \sin(\omega_d \min(t-\tau_1, t-\tau_2) + \varphi_1)$$

$$+A_2 L_{1010} e^{-\sigma \min(t-\tau_1, t-\tau_2)} \sin\big(\omega_d \min(t-\tau_1, t-\tau_2)\big)\big)$$

$$\times \big(A_1 K_{0010} e^{-\sigma \max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times \sin\big(\omega_d \max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big) + \varphi_1\big)$$

$$+A_2 L_{0010} e^{-\sigma \max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times \sin\Big(\omega_d \max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\big)\Big)\Big) \tag{3.67}$$

The steps explained above can also be used to setup and calculate the Volterra kernels for each of the components for $x_{1,2}^{bs1i1}$, $x_{1,2}^{bs2i1}$, $x_{1,2}^{bs1i2}$, and $x_{1,2}^{bs2i2}$, along with the corresponding $x_{2,2}$ components. They are all similar due to the substitution of a state solution into the second degree expansion along with an individual input. The calculated kernels for all of these components are provided in Appendix A.

3.4.3 Volterra Kernels of the Quadratic Input Type

The next kernel type would be an example of a quadratic input case. This includes the components $x_{1,2}^{qi1}$ and $x_{1,2}^{qi2}$, along with the corresponding $x_{2,2}$ components. It also includes the bilinear input 1-input 2 components, $x_{1,2}^{bi1i2}$ and $x_{2,2}^{bi1i2}$, since two inputs are also involved. Each of these second degree subsystems can be setup via the following equation.

$$\begin{bmatrix} x_{1,2}^{qi} \\ x_{2,2}^{qi} \end{bmatrix} = \int_0^t C\Phi(t-\tau) B_{qi} \begin{bmatrix} u_1^2 \\ u_2^2 \end{bmatrix} d\tau \tag{3.68}$$

For this kernel case, there is no substitution of any of the first degree linear responses. Therefore, the equations simply expand to

$$x_{1,2}^{qi} = \int_0^t \left(A_1 K_{0020} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau)+\varphi_1) + A_2 L_{0020} e^{-\sigma(t-\tau)} \sin\left(\omega_d(t-\tau)\right)\right)$$
$$\times u_1{}^2(\tau)\, d\tau$$
$$+ \cdots \tag{3.69}$$

Again, as an example, only the $x_{1,2}^{qi1}$ component response of the equation is shown. Specifically, the $h_{2_{\left(x_{1,2}^{qi1},u_1,u_1\right)}}$ kernel is focused on. This is accomplished by keeping only those terms with K_{0020} and L_{0020} gains (for quadratic input 1) and terms with only $u_1(\tau) \times u_1(\tau)$.

In order to generate the second degree kernel form, a Dirac Delta, or impulse, function is inserted into the equation. For reference, the Dirac Delta function is defined as $\delta(x - \mathrm{y})$, where the function equals zero everywhere where $x \neq \mathrm{y}$. When $x = \mathrm{y}$, the function can be thought of having an infinite value, although it is undefined. A property of the Dirac Delta function is that at the point where $x = \mathrm{y}$, any integral over an interval containing that point is equal to one, shown as $\int_{-\infty}^{\infty} \delta(x-y)dx = 1$. It is this property that is inserted into the equation to generate the Volterra form, which can be performed since it is a multiplication of one. The dummy variables are then changed to expand the equation as

$$x_{1,2}^{qi} = \int_0^t \int_0^t \left(A_1 K_{0020} e^{-\sigma(t-\tau_1)} \sin(\omega_d(t-\tau_1)+\varphi_1) + A_2 L_{0020} e^{-\sigma(t-\tau_1)} \sin\left(\omega_d(t-\tau_1)\right)\right)$$
$$\times \delta(\tau_1 - \tau_2) u_1(\tau_1) u_1(\tau_2)\, d\tau_1\, d\tau_2$$
$$+ \cdots \tag{3.70}$$

The $h_{2_{\left(x_{1,2}^{qi1},u_1,u_1\right)}}$ kernel is therefore

$$h_{2_{\left(x_{1,2}^{qi1},u_1,u_1\right)}} = \left(A_1 K_{0020} e^{-\sigma(t-\tau)} \sin(\omega_d(t-\tau)+\varphi_1) + A_2 L_{0020} e^{-\sigma(t-\tau)} \sin\left(\omega_d(t-\tau)\right)\right)$$
$$\times\, \delta\left(-(t-\tau_1)+(t-\tau_2)\right) \tag{3.71}$$

3.4.4 Remaining Volterra Kernel Types

The last remaining subsystem setup is the bilinear state 1-state 2 component and the bilinear input 1-input 2 component. These remaining two components are grouped together as such

$$\begin{bmatrix} x_{1,2}^{bsbi} \\ x_{2,2}^{bsbi} \end{bmatrix} = \int_0^t C\Phi(t-\tau) B_{bsbi} \begin{bmatrix} x_{1,1}x_{2,1} \\ u_1 u_2 \end{bmatrix} d\tau \tag{3.72}$$

As stated before, the kernels for the bilinear state 1-state 2 component are constructed similarly to the quadratic state kernels in the quadratic state example. The kernels for the bilinear input 1-input 2 component are constructed similarly to the quadratic input kernels in the previous example. This concludes the examples needed to produce the Volterra kernels for all the component responses. Solutions to all the kernels for the two degree of freedom, MIMO Volterra system is provided in Appendix A.

3.5 MIMO Volterra Kernel Surface Plots

Surface plots of the kernels were generated to promote investigation into the overall shape of each of the kernels. These plots are shown as a generic example that can be generated using the MIMO Volterra model. The system was set up to have a linear component damping ratio, ζ, of 0.1, with a natural frequency, ω_n, of 2. Nonlinear gains were chosen based on exaggerating nonlinear effects so that the amplitude of the nonlinear response is pronounced enough for investigation. The chosen parameters K_{abcd} and L_{abcd} for the example responses are provided in Table 3.2. Note, the units of the parameters K_{abcd} and L_{abcd} are dependent on the units of the two states and two inputs. For example, if state 1 has units of ft/s and input 1 has units of radians, then the K_{1010} parameter would have units of s/ft.

Due to the number of kernels, the plots are provided in Appendix B. The surface plots are left un-normalized. This is due to the nature of the system. Many of the kernels would look exactly alike if the plots were normalized since the only difference between some of the kernels is the amplitude of the oscillations. From the surface plots, overall intuition of the kernel functions can be built. The quadratic state components show the typical peaks and valleys appearing in all the kernels, and they are symmetric along the $\tau_1 = \tau_2$ diagonal.

There are a few things to note from the surface plots. First, all the cross kernels, those are kernels that have $u_1 u_2$, or $u_2 u_1$, have similar shapes between their respective $x_{1,2}$ and $x_{2,2}$ components. This makes intuitive sense in that the coupling will depend on the amplitude of the gains when real systems are modeled. For a generic model, it is expected that the kernels are similar until the couplings are more defined.

Table 3.2 Example Volterra Parameter Values

Gain Parameter	Value	Corresponding Matrix
K_{1000}	1.81	A
K_{0100}	2	A
L_{1000}	-4	A
L_{0100}	-2.21	A
K_{0010}	0.5	B
K_{0001}	1	B
L_{0010}	1.5	B
L_{0001}	2	B
K_{2000}	2	B_{qs}
L_{2000}	2.5	B_{qs}
K_{0200}	2	B_{qs}
L_{0200}	2.5	B_{qs}
K_{1100}	2	B_{bsbi}
L_{1100}	2.5	B_{bsbi}
K_{0011}	2	B_{bsbi}
L_{0011}	2.5	B_{bsbi}
K_{1010}	2	B_{bsi1}
L_{1010}	2.5	B_{bsi1}
K_{0110}	2	B_{bsi1}
L_{0110}	2.5	B_{bsi1}
K_{1001}	2	B_{bsi2}
L_{1001}	2.5	B_{bsi2}
K_{0101}	2	B_{bsi2}
L_{0101}	2.5	B_{bsi2}
K_{0020}	2	B_{qi2}
L_{0020}	2.5	B_{qi2}
K_{0002}	2	B_{qi2}
L_{0002}	2.5	B_{qi2}

* Units for K_{abcd} and L_{abcd} are dependent on the units of state 1, state 2, input 1, and input 2

Also note the kernels for the bilinear state 1-state 2 equations. These have asymmetric kernel shapes along the $\tau_1 = \tau_2$ diagonal, unlike the quadratic state kernels. This hints at the possible phase differences between the two states.

The quadratic input surface plots, show a value of zero for all points except $\tau_1 = \tau_2$. This shows the impulse function at work. When integrated, it is known that the quadratic input will be closely represented by the linear component shape, only varying in amplitude based on the nonlinear quadratic input gains.

3.6 Two Degree of Freedom, Second Order Volterra Step Response

The general Volterra kernels described above can be applied for a variety of inputs defined for $u_1(\tau)$ and $u_2(\tau)$. In order to solve for the desired input response, the input functions are first substituted into the equations. Then the kernels for h_1 are integrated once from 0 to t with respect to τ, while the kernels for h_2 are integrated twice with respect to τ_1 and τ_2. The component responses are then combined to arrive at the overall state response. That is,

$$x_1 = x_{1,1} + x_{1,2} \tag{3.73}$$

$$x_2 = x_{2,1} + x_{2,2} \tag{3.74}$$

where

$$x_{1,1} = \int_0^t h_{1_{(x_{1,1},u_1)}}(t-\tau)u_1(\tau)\,d\tau + \int_0^t h_{1_{(x_{1,1},u_2)}}(t-\tau)u_2(\tau)\,d\tau \tag{3.75}$$

$$x_{2,1} = \int_0^t h_{1_{(x_{2,1},u_1)}}(t-\tau)u_1(\tau)\,d\tau + \int_0^t h_{1_{(x_{2,1},u_2)}}(t-\tau)u_2(\tau)\,d\tau \tag{3.76}$$

and where

$$x_{1,2} = x_{1,2}^{qs1} + x_{1,2}^{bs1s2} + x_{1,2}^{qs2} + x_{1,2}^{bs1i1} + x_{1,2}^{bs2i1} + x_{1,2}^{bs1i2} + x_{1,2}^{bs2i2} + x_{1,2}^{qi1} + x_{1,2}^{bi1i2} + x_{1,2}^{qi2} \tag{3.77}$$

$$x_{2,2} = x_{2,2}^{qs1} + x_{2,2}^{bs1s2} + x_{2,2}^{qs2} + x_{2,2}^{bs1i1} + x_{2,2}^{bs2i1} + x_{2,2}^{bs1i2} + x_{2,2}^{bs2i2} + x_{2,2}^{qi1} + x_{2,2}^{bi1i2} + x_{2,2}^{qi2} \tag{3.78}$$

The individual component responses are defined as

$$x_{1,2}^{qs1} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs1},u_1,u_1\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2$$
$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs1},u_1,u_2\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_2(\tau_2)\, d\tau_1 d\tau_2$$
$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs1},u_2,u_1\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2$$
$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs1},u_2,u_2\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_2(\tau_2)\, d\tau_1 d\tau_2$$

(3.79)

$$x_{1,2}^{bs1s2} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1s2},u_1,u_1\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2$$
$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1s2},u_1,u_2\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_2(\tau_2)\, d\tau_1 d\tau_2$$
$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1s2},u_2,u_1\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2$$
$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1s2},u_2,u_2\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_2(\tau_2)\, d\tau_1 d\tau_2$$

(3.80)

$$x_{1,2}^{qs2} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs2},u_1,u_1\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs2},u_1,u_2\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_2(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs2},u_2,u_1\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs2},u_2,u_2\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_2(\tau_2)\, d\tau_1 d\tau_2$$

(3.81)

$$x_{1,2}^{bs1i1} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1i1},u_1,u_1\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1i1},u_2,u_1\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2$$

(3.82)

$$x_{1,2}^{bs2i1} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs2i1},u_1,u_1\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs2i1},u_2,u_1\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2$$

(3.83)

$$x_{1,2}^{bs1i2} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1i2},u_1,u_2\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_2(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1i2},u_2,u_2\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_2(\tau_2)\, d\tau_1 d\tau_2$$

(3.84)

$$x_{1,2}^{bs2i2} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs2i2},u_1,u_2\right)}}(t-\tau_1,t-\tau_2)u_1(\tau_1)u_2(\tau_2)\,d\tau_1 d\tau_2$$
$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs2i2},u_2,u_2\right)}}(t-\tau_1,t-\tau_2)u_2(\tau_1)u_2(\tau_2)\,d\tau_1 d\tau_2$$

(3.85)

$$x_{1,2}^{qi1} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qi1},u_1,u_1\right)}}(t-\tau_1,t-\tau_2)u_1(\tau_1)u_1(\tau_2)\,d\tau_1 d\tau_2 \qquad (3.86)$$

$$x_{1,2}^{bi1i2} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bi1i2},u_1,u_2\right)}}(t-\tau_1,t-\tau_2)u_1(\tau_1)u_2(\tau_2)\,d\tau_1 d\tau_2 \qquad (3.87)$$

$$x_{1,2}^{qi2} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qi2},u_2,u_2\right)}}(t-\tau_1,t-\tau_2)u_2(\tau_1)u_2(\tau_2)\,d\tau_1 d\tau_2 \qquad (3.88)$$

with similar terms for $x_{2,2}$. Each of the kernels requiring integration are listed in Appendix A.

Due to the size of the kernels and the number of integrations performed, especially for the quadratic state and bilinear state 1-state 2 components, integration software was utilized to perform the operations. After integration, additional mathematical rearrangements were performed to simplify and organize the responses.

Note, the second degree components are computed one level lower and then compiled. That is, a response can be computed at each individual component kernel level, such as the response attributed to the $h_{2_{\left(x_{1,2}^{qs1},u_1,u_1\right)}}$ kernel only, or notation-wise, a $x_{1,2}^{qs1,u_1,u_1}$ component response. This will give one additional level of fidelity and allow for the analysis of each component's contribution to the total state response. This is similar to how various commercial linear numerical methods work, such as step(sys) in Matlab. While many compute only linear responses, they compute output components for each input-output pair. For a two-input and two-output system, commercial software yields four responses: input 1 to output 1, input 1 to output 2, input 2 to output 1, and input 2 to output 2. This is synonymous to the linear subcomponents for this MIMO Volterra model. The MIMO Volterra model now adds additional input-output components for the second degree, nonlinear terms.

For this thesis research, the analytical step response was computed by setting $u_1(\tau)$ and $u_2(\tau)$ as constants u_1 and u_2 and integrating. The step response solution to the first order variational expansion with respect to each input is

$$x_{1,1}(t) = x_{1,1}^{u_1}(t) \times u_1 + x_{1,1}^{u_2}(t) \times u_2 \tag{3.89}$$

$$\begin{aligned} x_{1,1}^{u_1}(t) = \frac{1}{\sigma^2 + {\omega_d}^2} e^{-\sigma t}\big(&A_2 L_{0010}\omega_d e^{\sigma t} + A_1 K_{0010}\omega_d e^{\sigma t}\cos(\varphi_1) - A_2 L_{0010}\omega_d \cos(t\omega_d) \\ &- A_1 K_{0010}\omega_d \cos(\varphi_1 + t\omega_d) + A_1 K_{0010}\sigma e^{\sigma t}\sin(\varphi_1) - A_2 L_{0010}\sigma\sin(t\omega_d) \\ &- A_1 K_{0010}\sigma\sin(\varphi_1 + t\omega_d)\big) \end{aligned} \tag{3.90}$$

$$\begin{aligned} x_{1,1}^{u_2}(t) = \frac{1}{\sigma^2 + {\omega_d}^2} e^{-\sigma t}(&A_2 L_{0001}\omega_d e^{\sigma t} + A_1 K_{0001}\omega_d e^{\sigma t}\cos(\varphi_1) - A_2 L_{0001}\omega_d \cos(t\omega_d) \\ &- A_1 K_{0001}\omega_d \cos(\varphi_1 + t\omega_d) + A_1 K_{0001}\sigma e^{\sigma t}\sin(\varphi_1) - A_2 L_{0001}\sigma\sin(t\omega_d) \\ &- A_1 K_{0001}\sigma\sin(\varphi_1 + t\omega_d)) \end{aligned} \tag{3.91}$$

and

$$x_{2,1}(t) = x_{2,1}^{u_1}(t) \times u_1 + x_{2,1}^{u_2}(t) \times u_2 \tag{3.92}$$

$$x_{2,1}^{u_1}(t) = \frac{1}{\sigma^2 + {\omega_d}^2} e^{-\sigma t}\big(A_3 K_{0010}\omega_d e^{\sigma t} + A_4 L_{0010}\omega_d e^{\sigma t}\cos(\varphi_2) - A_3 K_{0010}\omega_d \cos(t\omega_d)$$
$$- A_4 L_{0010}\omega_d \cos(\varphi_2 + t\omega_d) + A_4 L_{0010}\sigma e^{\sigma t}\sin(\varphi_2) - A_3 K_{0010}\sigma \sin(t\omega_d)$$
$$- A_4 L_{0010}\sigma \sin(\varphi_2 + t\omega_d)\big) \tag{3.93}$$

$$x_{2,1}^{u_2}(t) = \frac{1}{\sigma^2 + {\omega_d}^2} e^{-\sigma t}(A_3 K_{0001}\omega_d e^{\sigma t} + A_4 L_{0001}\omega_d e^{\sigma t}\cos(\varphi_2) - A_3 K_{0001}\omega_d \cos(t\omega_d)$$
$$- A_4 L_{0001}\omega_d \cos(\varphi_2 + t\omega_d) + A_4 L_{0001}\sigma e^{\sigma t}\sin(\varphi_2) - A_3 K_{0001}\sigma \sin(t\omega_d)$$
$$- A_4 L_{0001}\sigma \sin(\varphi_2 + t\omega_d)) \tag{3.94}$$

For the second order variational expansion, the analytical solutions are provided in Appendix C. Figures 3.4 through 3.9 show component plots of the responses from the analytical equations. The following component response plots use a generic step value of 1 for both inputs.

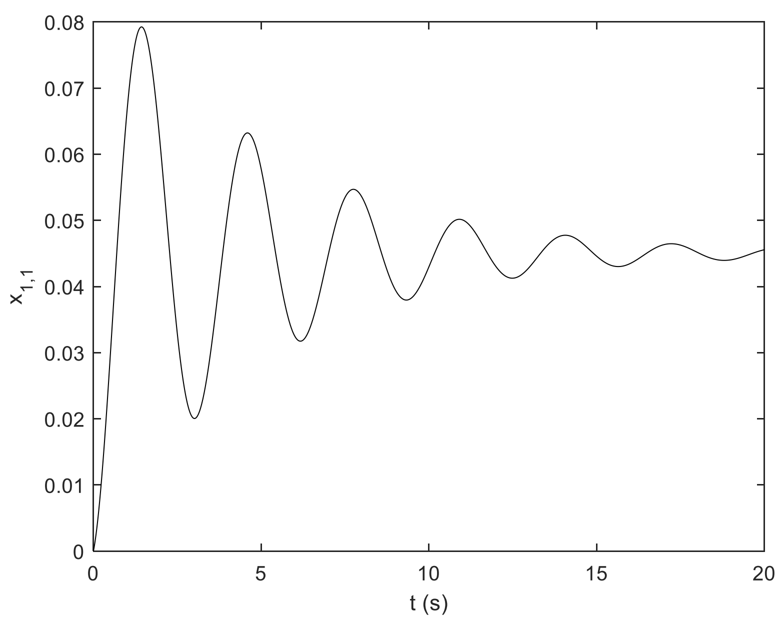

Figure 3.4 State Output 1 Linear Step Response

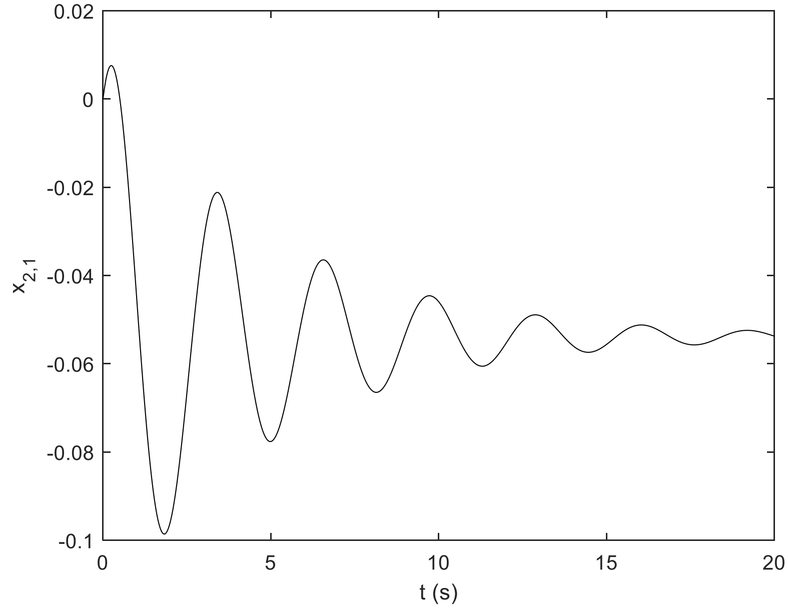

Figure 3.5 State Output 2 Linear Step Response

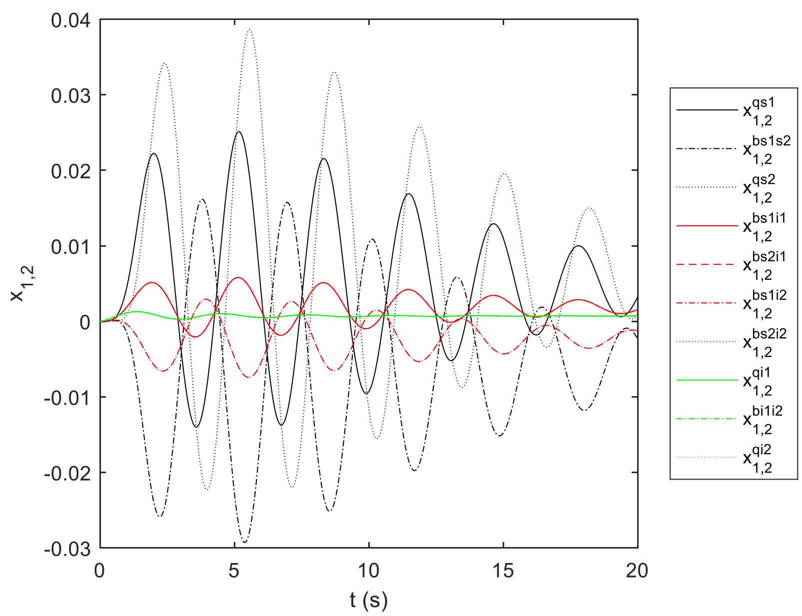

Figure 3.6 State Output 1 Nonlinear Components Step Response

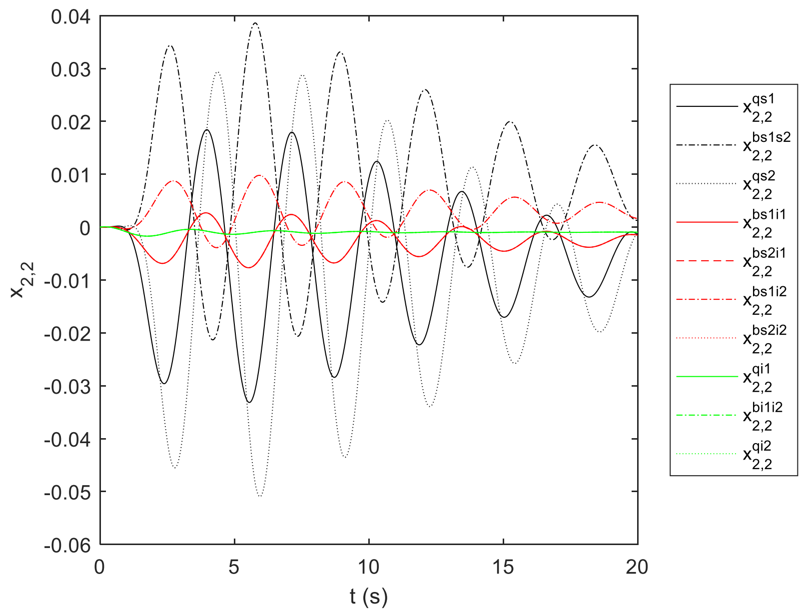

Figure 3.7 State Output 2 Nonlinear Components Step Response

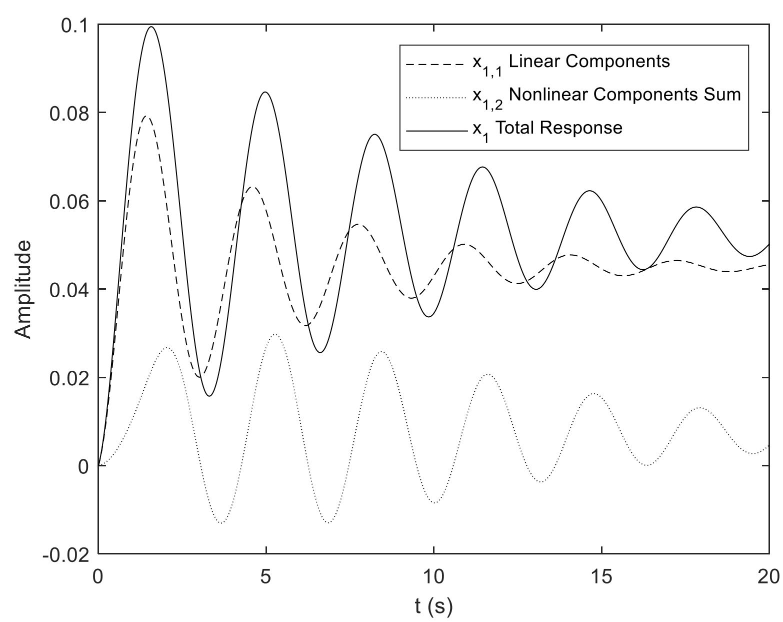

Figure 3.8 State Output 1 Linear, Nonlinear, and Total MIMO Volterra Model Step Response

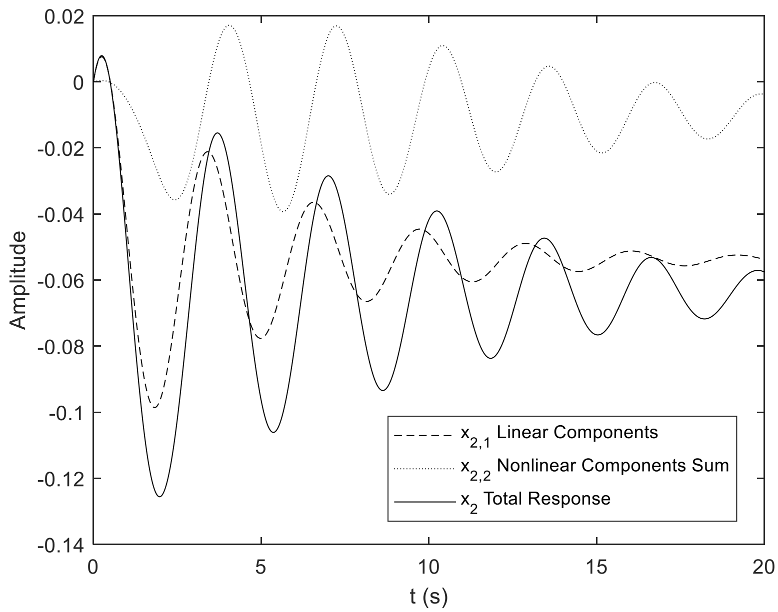

Figure 3.9 State Output 2 Linear, Nonlinear, and Total MIMO Volterra Model Step Response

From the responses, the system in the example has dominant linear components with high amplitude, and the nonlinear components are noticeably weaker. However, it is apparent from the total response graphs that the nonlinearities can deviate the total response away from a linear model. It is also seen that the MIMO Volterra model allows specific components of the nonlinear response to be analyzed. The nonlinearities can be further analyzed at the subcomponent level; that is, at each nonlinearity with respect to each input. For example, the quadratic state 1 component can be studied at the input 1 – input 1 term, $x_{1,2}^{qs1,u_1,u_1}$, input 1 – input 2 term, $x_{1,2}^{qs1,u_1,u_2}$, input 2 – input 1 term, $x_{1,2}^{qs1,u_2,u_1}$, and input 2 – input 2 term, $x_{1,2}^{qs1,u_2,u_2}$. These subcomponents are solved naturally from the kernel terms in Equations (3.79) through (3.88). This allows the individual cause and effect response caused by each nonlinear term to be separated and observed.

CHAPTER 4

AIRCRAFT DYNAMICS OF STUDY

4.1 Overview

This chapter derives the aircraft equations of motion that will be used for study. This involves the initial assumptions and setup of the full, six degrees of freedom, equations of motion. This chapter then proceeds with a description of the experimental data used, that of a F-16 fighter aircraft, and all the control and aerodynamic limits. Next, the derivation of the two degree of freedom, reduced order, longitudinal model is reviewed. This reduced order longitudinal model describes the short period natural mode. After the longitudinal dynamics of study are defined, latitudinal dynamics are explored with derivation of a two degree of freedom, reduced order model, for the dutch roll natural mode. These dynamics are conducive to studying both multivariable coupling and nonlinear characteristics for the MIMO Volterra model.

4.2 Aircraft Equations of Motion

The general six degrees of freedom translational and rotational aircraft equations result from Newton's laws of motion. There are twelve first order, nonlinear, coupled, ordinary differential equations which represent the navigation, force, kinematic, and moment dynamics. Assumptions made during their formulation include a flat earth, constant gravity, constant mass, symmetry about the X-Z planes, rigid body, and constant spinning internal turbofan engine. The equations below are described in the aircraft body frame.

Navigation Equations

$$\dot{x} = u\cos\theta\cos\psi + v(\sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi) + w(\sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi)$$

$$\dot{y} = u\cos\theta\sin\psi + v(\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi) + w(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)$$

$$\dot{z} = u\sin\theta - v\sin\phi\cos\theta - w\cos\phi\cos\theta \qquad (4.1)$$

Kinematic Equations

$$\dot{\phi} = p + q\frac{\sin\theta\sin\phi}{\cos\theta} + r\frac{\sin\theta\cos\phi}{\cos\theta}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi$$

$$\dot{\psi} = q\frac{\sin\phi}{\cos\theta} + r\frac{\cos\phi}{\cos\theta} \qquad (4.2)$$

Force Equations

$$\dot{u} = rv - qw - g\sin\theta + \frac{\bar{q}\bar{S}}{m}C_{x,t} + \frac{T}{m}$$

$$\dot{v} = pw - ru + g\sin\phi\cos\theta + \frac{\bar{q}\bar{S}}{m}C_{y,t}$$

$$\dot{w} = qu - pv + g\cos\phi\cos\theta + \frac{\bar{q}\bar{S}}{m}C_{z,t} \qquad (4.3)$$

Moment Equations

$$\dot{p} = {I_{xx}}^{-1}\dot{H}_x^B - {I_{xy}}^{-1}\dot{H}_y^B - {I_{zx}}^{-1}\dot{H}_z^B$$

$$\dot{q} = -{I_{xy}}^{-1}\dot{H}_x^B + {I_{yy}}^{-1}\dot{H}_y^B - {I_{yz}}^{-1}\dot{H}_z^B - H_e r$$

$$\dot{r} = -{I_{zx}}^{-1}\dot{H}_x^B - {I_{yz}}^{-1}\dot{H}_y^B + {I_{zz}}^{-1}\dot{H}_z^B + H_e q \qquad (4.4)$$

where

$$\dot{H}_x^B = (I_{yy} - I_{zz})qr + I_{yz}(q^2 - r^2) + (I_{zx}q - I_{xy}r)p + \bar{q}\bar{S}\bar{b}C_{l,t}$$

$$\dot{H}_y^B = (I_{zz} - I_{xx})rp + I_{zx}(r^2 - p^2) + (I_{xy}r - I_{yz}p)q + \bar{q}\bar{S}\bar{c}C_{m,t}$$

$$\dot{H}_z^B = (I_{xx} - I_{yy})pq + I_{xy}(p^2 - q^2) + (I_{yz}p - I_{zx}q)r + \bar{q}\bar{S}\bar{b}C_{n,t}$$

The first three equations, the navigation equations, are referenced to the inertial frame positions of the Earth pointing north (X), east (Y), and down (Z). These equations transform the body frame velocities u, v, and w, to the inertial frame about the Euler angles. The position states x and y, while dependent on the Euler equations, are not coupled to any of the other equations. Only the $\dot{z}$ equation, which carries the altitude h information for the calculation of the dynamic pressure $\bar{q}$ (via atmospheric density ρ), is used. These navigation equations are often omitted from the equation set if the navigation information is not needed, and the overall change in altitude is negligible. This is the case for the dynamics studied in this thesis; therefore, these equations are dropped from the reduced order models derived later in the chapter.

The kinematic equations describe the Euler angles, or the angles from the inertial frame fixed to the Earth, to the aircraft body-fixed frame. In other words, they provide the aircraft attitude and orientation, the roll angle ϕ, the pitch angle θ, and the yaw angle ϕ. These rotation angles are needed for the gravitational terms in the force equations, which give the gravitational acceleration components in the body frame.

The force equations contain the contributing components of the aerodynamic forces and propulsive forces exerted on the aircraft. These are provided by the total aerodynamic coefficients $C_{x,t}$, $C_{y,t}$, and $C_{z,t}$. The thrust T is only exerted along the x axis in this case. This is due to the assumption that the single, turbofan engine of the F-16 lies solely along this axis. Note, the full, generalized form of the force equations will contain thrust terms in the $\dot{v}$ and $\dot{w}$ equations to capture the propulsive forces exerted if the aircraft engine is off the centerline of the x-axis, or if there is some sort of directional thrust vectoring employed. Note that the force equations, currently described in the body axis, can be transformed into the wind axis (and used in the stability axis) via the following equations.

$$\dot{V}_T = \frac{u\dot{u} + v\dot{v} + w\dot{w}}{V_T}$$

$$\dot{\alpha} = \frac{u\dot{w} - w\dot{u}}{u^2 + w^2}$$

$$\dot{\beta} = \frac{V_T\dot{v} - v\dot{V}_T}{{V_T}^2\sqrt{1 - \left(\frac{v}{V_T}\right)^2}} \tag{4.5}$$

where

$$V_T = \sqrt{u^2 + v^2 + w^2}$$

These transformations will be used later for defining the longitudinal and latitudinal dynamics under investigation.

The final three equations are the moment equations. Like the force equations, the moment equations are dependent on dimensionless aerodynamic coefficients, in this case the aerodynamic moment coefficients $C_{l,t}$, $C_{m,t}$, and $C_{n,t}$ in the angular momentum rate of change components $\dot{H}_x^B$, $\dot{H}_y^B$, and $\dot{H}_z^B$. Here, the equations can be further simplified and tailored to the F-16 aircraft. The F-16 is symmetrical about the x- z plane; therefore, the products of inertia, I_{xy} and I_{yz}, are zero. The moment equations contain the angular momentum component H_e, which captures the angular momentum of an assumed, constant spinning turbofan engine. A further simplification is taken to neglect these terms, as the effect of the engine is relatively negligible.

In classical linear theory, these equations are linearized based on the assumption that the aircraft is being perturbed from an equilibrium point. One type of equilibrium point is found when the aircraft is flying steady and wings-level, in rectilinear flight. Mathematically, this is when the aircraft moments p, q, and r (therefore, $\dot{p}$, $\dot{q}$, and $\dot{r}$) are zero. Also, the force accelerations and Euler angle rates $\dot{u}$, $\dot{v}$, $\dot{w}$, and $\dot{\phi}$, $\dot{\theta}$, and $\dot{\psi}$ are zero. Finally, the flight path angle $\gamma = \theta - \alpha$. This state decouples the longitudinal equations of motion. Since the MIMO Volterra model was developed with the variational expansion method, the model will describe weak nonlinearities about an equilibrium point; therefore, perturbation theory will be used much like that for a linear system.

4.3 F-16 Experimental Data

As discussed previously, the force equations and moment equations depend on non-dimensional aerodynamic coefficient data in the body-axis system. These are $C_{x,t}$, $C_{y,t}$, and $C_{z,t}$ for the total aerodynamic force coefficients in reference to the x, y, and z body axis; and $C_{l,t}$, $C_{m,t}$, and $C_{n,t}$ for the total aerodynamic moment coefficients. In reference to the F-16 experimental data captured by NASA Langley, these coefficients are defined as

X-axis aerodynamic force coefficient

$$C_{x,t} = C_x(\alpha, \beta, \delta_{hrzt}) + \Delta C_{x,\delta_{lef}}(\alpha, \beta)\left(1 - \frac{\delta_{lef}}{25}\right) + C_{x,spbr}(\alpha)\left(\frac{\delta_{spbr}}{60}\right)$$
$$+ \frac{\bar{c}q}{2V_T}\left[C_{x_q}(\alpha) + \Delta C_{x_{q,\delta_{lef}}}(\alpha)\left(1 - \frac{\delta_{lef}}{25}\right)\right] \tag{4.6}$$

where

$$\Delta C_{x,\delta_{lef}} = C_{x,\delta_{lef}}(\alpha, \beta) - C_x(\alpha, \beta, \delta_{hrzt} = 0^o)$$

Y-axis aerodynamic force coefficient

$$C_{y,t} = C_y(\alpha,\beta) + \Delta C_{y,\delta_{lef}}(\alpha,\beta)\left(1 - \frac{\delta_{lef}}{25}\right)$$
$$+ \left[\Delta C_{y,\delta_{alrn}=20^o}(\alpha,\beta) + \Delta C_{y,\delta_{alrn}=20^o,\delta_{lef}}(\alpha,\beta)\left(1 - \frac{\delta_{lef}}{25}\right)\right]\left(\frac{\delta_{alrn}}{20}\right)$$
$$+ \Delta C_{y,\delta_{rdr}=30^o}(\alpha,\beta)\left(\frac{\delta_{rdr}}{30}\right)$$
$$+ \frac{b}{2V_T}\left\{\left[C_{y_r}(\alpha) + C_{y_{r,\delta_{lef}}}(\alpha)\left(1 - \frac{\delta_{lef}}{25}\right)\right] r\right.$$
$$\left. + \left[C_{y_p}(\alpha) + C_{y_{p,\delta_{lef}}}(\alpha)\left(1 - \frac{\delta_{lef}}{25}\right)\right] p\right\} \tag{4.7}$$

where

$$\Delta C_{y,\delta_{lef}} = C_{y,\delta_{lef}}(\alpha,\beta) - C_y(\alpha,\beta)$$

$$\Delta C_{y,\delta_{alrn}=20^o} = C_{y,\delta_{alrn}=20^o}(\alpha,\beta) - C_y(\alpha,\beta)$$

$$\Delta C_{y,\delta_{alrn}=20^o,\delta_{lef}} = C_{y,\delta_{alrn}=20^o,\delta_{lef}}(\alpha,\beta) - C_{y,\delta_{lef}}(\alpha,\beta) - \Delta C_{y,\delta_{alrn}=20^o}$$

$$\Delta C_{y,\delta_{rdr}=30^o} = C_{y,\delta_{rdr}=30^o}(\alpha,\beta) - C_y(\alpha,\beta)$$

Z-axis aerodynamic force coefficient

$$C_{z,t} = C_z(\alpha,\beta,\delta_{hrzt}) + \Delta C_{z,\delta_{lef}}(\alpha,\beta)\left(1 - \frac{\delta_{lef}}{25}\right) + C_{z,spbr}(\alpha)\left(\frac{\delta_{spbr}}{60}\right)$$
$$+ \frac{\bar{c}q}{2V_T}\left[C_{z_q}(\alpha) + C_{z_{q,\delta_{lef}}}(\alpha)\left(1 - \frac{\delta_{lef}}{25}\right)\right] \tag{4.8}$$

where

$$\Delta C_{z,\delta_{lef}} = C_{z,\delta_{lef}}(\alpha,\beta) - C_z(\alpha,\beta,\delta_{hrzt} = 0^o)$$

x-axis rolling moment coefficient

$$C_{l,t} = C_l(\alpha,\beta,\delta_{hrzt}) + \Delta C_{l,\delta_{lef}}(\alpha,\beta)\left(1 - \frac{\delta_{lef}}{25}\right)$$
$$+ \left[\Delta C_{l,\delta_{alrn}=20^o}(\alpha,\beta) + \Delta C_{l,\delta_{alrn}=20^o,\delta_{lef}}(\alpha,\beta)\left(1 - \frac{\delta_{lef}}{25}\right)\right]\left(\frac{\delta_{alrn}}{20}\right)$$
$$+ \Delta C_{l,\delta_{rdr}=30^o}(\alpha,\beta)\left(\frac{\delta_{rdr}}{30}\right)$$
$$+ \frac{\bar{b}}{2V_T}\left\{\left[C_{l_r}(\alpha) + C_{l_{r,\delta_{lef}}}(\alpha)\left(1 - \frac{\delta_{lef}}{25}\right)\right]r\right.$$
$$\left. + \left[C_{l_p}(\alpha) + C_{l_{p,\delta_{lef}}}(\alpha)\left(1 - \frac{\delta_{lef}}{25}\right)\right]p\right\} + C_{l,\beta}(\alpha)\beta$$

(4.9)

where

$$\Delta C_{l,\delta_{lef}} = C_{l,\delta_{lef}}(\alpha,\beta) - C_l(\alpha,\beta,\delta_{hrzt} = 0^o)$$

$$\Delta C_{l,\delta_{alrn}=20^o} = C_{l,\delta_{alrn}=20^o}(\alpha,\beta) - C_l(\alpha,\beta,\delta_{hrzt} = 0^o)$$

$$\Delta C_{l,\delta_{alrn}=20^o,\delta_{lef}} = C_{l,\delta_{alrn}=20^o,\delta_{lef}}(\alpha,\beta) - C_{l,\delta_{lef}}(\alpha,\beta) - \Delta C_{y,\delta_{alrn}=20^o}$$

$$\Delta C_{l,\delta_{rdr}=30^o} = C_{l,\delta_{rdr}=30^o}(\alpha,\beta) - C_l(\alpha,\beta,\delta_{hrzt} = 0^o)$$

y-axis pitching moment coefficient

$$C_{m,t} = C_m(\alpha,\beta,\delta_{hrzt})\eta_{\delta_{hrzt}}(\delta_{hrzt}) + C_{z,t}(x_{cmr} - x_{cm}) + \Delta C_{m,\delta_{lef}}\left(1 - \frac{\delta_{lef}}{25}\right)$$

$$+C_{m,spbr}(\alpha)\left(\frac{\delta_{spbr}}{60}\right) + \frac{\bar{c}q}{2V_T}\left[C_{m_q}(\alpha) + C_{m_{q,\delta_{lef}}}(\alpha)\left(1 - \frac{\delta_{lef}}{25}\right)\right]$$

$$+C_m(\alpha) + C_m(\alpha,\delta_{hrzt}) \tag{4.10}$$

where

$$\Delta C_{m,\delta_{lef}} = C_{m,\delta_{lef}}(\alpha,\beta) - C_m(\alpha,\beta,\delta_{hrzt} = 0^o)$$

z-axis yawing moment coefficient

$$C_{n,t} = C_n(\alpha,\beta,\delta_{hrzt}) + \Delta C_{n,\delta_{lef}}(\alpha,\beta)\left(1 - \frac{\delta_{lef}}{25}\right) - C_{y,t}(x_{cmr} - x_{cm})\left(\frac{\bar{c}}{\bar{b}}\right)$$

$$+\left[\Delta C_{n,\delta_{alrn}=20^o}(\alpha,\beta) + \Delta C_{n,\delta_{alrn}=20^o,\delta_{lef}}(\alpha,\beta)\left(1 - \frac{\delta_{lef}}{25}\right)\right]\left(\frac{\delta_{alrn}}{20}\right)$$

$$+\Delta C_{n,\delta_{rdr}=30^o}(\alpha,\beta)\left(\frac{\delta_{rdr}}{30}\right)$$

$$+\frac{\bar{b}}{2V_T}\left\{\left[C_{n_r}(\alpha) + C_{n_{r,\delta_{lef}}}(\alpha)\left(1 - \frac{\delta_{lef}}{25}\right)\right]r\right.$$

$$\left.+\left[C_{n_p}(\alpha) + C_{n_{p,\delta_{lef}}}(\alpha)\left(1 - \frac{\delta_{lef}}{25}\right)\right]p\right\} + C_{n,\beta}(\alpha)\beta$$

(4.11)

where

$$\Delta C_{n,\delta_{lef}} = C_{n,\delta_{lef}}(\alpha,\beta) - C_n(\alpha,\beta,\delta_{hrzt} = 0^o)$$

$$\Delta C_{n,\delta_{alrn}=20^o} = C_{n,\delta_{alrn}=20^o}(\alpha,\beta) - C_n(\alpha,\beta,\delta_{hrzt} = 0^o)$$

$$\Delta C_{n,\delta_{alrn}=20^o,\delta_{lef}} = C_{n,\delta_{alrn}=20^o,\delta_{lef}}(\alpha,\beta) - C_{n,\delta_{lef}}(\alpha,\beta) - \Delta C_{n,\delta_{alrn}=20^o}$$

$$\Delta C_{n,\delta_{rdr}=30^o} = C_{n,\delta_{rdr}=30^o}(\alpha,\beta) - C_n(\alpha,\beta,\delta_{hrzt} = 0^o)$$

The coefficients are built up from the experimental data provided by Nguyen[28] and substituted into the equations of motion. The data sets are comprised of 1, 2, and 3 dimensional tables that must be interpolated between for each of the independent variables as shown. This experimental data is comprised of multiple datasets performed at varying angles of attack, sideslip angles, and control inputs. Note that the thrust T is a direct lookup based on the input thrust power level, the mach number, and the altitude. The control input limits are presented in the table below.

Table 4.1 Control Input Limits

Control Input	Notation	Range	Reference
Horizontal Tail	δ_{hrzt}	-25 to 25 degrees	Down Is Positive
Aileron	δ_{alrn}	-21.5 to 21.5 degrees	Down Is Positive
Rudder	δ_{rdr}	-30 to 30 degrees	Left Is Positive
Leading Edge Flap	δ_{lef}	0 to 25 degrees	Down Is Positive
Speed Brake	δ_{spbr}	0 to 60 degrees	Up Is Positive
Thrust Power Level	δ_{th}	0 to 1	N/A

The data was captured across the following aerodynamic flight parameters and have the following limits.

Table 4.2 Aerodynamic Limits

State	**Notation**	**Range**
Angle of Attack	α	-10 to +90 degrees
Sideslip Angle	β	-30 to +30 degrees

4.4 Two Degree of Freedom Longitudinal Reduced Order Model

When operating at the equilibrium conditions of steady, wings-level, rectilinear flight, the longitudinal and latitudinal equations of motion can be decoupled. The longitudinal, nonlinear equations become (assuming $v = 0$, $\phi = 0$, $\varphi = 0$, $p = 0$, $r = 0$, and $\dot{v} = 0$, $\dot{\phi} = 0$, $\dot{\varphi} = 0$, $\dot{p} = 0$, $\dot{r} = 0$)

$$\dot{u} = -qw - g\sin\theta + \frac{\bar{q}\bar{S}}{m}C_{x,t} + \frac{T}{m}$$

$$\dot{w} = qu + g\cos\theta + \frac{\bar{q}\bar{S}}{m}C_{z,t}$$

$$\dot{\theta} = q \tag{4.12}$$

$$\dot{q} = \frac{\bar{q}\bar{S}\bar{c}C_{m,t}}{I_{yy}}$$

$$\dot{V}_T = \frac{u\dot{u} + v\dot{v} + w\dot{w}}{\sqrt{u^2 + w^2}}$$

$$\dot{\alpha} = \frac{u\dot{w} - w\dot{u}}{u^2 + w^2}$$

This decoupled longitudinal model is of order three. Further assumptions can be made in order to reduce the dynamics to two degrees of freedom, which corresponds to the developed MIMO Volterra model. The three degree of freedom model would capture all the dynamics for purely longitudinal motion, and any additional assumptions would reduce the analysis to a specific maneuver or subset of the dynamics.

The short period natural mode is of interest to engineers studying longitudinal dynamic. This mode is a natural, rapid, oscillatory motion that is highly damped. A characteristic of this oscillation is that it occurs immediately after a deviation from equilibrium and damps out quickly. Analysis has shown that during these first few seconds of a longitudinal maneuver, the angle of attack and the pitch angle vary together; therefore, the aircraft experiences only a very small change in the flight path angle. The short period can be considered the immediate dynamics induced by the pilot as the pilot performs a longitudinal only maneuver. The response over a greater period of time is known as the Phugoid mode, which is a very lightly damped oscillation with small amplitude. This mode occurs after the short period mode dies out and is generally easy to compensate for by the pilot. Thus, the short period is the focus for the two degree of freedom reduced order model.

Since the angle of attack is generally measured and worthwhile knowledge for the pilot, the response to the angle of attack, α, and the pitch rate, q, is sought. This requires an axis change from the body-fixed frame to the stability frame. This can be accomplished by substituting the following transform

$$u = V_T \cos\alpha \cos\beta$$
$$w = V_T \sin\alpha \cos\beta \tag{4.13}$$

into the $\dot{\alpha}$ equation, along with the $\dot{u}$ and $\dot{w}$ equations. This yields the following for $\dot{\alpha}$.

$$\dot{\alpha} = q + \frac{\bar{q}\bar{S}C_{z,t}\cos\alpha - \left(\bar{q}\bar{S}C_{x,t} + T\right)\sin\alpha + gm\cos(\alpha - \theta)}{mV_T} \tag{4.14}$$

Additional assumptions are required to make α and q the only time dependent variables in the model. Since these equations describe longitudinal motion only, the sideslip angle, β, from the transform was set to zero. Another assumption is that since the mode being studied is only of short duration, the total velocity, V_T, remains relatively constant. Note, this constraint still allows u and w to change (and therefore α), as it is only the total velocity vector of the two components that remains constant. Also, it is known that for the short period mode, the angle of attack and pitch angle vary closely together. Therefore, there is little change in the flight path angle and we can assume the flight path angle, $\gamma = \alpha - \theta$, is zero. Note that this also eliminates our dependence on the $\dot{\theta}$ equation, as $\dot{\theta}$ equals the pitch rate q (which we are capturing), and the dependence on the pitch angle, $\theta,$ has effectively been removed with the flight path assumption. This reduces the $\dot{\alpha}$ equation, and the final two degree of freedom motion for the short period is described by the two-equation set

$$\dot{\alpha} = q + \frac{\bar{q}\bar{S}C_{z,t}\cos\alpha - (\bar{q}\bar{S}C_{x,t} + T)\sin\alpha + mg}{mV_{T_o}}$$

$$\dot{q} = \frac{\bar{q}\bar{S}\bar{c}C_{m,t}}{I_{yy}} \tag{4.15}$$

Note that this is a set of equations that describe two outputs based on four inputs. The thrust, T, is a function of the input throttle, δ_{th}, while the aerodynamic coefficients, $C_{x,t}$ and $C_{z,t}$, are functions of the input to the horizontal tail, δ_{hrzt}, leading edge flap, δ_{lef}, and speed brake, δ_{spbr}. Two of these inputs can be selected for a two-input and two-output system. However, due to the assumption of constant total velocity used to reduce the equations, the response is predicted to substantially deviate from the full order solution when there is a change in the throttle input or speed brake input.

Finally, the use of Equation (4.13) can be used to transform α and the constant V_T to the body axis velocities. Therefore, the heave w and pitch rate q outputs are available as a two degree of freedom system as well from Equation (4.15).

4.5 Two Degree of Freedom Latitudinal Reduced Order Model

In addition to the longitudinal analysis, a latitudinal, two degree of freedom approximation is sought. Since the longitudinal dynamics revolve around rotation of only one axis, the y axis, the simplification is relatively straightforward. A reduced three degree of freedom model is natural, with all latitudinal states and time derivatives set to zero. However, for analyzing responses of the latitudinal states, the reduction is more complex and more assumptions must be made. This means that a two degree of freedom approximation of latitudinal dynamics will be somewhat inaccurate compared to the full ordered model. This is seen with the multitude of published reduced order models on the subject, each with its own assumptions and flight conditions where the model can be utilized.

An accurate description of the latitudinal dynamics is needed in order to study the additional nonlinearities that can be captured using the Volterra model. Therefore, the initial response immediately after a perturbation of the system, at time less than 10 seconds for example, may be accurate enough to show the model's strength.

There are three classical natural modes to latitudinal dynamics: the dutch roll mode, the roll subsidence mode, and the spiral mode. Of the three, the dutch roll mode is the mode that is oscillatory in nature. It has been so named due to the similarities of the motion with that of an ice skater. It is a combined side slipping, banking, yawing, back and forth motion after a rudder or aileron control input has been performed.

The dutch roll includes both roll rate and yaw rate. It is known that a large dihedral in the aircraft produces a larger roll rate when the maneuver is performed. Since the F-16 aircraft under study has no dihedral (or may even have a slight anhedral), the roll angle and roll rate can be assumed to be small and negligible. Therefore, the two states of focus (and assumed to be most dominant) are the side slip angle, β and the yaw moment, r.

From the full ordered model given by Equations (4.1) through (4.4), the pitch angle and pitching moment equations can be assumed to be constant during the duration being studied. They are not expected to change much during the small-time duration studied; therefore, their time derivatives are zero: $\dot{\theta} = 0$, q and $\dot{q} = 0$. Also, while the yaw angle, ψ, is dependent on the yaw rate, the reverse is not true. In fact, the yaw angle is not fed back into any of the other state equations. Note, it is fed into the navigation equations; however, those are not being studied and the navigation states are not fed back into the other nine equations either. Thus, the reduction can remain. Therefore, if the yaw angle is not specifically being studied as an output, then this equation can be dropped altogether. Finally, it has been stated that the yaw rate can be assumed to be more dominant than the roll rate; thus, the roll angle and rate and the roll moment and rate can be assumed to be zero: $\phi = 0$, $\dot{\phi} = 0$, p =0, and $\dot{p} = 0$. This is an assumption of pure yawing motion and is arguably the biggest assumption made here. These assumptions reduce the dynamics to four nonlinear equations.

$$\dot{u} = rv - g\sin\theta + \frac{\bar{q}\bar{S}}{m}C_{x,t} + \frac{T}{m}$$

$$\dot{v} = -ru + \frac{\bar{q}\bar{S}}{m}C_{y,t}$$

$$\dot{w} = g\cos\phi\cos\theta + \frac{\bar{q}\bar{S}}{m}C_{z,t}$$

$$\dot{r} = \bar{q}\bar{S}\bar{b}\left(\frac{C_{n,t}}{I_{zz}} - \frac{C_{l,t}}{I_{zx}}\right) \tag{4.16}$$

An axis change is required to change from the body-fixed frame to the stability frame. This can be accomplished by substituting the following transform

$$u = V_T \cos\alpha \cos\beta$$

$$v = V_T \sin\beta$$

$$w = V_T \sin\alpha \cos\beta \tag{4.17}$$

into the $\dot{\beta}$ equation, along with the $\dot{u}$, $\dot{v}$, and $\dot{w}$ equations. This yields the following for $\dot{\beta}$.

$$\dot{\beta} = \frac{1}{m\sqrt{{V_T}^2 \cos^2\beta}} \cos\beta \left(\bar{q}\bar{S}C_{y,t}\cos\beta - \left(\bar{q}\bar{S}C_{z,t} + mg\cos\phi\cos\theta\right)\sin\alpha\sin\beta\right.$$

$$\left. - \left(mV_T r + \sin\beta\left(\bar{q}\bar{S}C_{x,t} + T - mg\sin\theta\right)\right)\cos\alpha\right) \tag{4.18}$$

A few assumptions need to be made in order to reduce the equations. The roll angle ϕ from the transform equations, is zero based on the non-roll assumption made earlier. The total velocity V_T, the pitch angle θ, and the angle of attack α are assumed to undergo very little change during the shorter time period under analysis; therefore, these states are kept constant. The resulting assumptions to the $\dot{\beta}$ equation lead to the two-equation set below which describe a reduced order, two degree of freedom, model for the dutch roll mode.

$$\dot{\beta} = \frac{1}{m\sqrt{{V_{T_o}}^2 \cos^2\beta}} \cos\beta \left(\bar{q}\bar{S}C_{y,t}\cos\beta - \left(\bar{q}\bar{S}C_{z,t} + mg\cos\theta_o\right)\sin\alpha_o \sin\beta\right.$$
$$\left. - \left(mV_{T_o}r + \sin\beta\left(\bar{q}\bar{S}C_{x,t} + T - mg\sin\theta_o\right)\right)\cos\alpha_o\right)$$

$$\dot{r} = \bar{q}\bar{S}\bar{b}\left(\frac{C_{n,t}}{I_{zz}} - \frac{C_{l,t}}{I_{zx}}\right) \tag{4.19}$$

Note that the use of Equation (4.17) can be used to transform β and the constant V_T to the body axis velocities. Therefore, the sway v and yaw rate r outputs are available as a two degree of freedom system as well from Equation (4.19).

CHAPTER 5

NONLINEAR FLIGHT DYNAMICS ANALYSIS

5.1 Overview

This chapter describes the initialization of the gain parameters used in the MIMO Volterra model. Then, the nonlinearities of the F-16 experimental data is investigated. The results of the MIMO Volterra model are then presented in comparison to numerical simulation. The key nonlinearities and the total state responses are discussed for both the short period and dutch roll reduced order models presented earlier.

5.2 Short Period MIMO Volterra Scenario 1

As presented in the previous chapter, the short period motion has been described by the reduced order model in Equation (4.15). The MIMO Volterra gains $K_{1000} \cdots L_{0002}$, are initialized around an equilibrium point with the horizontal tail and thrust power level as inputs. This is described by the multivariable Taylor series expansion as follows

$$\begin{aligned}\Delta\dot{\alpha} \approx \frac{\partial f}{\partial \alpha}\Delta\alpha + \frac{\partial f}{\partial q}\Delta q + \frac{\partial f}{\partial \delta_{hrzt}}\Delta\delta_{hrzt} + \frac{\partial f}{\partial \delta_{th}}\Delta\delta_{th} + \frac{1}{2!}\frac{\partial^2 f}{\partial \alpha^2}\Delta\alpha^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial q^2}\Delta q^2 \\ + \frac{1}{2!}\frac{\partial^2 f}{\partial {\delta_{hrzt}}^2}\Delta{\delta_{hrzt}}^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial {\delta_{th}}^2}\Delta{\delta_{th}}^2 + \frac{\partial^2 f}{\partial \alpha \partial q}\Delta\alpha\Delta q + \frac{\partial^2 f}{\partial \alpha \partial \delta_{hrzt}}\Delta\alpha\Delta\delta_{hrzt} \\ + \frac{\partial^2 f}{\partial \alpha \partial \delta_{th}}\Delta\alpha\Delta\delta_{th} + \frac{\partial^2 f}{\partial q \partial \delta_{hrzt}}\Delta q\Delta\delta_{hrzt} + \frac{\partial^2 f}{\partial q \partial \delta_{th}}\Delta q\Delta\delta_{th} \\ + \frac{\partial^2 f}{\partial \delta_{th} \partial \delta_{hrzt}}\Delta\delta_{th}\Delta\delta_{hrzt} \\ + \cdots \qquad (5.1)\end{aligned}$$

$$\Delta\dot{q} \approx \frac{\partial f}{\partial \alpha}\Delta\alpha + \frac{\partial f}{\partial q}\Delta q + \frac{\partial f}{\partial \delta_{hrzt}}\Delta\delta_{hrzt} + \frac{\partial f}{\partial \delta_{th}}\Delta\delta_{th} + \frac{1}{2!}\frac{\partial^2 f}{\partial \alpha^2}\Delta\alpha^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial q^2}\Delta q^2$$
$$+ \frac{1}{2!}\frac{\partial^2 f}{\partial {\delta_{hrzt}}^2}\Delta{\delta_{hrzt}}^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial {\delta_{th}}^2}\Delta{\delta_{th}}^2 + \frac{\partial^2 f}{\partial \alpha \partial q}\Delta\alpha\Delta q + \frac{\partial^2 f}{\partial \alpha \partial \delta_{hrzt}}\Delta\alpha\Delta\delta_{hrzt}$$
$$+ \frac{\partial^2 f}{\partial \alpha \partial \delta_{th}}\Delta\alpha\Delta\delta_{th} + \frac{\partial^2 f}{\partial q \partial \delta_{hrzt}}\Delta q\Delta\delta_{hrzt} + \frac{\partial^2 f}{\partial q \partial \delta_{th}}\Delta q\Delta\delta_{th}$$
$$+ \frac{\partial^2 f}{\partial \delta_{th} \partial \delta_{hrzt}}\Delta\delta_{th}\Delta\delta_{hrzt}$$
$$+ \cdots \tag{5.2}$$

The partial derivatives correspond to the Volterra model gains $K_{1000} \cdots L_{0002}$ based on the states and/or inputs of each of the terms (e.g. $\Delta\alpha\Delta\delta_{th}$ term for $\dot{\alpha}$ corresponds to K_{1001}, for state 1 – input 2). They were calculated by the central finite difference method utilizing the experimental data and the reduced order model equations.

Various scenarios were run to discover significant nonlinearities. The F-16 data was found to be highly linear for most of the flight envelope. Figure 5.1 shows a plot of the aerodynamic C_z data across the angle of attack and sideslip angle range. This coefficient data is closely related to an aircraft's lift-curve slope. The coefficient value ramps up approximately linearly until the angle of attack reaches near forty degrees. Around this point, a curve is seen which levels out for the rest of the range. At this curve, a significant quadratic derivative is expected. Therefore, this area of the flight condition will be explored.

In addition, Figure 5.2 shows a plot of the relevant aerodynamic C_m data across the angle of attack and sideslip angle range. Like the C_z data, a curve in the data can be seen, occurring at roughly an angle of attack of sixty degrees. Unfortunately, an equilibrium solution near this high an angle of attack could not be found; therefore, the MIMO Volterra model could not be used here.

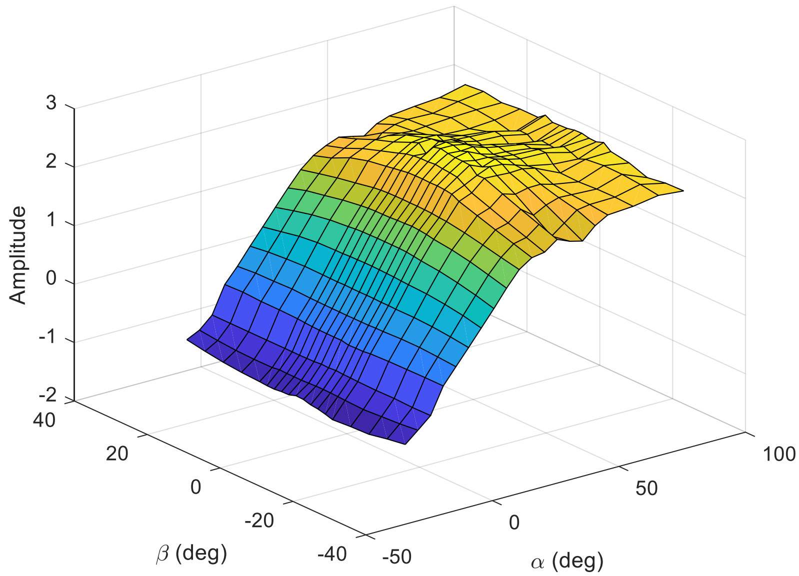

Figure 5.1 Aerodynamic Data for $-C_z(\alpha, \beta, \delta_{hrzt}=0)$

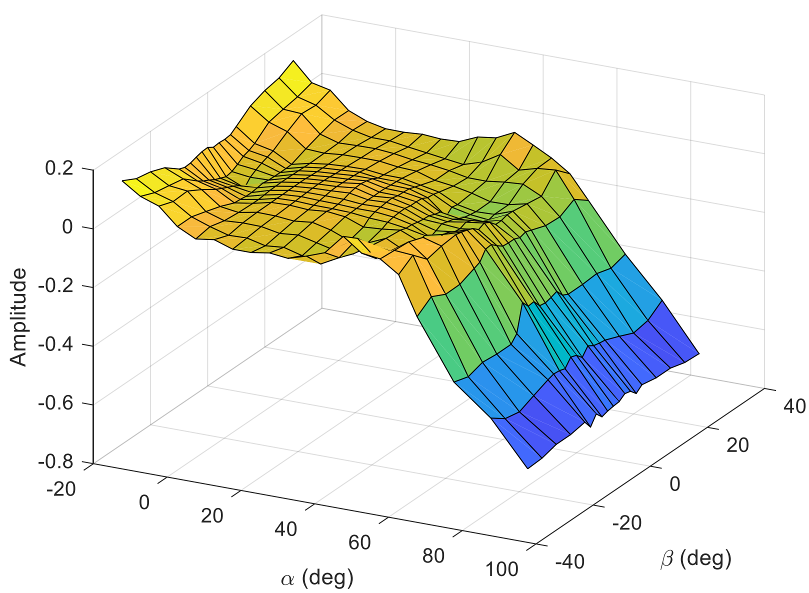

Figure 5.2 Aerodynamic Data for $C_m(\alpha, \beta, \delta_{hrzt}=0)$

5.2.1 One Input Short Period Analysis

Results were plotted to compare four responses: the linear component response of the MIMO Volterra model, the total response (linear and nonlinear) of the MIMO Volterra model, a numerical simulation of the reduced order model (via fourth order Runga Kutta), and a numerical simulation of the full, twelve equations of motion (via fourth order Runga Kutta). The full order simulation is shown so one can observe how the reduced order assumptions have changed the response.

As previously explained, the F-16 data was found to be linear for a majority of the flight envelope until an angle of attack of around forty degrees is reached. To show this characteristic, Figures 5.3 and 5.4 show the angle of attack and pitch rate response at a low initial angle of attack of about 3.4 degrees (at an altitude of 10,000 feet and total velocity of 500 feet per second). Figures 5.5 and 5.6 show the response when the total velocity was reduced to 225 feet per second, resulting in an increased, initial angle of attack around 22 degrees.

Based on the responses, the linear and total MIMO Volterra model, along with the reduced order model, align with each other nearly exactly. The full order model deviates due to the nonlinear coupling from incorporating the full twelve equations. Note, the approximations taken for the two degree of freedom model eliminated all of the explicit nonlinear terms in the $\dot{\alpha}$ and $\dot{q}$ equations; therefore, the only nonlinearities left are those present in the coefficient values captured in the F-16 data through Equations (4.6) through (4.11). The full order model retains the explicit, nonlinear equations terms, while also incorporating the full coupling and results in the deviations seen.

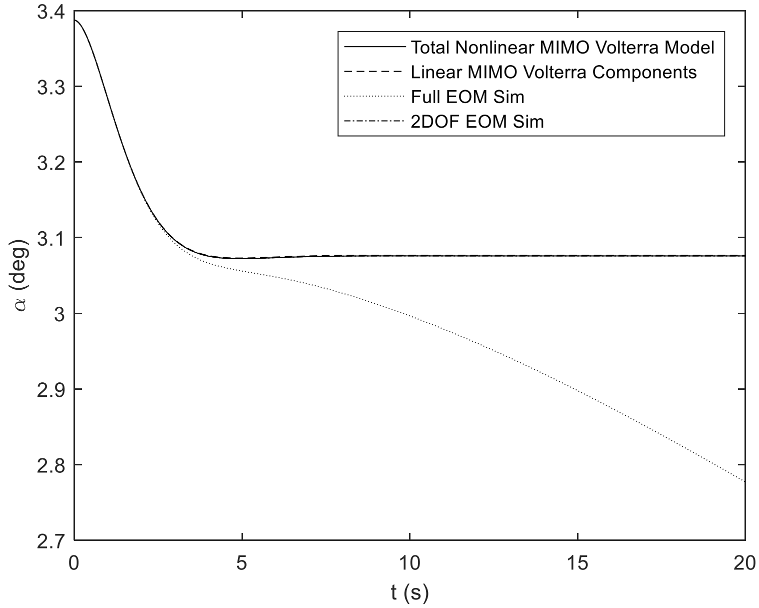

Figure 5.3 Angle of Attack α Step Response of δ_{hrzt}=+0.05 deg from Equilibrium at V_T=500 ft/s

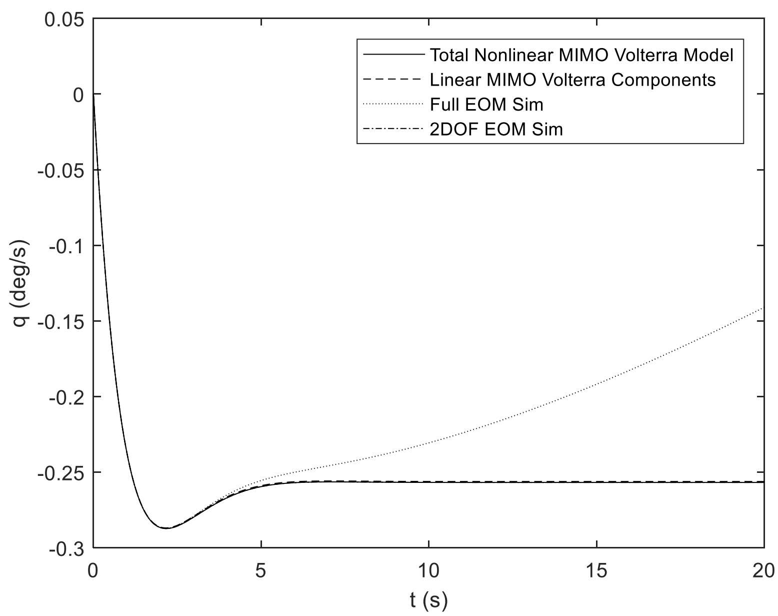

Figure 5.4 Pitch Rate q Step Response of δ_{hrzt}=+0.05 deg from Equilibrium at V_T=500 ft/s

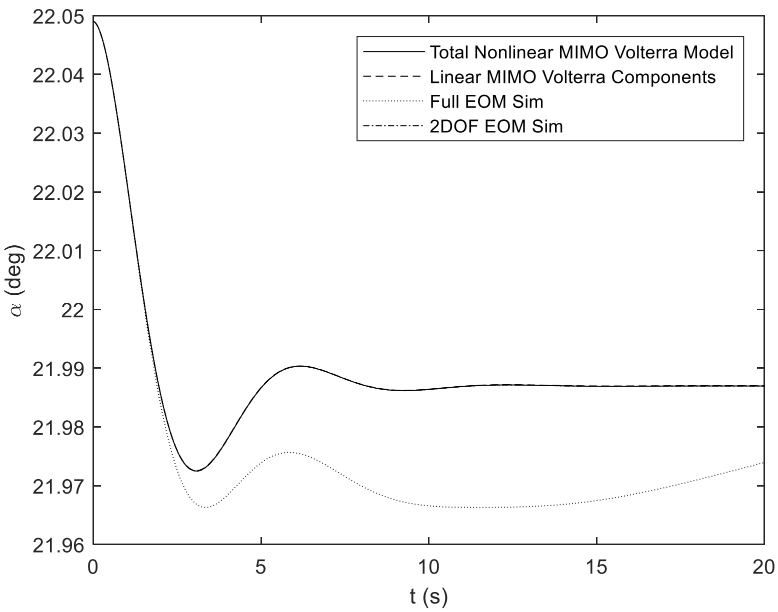

Figure 5.5 Angle of Attack α Step Response of δ_{hrzt}=+0.05 deg from Equilibrium at V_T=225 ft/s

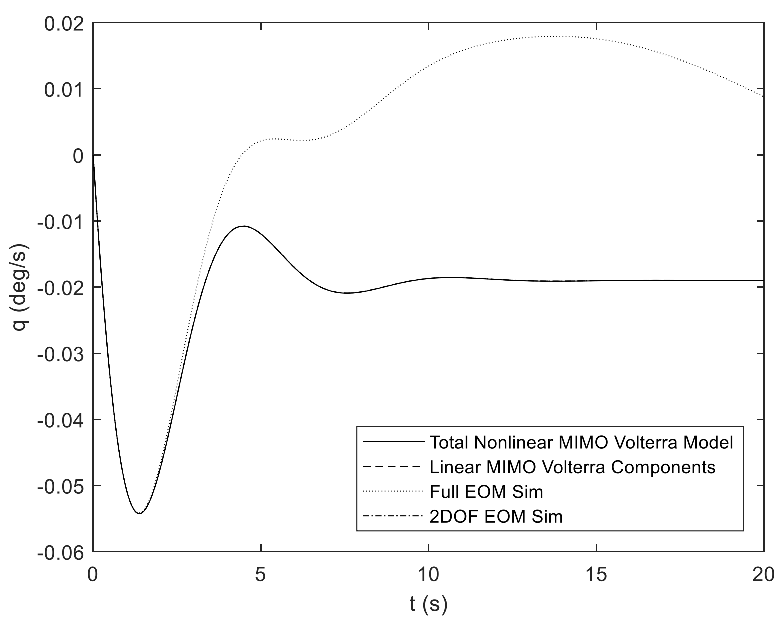

Figure 5.6 Pitch Rate q Step Response of δ_{hrzt}=+0.05 deg from Equilibrium at V_T=225 ft/s

Reducing the total velocity to 173 feet per second yields a trimmed angle of attack of 39.87 degrees, near the targeted 40 degrees as seen in the experimental data. The gains in Table 5.1 were calculated based on this velocity for rectilinear, wings-level flight, along with: an altitude of 10,000 feet, an angle of attack of 39.87 degrees, a pitch angle of 39.87 degrees, an initial horizontal tail deflection of -17.74 degrees, an initial thrust power of 0.90, and a center of mass at the nominal placement of 0.3. Note, the parameters K_{abcd} and L_{abcd} have units dependent on the two states, angle of attack (radians) and pitch rate (radians per second), and the two inputs, horizontal tail deflection (radians) and thrust power level (%). For example, K_{0110} has the units of 1/rad.

Unlike the lower angles of attack, nonlinearities arise as predicted. Figures 5.7 and 5.8 show the angle of attack response and the pitch rate response respectively, for a horizontal tail input of +0.05 degrees from equilibrium. The results show that the MIMO Volterra model offers greater accuracy to the reduced order model than the linear model alone. As can be seen, the full order model deviates from the reduced order model. This is due to the coupling and nonlinearities present in the full order model as explained previously.

The accuracy of the MIMO Volterra model is confirmed and insight into the nonlinear components can now be extracted. Figures 5.9 and 5.10 show a breakdown of the angle of attack and pitch rate into their linear and total nonlinear components respectively. Here, the total summation of the nonlinear components can be used to show the overall impact of the nonlinearities to the total response. The strength of the nonlinearity can now be measured, which is a capability that cannot be extracted from numerical simulation.

Table 5.1 Short Period MIMO Volterra Parameter Values for α, q, δ_{hrzt}, and δ_{th} at V_T=173 ft/s

Gain Parameter	Value	Corresponding Matrix
K_{1000}	-8.96e-02	A
K_{0100}	9.28e-01	A
L_{1000}	-4.11e-01	A
L_{0100}	-5.11e-01	A
K_{0010}	-3.86e-02	B
K_{0001}	-1.62e-01	B
L_{0010}	-9.54e-01	B
L_{0001}	5.16e-33	B
K_{2000}	-2.30e+00	B_{qs}
L_{2000}	-3.29e+01	B_{qs}
K_{0200}	-3.08e-11	B_{qs}
L_{0200}	9.17e-12	B_{qs}
K_{1100}	-7.28e-02	B_{bsbi}
L_{1100}	3.00e-03	B_{bsbi}
K_{0011}	-4.97e-15	B_{bsbi}
L_{0011}	0	B_{bsbi}
K_{1010}	4.95e-02	B_{bsi1}
L_{1010}	4.55e-01	B_{bsi1}
K_{0110}	2.62e-12	B_{bsi1}
L_{0110}	-3.67e-12	B_{bsi1}
K_{1001}	-1.94e-01	B_{bsi2}
L_{1001}	0	B_{bsi2}
K_{0101}	-2.48e-15	B_{bsi2}
L_{0101}	0	B_{bsi2}
K_{0020}	-3.70e-11	B_{qi1}
L_{0020}	-4.75e-14	B_{qi1}
K_{0002}	-4.72e-14	B_{qi2}
L_{0002}	-7.19e-31	B_{qi2}

*Units for K_{abcd} and L_{abcd} are dependent on α, q, δ_{hrzt}, and δ_{th}

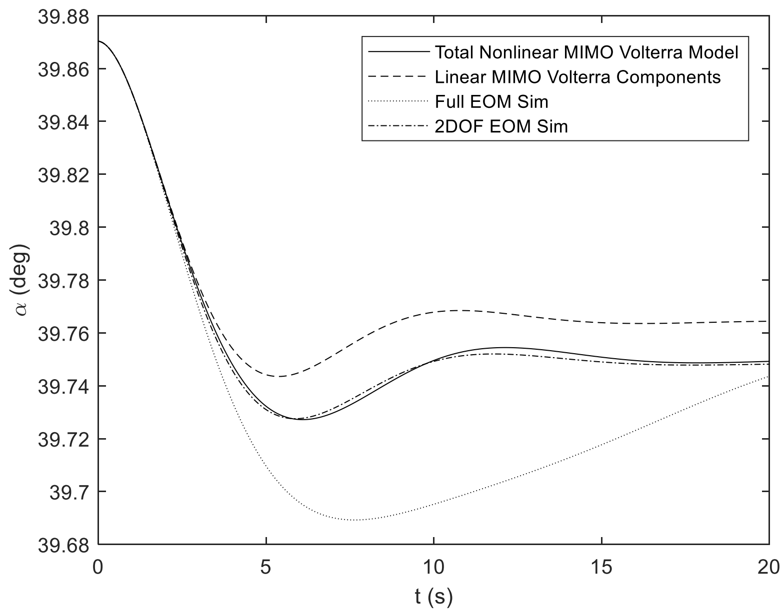

Figure 5.7 Angle of Attack α Step Response of δ_{hrzt}=+0.05 deg from Equilibrium at V_T=173 ft/s

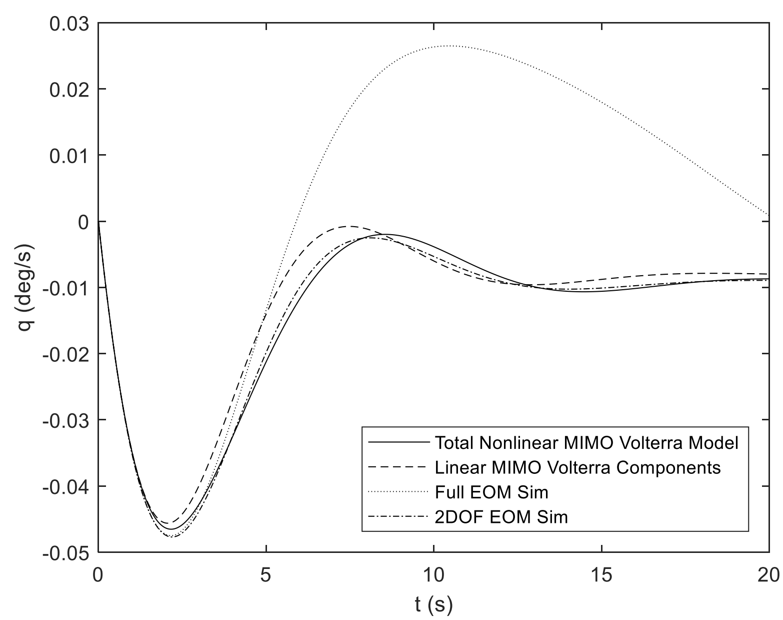

Figure 5.8 Pitch Rate q Step Response of δ_{hrzt}=+0.05 deg from Equilibrium at V_T=173 ft/s

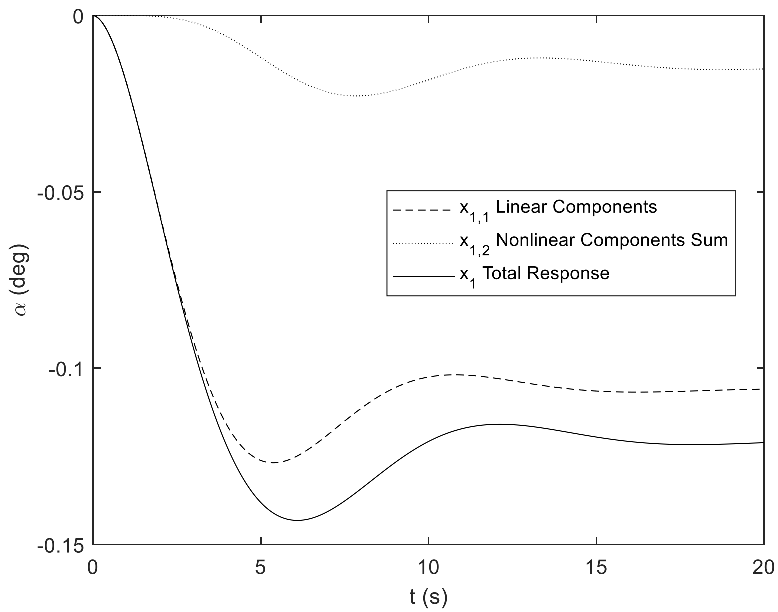

Figure 5.9 MIMO Volterra Model Angle of Attack α Step Response of δ_{hrzt}=+0.05 deg from Equilibrium at V_T=173 ft/s

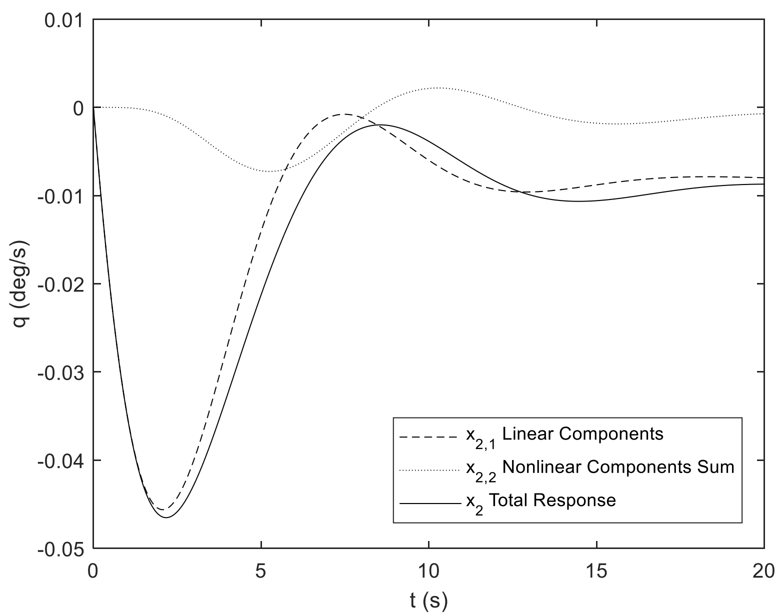

Figure 5.10 MIMO Volterra Model Pitch Rate q Step Response of δ_{hrzt}=+0.05 deg from Equilibrium at V_T=173 ft/s

Figures 5.11 and 5.12 decompose the nonlinear component further, into the quadratic state components, bilinear state-input components, etc., for the angle of attack and pitch rate, respectively. The strongest nonlinear component is the quadratic state 1 component, or the second derivative effect of the change in angle of attack. The quadratic state 1 component can be further broken down into its subcomponents, representing the terms with respect to each of the inputs. These are shown in Figures 5.13 and 5.14 for the angle of attack and pitch rate, respectively. The results directly match the intuitive prediction made previously about quadratic state nonlinearities occurring near perturbations near forty degrees alpha. These plots show the capability of the MIMO Volterra model to capture the responses of individual nonlinear components and their overall effects on total system response. This is a capability that is absent from numerical simulation.

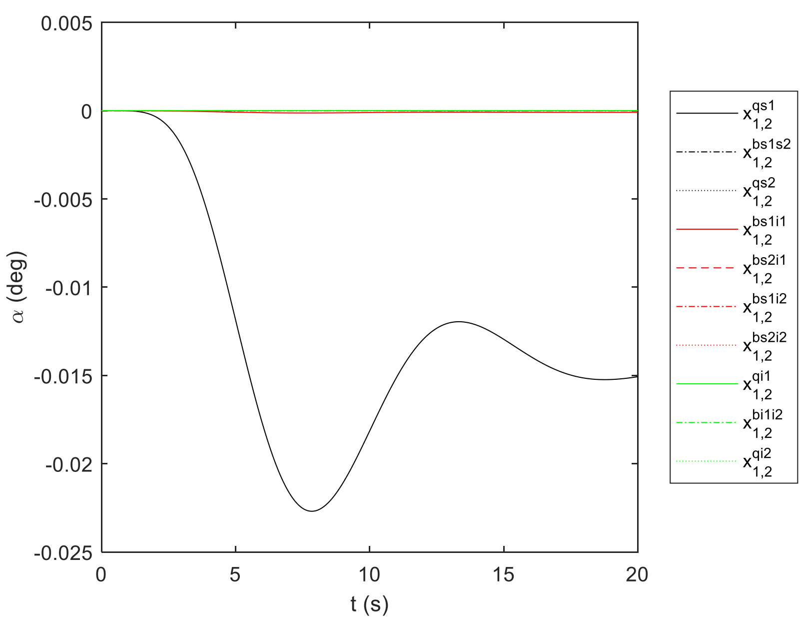

Figure 5.11 Nonlinear Components of Angle of Attack α Step Response of δ_{hrzt}=+0.05 deg from Equilibrium at V_T=173 ft/s

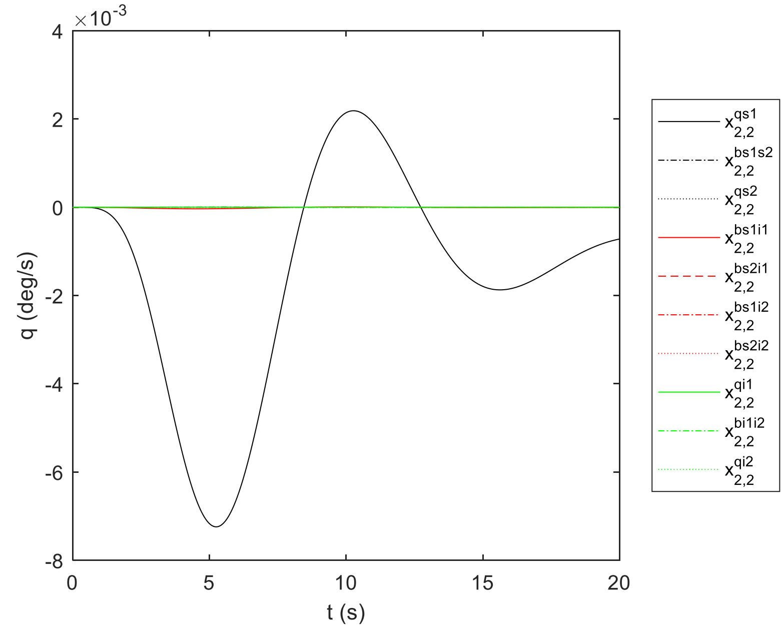

Figure 5.12 Nonlinear Components of Pitch Rate q Step Response of δ_{hrzt}=+0.05 deg from Equilibrium at V_T=173 ft/s

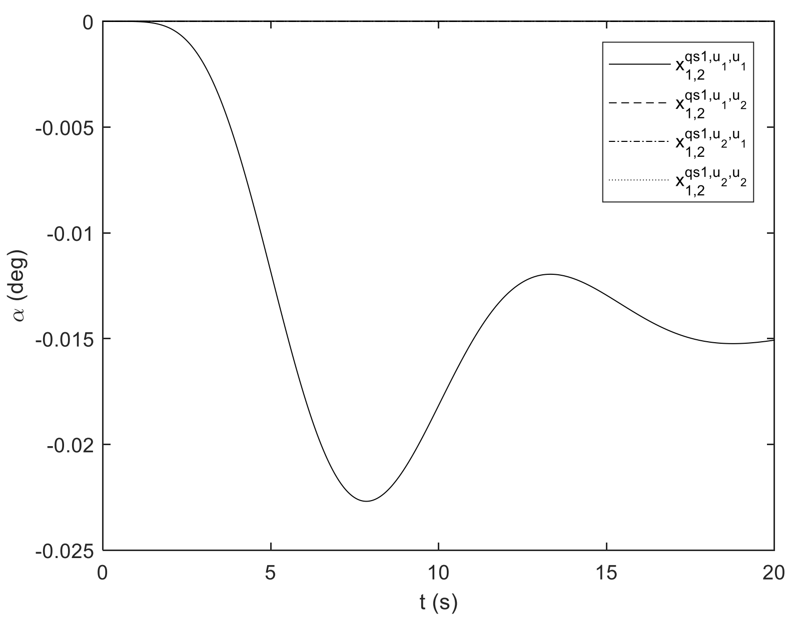

Figure 5.13 Quadratic State 1 Subcomponents of Angle of Attack α Step Response of δ_{hrzt}=+0.05 deg from Equilibrium at V_T=173 ft/s

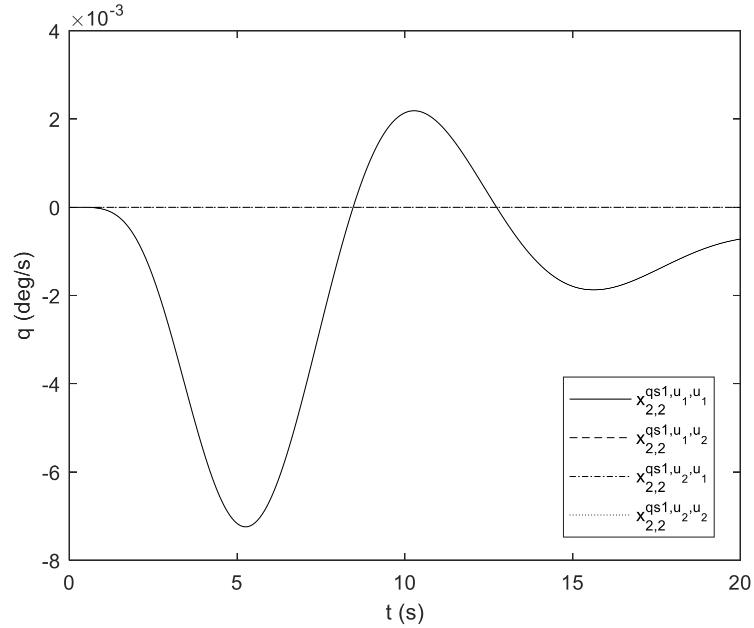

Figure 5.14 Quadratic State 1 Subcomponents of Pitch Rate q Step Response of δ_{hrzt}=+0.05 deg from Equilibrium at V_T=173 ft/s

5.2.2 Two Input Short Period Analysis

The MIMO Volterra model, using the gains from Table 5.1, is based on the approximations taken to derive the reduced order model. Therefore, inputs for deflections of the horizontal tail, δ_{hrzt}, are expected to match the reduced order model effectively from the model in Section 5.2.1. The results of incorporating the second input is investigated to show the constraints due to the assumptions made. The second input in this case is the thrust power level, δ_{th}.

Upon inspection of Equation (4.15), the thrust term T is a function of δ_{th}; therefore, a change to this input is incorporated into the reduced order model. However, the approximation of constraining the total velocity derivative to zero, means that the coupling of the change in thrust to the total velocity will not be captured. This limits the reduced order model's accuracy for this input.

Considering this, Figures 5.15 and 5.16 show the angle of attack and pitch rate responses change in both the horizontal tail input δ_{hrzt} and the thrust power level δ_{th}. The system is again initialized using the initial conditions and Volterra gains from Table 5.1.

The MIMO Volterra model is able to handle the two-input, two-output scenario; however, the responses deviate from the full order model due to the constraints noted earlier. The next section will attempt to provide a longitudinal, two-input, two-output scenario that better represents both the reduced and full order model.

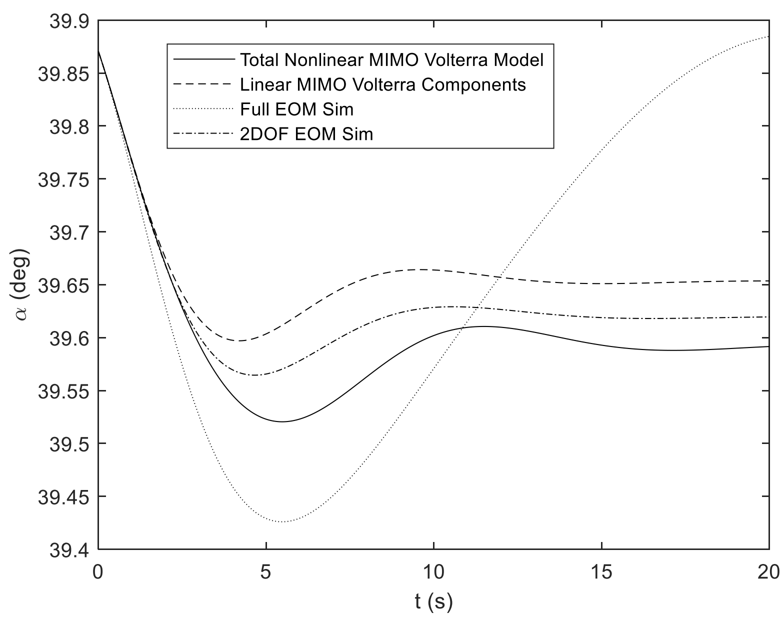

Figure 5.15 Angle of Attack α Step Response of δ_{hrzt}=+0.05 deg and δ_{th}=+0.01 from Equilibrium at V_T=173 ft/s

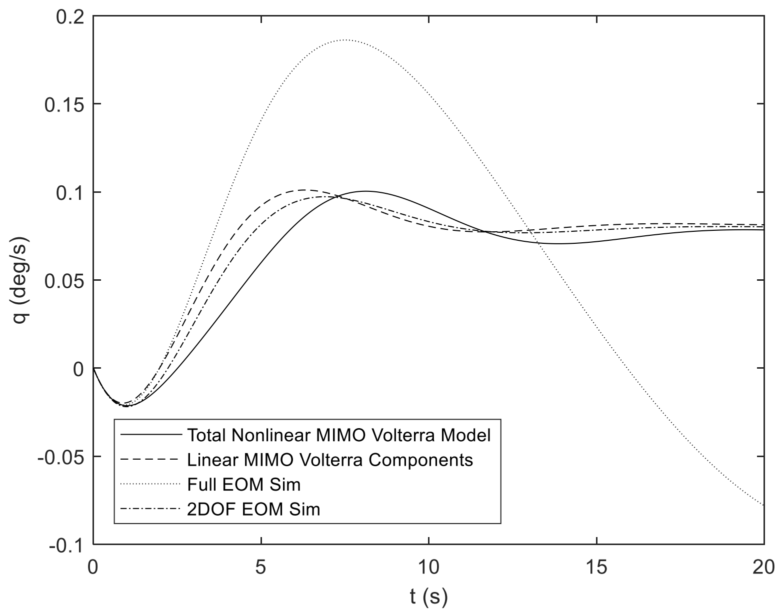

Figure 5.16 Pitch Rate q Step Response of δ_{hrzt}=+0.05 deg and δ_{th}=+0.01 from Equilibrium at V_T=173 ft/s

5.3 Short Period MIMO Volterra Scenario 2

5.3.1 Two Input Short Period Analysis

The previous section, Section 5.2.2, shows an analysis of the short period motion based on two inputs, δ_{hrzt} and δ_{th}. Due to the constraints placed upon V_T in the reduced order model (from Equation (4.15)), an input to δ_{th} will not result in accurate results compared to that of the reduced and full order model. To show a multiple input scenario that more closely matches these models, the leading edge flap input, δ_{lef}, is substituted in place of the thrust power input, δ_{th}. Again, the MIMO Volterra gains $K_{1000} \cdots L_{0002}$, are initialized around an equilibrium point, but now with the horizontal tail and leading edge flap deflections as inputs. This is described by the multivariable Taylor series expansion as follows

$$\Delta\dot{\alpha} \approx \frac{\partial f}{\partial \alpha}\Delta\alpha + \frac{\partial f}{\partial q}\Delta q + \frac{\partial f}{\partial \delta_{hrzt}}\Delta\delta_{hrzt} + \frac{\partial f}{\partial \delta_{lef}}\Delta\delta_{lef} + \frac{1}{2!}\frac{\partial^2 f}{\partial \alpha^2}\Delta\alpha^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial q^2}\Delta q^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial {\delta_{hrzt}}^2}\Delta{\delta_{hrzt}}^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial {\delta_{lef}}^2}\Delta{\delta_{lef}}^2 + \frac{\partial^2 f}{\partial \alpha \partial q}\Delta\alpha\Delta q + \frac{\partial^2 f}{\partial \alpha \partial \delta_{hrzt}}\Delta\alpha\Delta\delta_{hrzt} + \frac{\partial^2 f}{\partial \alpha \partial \delta_{lef}}\Delta\alpha\Delta\delta_{lef} + \frac{\partial^2 f}{\partial q \partial \delta_{hrzt}}\Delta q\Delta\delta_{hrzt} + \frac{\partial^2 f}{\partial q \partial \delta_{lef}}\Delta q\Delta\delta_{lef} + \frac{\partial^2 f}{\partial \delta_{lef} \partial \delta_{hrzt}}\Delta\delta_{lef}\Delta\delta_{hrzt} + \cdots \quad (5.3)$$

$$\Delta\dot{q} \approx \frac{\partial f}{\partial \alpha}\Delta\alpha + \frac{\partial f}{\partial q}\Delta q + \frac{\partial f}{\partial \delta_{hrzt}}\Delta\delta_{hrzt} + \frac{\partial f}{\partial \delta_{lef}}\Delta\delta_{lef} + \frac{1}{2!}\frac{\partial^2 f}{\partial \alpha^2}\Delta\alpha^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial q^2}\Delta q^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial {\delta_{hrzt}}^2}\Delta{\delta_{hrzt}}^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial {\delta_{lef}}^2}\Delta{\delta_{lef}}^2 + \frac{\partial^2 f}{\partial \alpha \partial q}\Delta\alpha\Delta q + \frac{\partial^2 f}{\partial \alpha \partial \delta_{hrzt}}\Delta\alpha\Delta\delta_{hrzt} + \frac{\partial^2 f}{\partial \alpha \partial \delta_{lef}}\Delta\alpha\Delta\delta_{lef} + \frac{\partial^2 f}{\partial q \partial \delta_{hrzt}}\Delta q\Delta\delta_{hrzt} + \frac{\partial^2 f}{\partial q \partial \delta_{lef}}\Delta q\Delta\delta_{lef} + \frac{\partial^2 f}{\partial \delta_{lef} \partial \delta_{hrzt}}\Delta\delta_{lef}\Delta\delta_{hrzt} + \cdots \quad (5.4)$$

The gains in Table 5.2 were calculated based on rectilinear, wings-level flight at an altitude of 10,000 feet, a total velocity of 220 feet per second, an angle of attack and pitch angle of 23 degrees, a horizontal tail deflection of -6.14 degrees, a thrust power of 0.64, and a center of mass at the nominal placement of 0.3. Note, the parameters K_{abcd} and L_{abcd} have units dependent on the two states, angle of attack (radians) and pitch rate (radians per second), and the two inputs, horizontal tail deflection (radians) and leading edge flap deflection (radians). For example, K_{0101} has the units of 1/rad.

Table 5.2 Short Period MIMO Volterra Parameter Values for α, q, δ_{hrzt}, and δ_{lef} at V_T=220 ft/s

Gain Parameter	Value	Corresponding Matrix
K_{1000}	-3.15e-01	A
K_{0100}	9.35e-01	A
L_{1000}	-1.09e+00	A
L_{0100}	-6.24e-01	A
K_{0010}	-3.83e-02	B
K_{0001}	-1.42e-02	B
L_{0010}	-1.55e+00	B
L_{0001}	-7.03e-02	B
K_{2000}	2.45e-01	B_{qs}
L_{2000}	1.39e-11	B_{qs}
K_{0200}	5.62e-12	B_{qs}
L_{0200}	-1.56e-11	B_{qs}
K_{1100}	1.26e-01	B_{bsbi}
L_{1100}	-1.04e+00	B_{bsbi}
K_{0011}	2.59e-13	B_{bsbi}
L_{0011}	0	B_{bsbi}
K_{1010}	3.00e-01	B_{bsi1}
L_{1010}	6.96e-01	B_{bsi1}
K_{0110}	-2.85e-13	B_{bsi1}
L_{0110}	0	B_{bsi1}
K_{1001}	1.07e-01	B_{bsi2}
L_{1001}	1.06e+00	B_{bsi2}
K_{0101}	5.92e-03	B_{bsi2}
L_{0101}	3.21e-01	B_{bsi2}
K_{0020}	5.78e-12	B_{qi1}
L_{0020}	-1.04e-12	B_{qi1}
K_{0002}	-1.36e-11	B_{qi2}
L_{0002}	-3.53e-11	B_{qi2}

*Units for K_{abcd} and L_{abcd} are dependent on α, q, δ_{hrzt}, and δ_{lef}

Figures 5.17 and 5.18 show the angle of attack and pitch rate responses to a horizontal tail deflection input of +2 degrees and a leading edge flap deflection input of +2 degrees from equilibrium. The responses show that the MIMO Volterra model is able to accurately track the reduced order model. Deviations from the full order model are still seen, which is likely due to the phugoid natural mode dynamics that are not captured in the reduced and Volterra models.

Figures 5.19 and 5.20 break down the MIMO Volterra model into the linear and total nonlinear components. Here, the total summation of the nonlinear components can be used to show the overall impact of the nonlinearities to the total response. Note that the contribution of the nonlinear component is small, but accounts for the difference between the linear and total Volterra response from Figures 5.17 and 5.18.

Further breakdowns of the nonlinear components are shown in Figures 5.21 and 5.22. These plots show the contributions from the individual nonlinear terms. The nonlinearities from these responses are shown to involve the $x_{1,2}^{qs1}$, $x_{1,2}^{bs1s2}$, $x_{1,2}^{bs1i1}$, $x_{1,2}^{bs1i2}$, and $x_{1,2}^{bs2i2}$ components for the angle of attack response and, similarly, the $x_{2,2}^{qs1}$, $x_{2,2}^{bs1s2}$, $x_{2,2}^{bs1i1}$, $x_{2,2}^{bs1i2}$, and $x_{2,2}^{bs2i2}$ components for the pitch rate response. Compared to the single input scenario from Section 5.2.1, the addition of the leading edge flap input is shown to activate the bilinear state 1 – state 2 terms, the bilinear state 1 – input 2 terms, and the bilinear state 2 – input 2 terms. The summation of each of these component responses in Figures 5.21 and 5.22 result in the total nonlinear effects on the system.

The nonlinearities are further investigated at the subcomponent level, with each subcomponent representing the nonlinearity with respect to each of the inputs, u_1 and u_2 (δ_{hrzt} and δ_{lef} respectively). Figures 5.23 through 5.28 show the subcomponents for the angle of attack nonlinear response while Figures 5.29 through 5.34 show the subcomponents for the pitch rate nonlinear response. These subcomponents are the main contributors to the overall nonlinear effect.

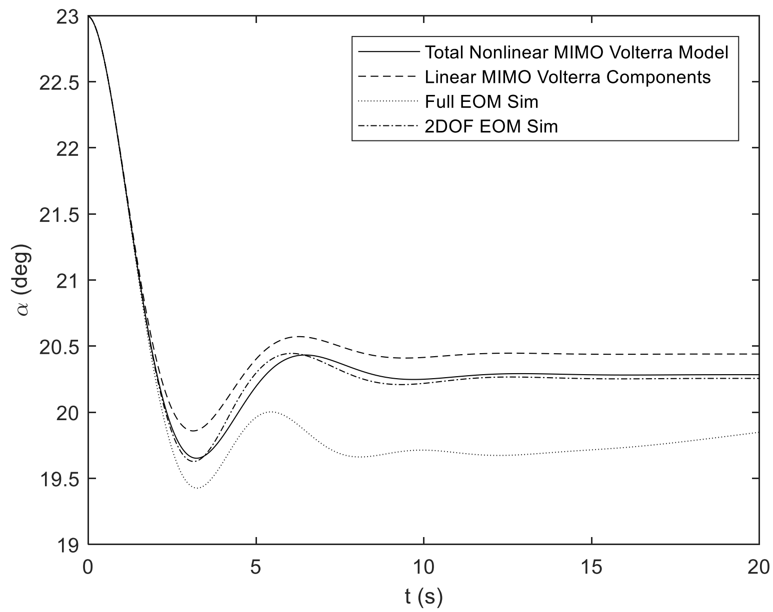

Figure 5.17 Angle of Attack α Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

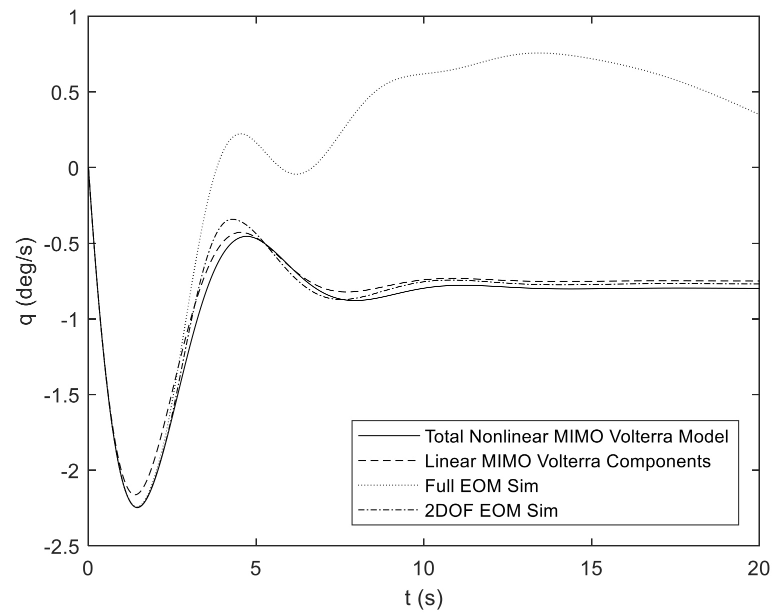

Figure 5.18 Pitch Rate q Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

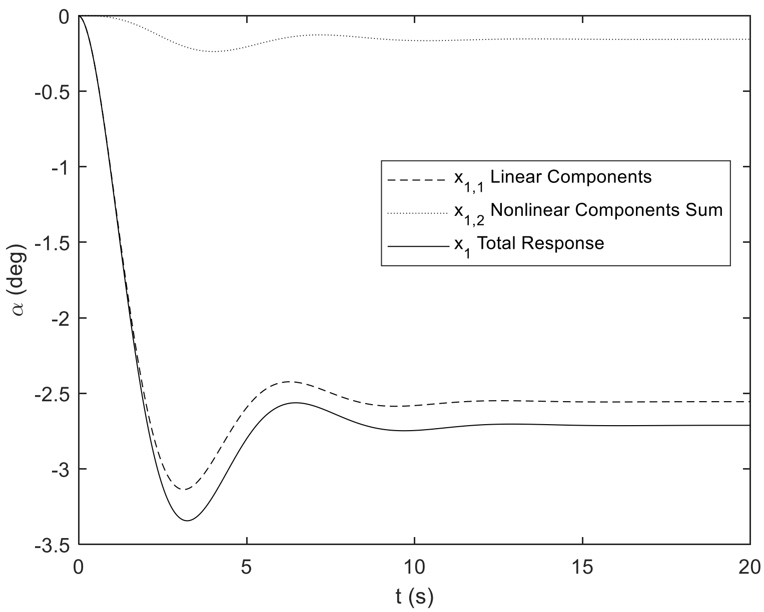

Figure 5.19 MIMO Volterra Model Angle of Attack α Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

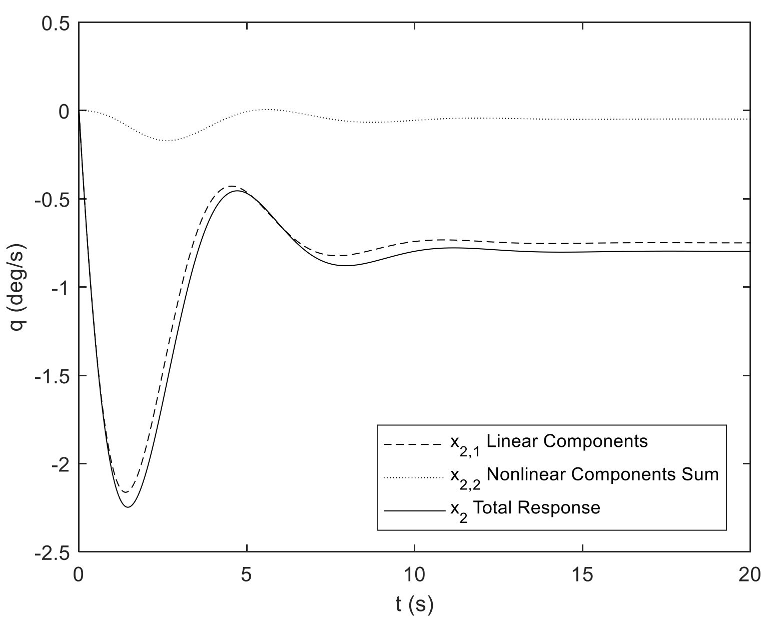

Figure 5.20 MIMO Volterra Model Pitch Rate q Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

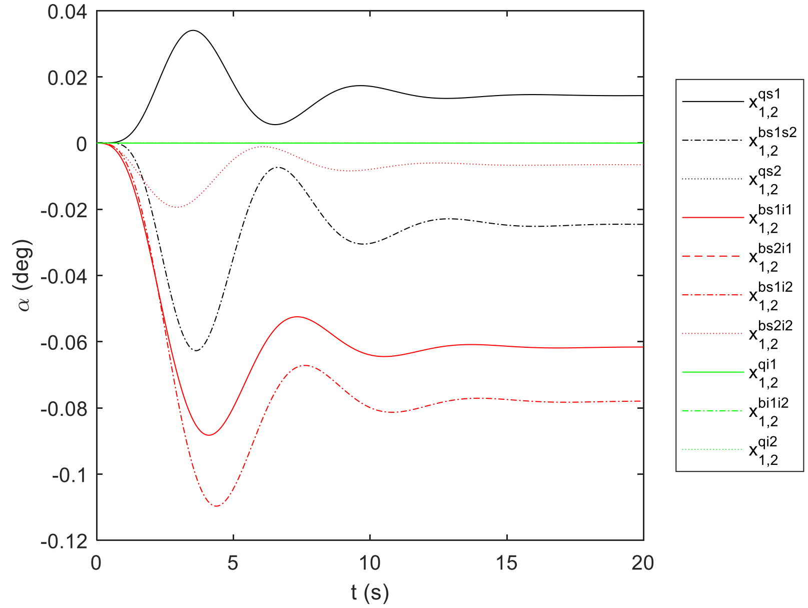

Figure 5.21 Nonlinear Components of Angle of Attack α Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

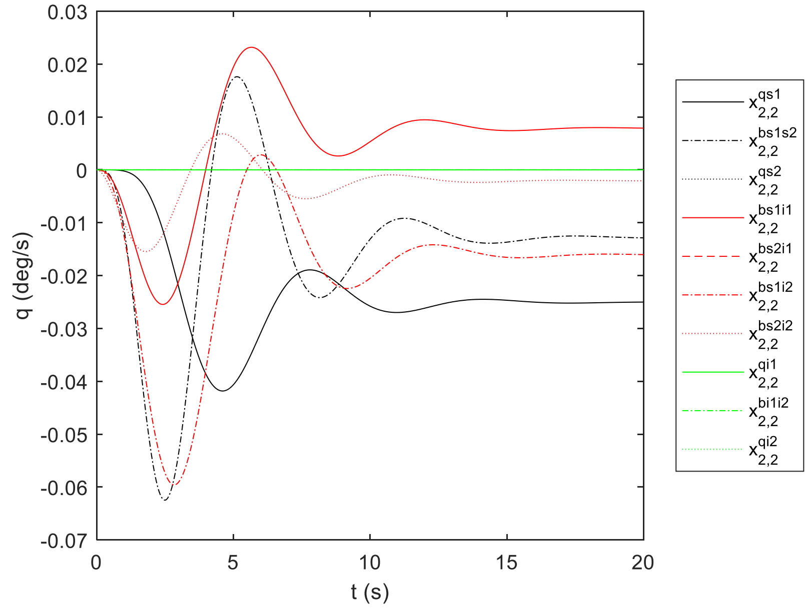

Figure 5.22 Nonlinear Components of Pitch Rate q Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

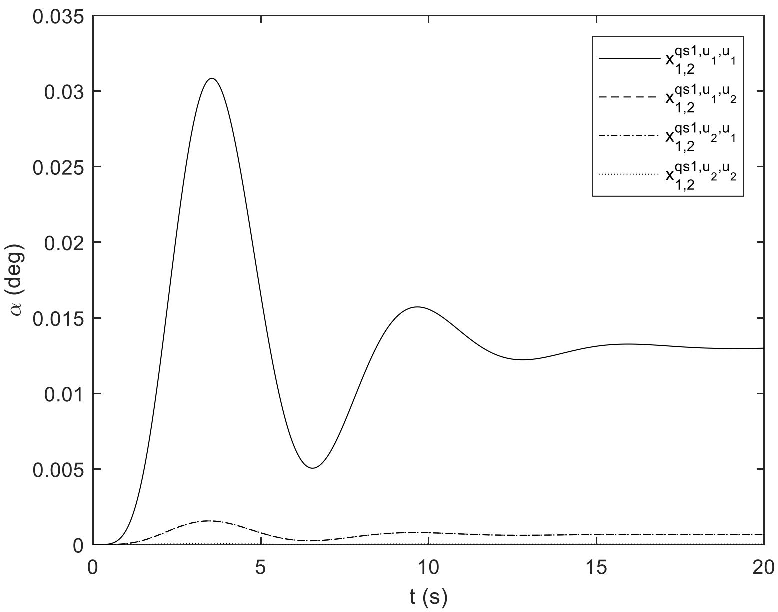

Figure 5.23 Quadratic State 1 Subcomponents of Angle of Attack α Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

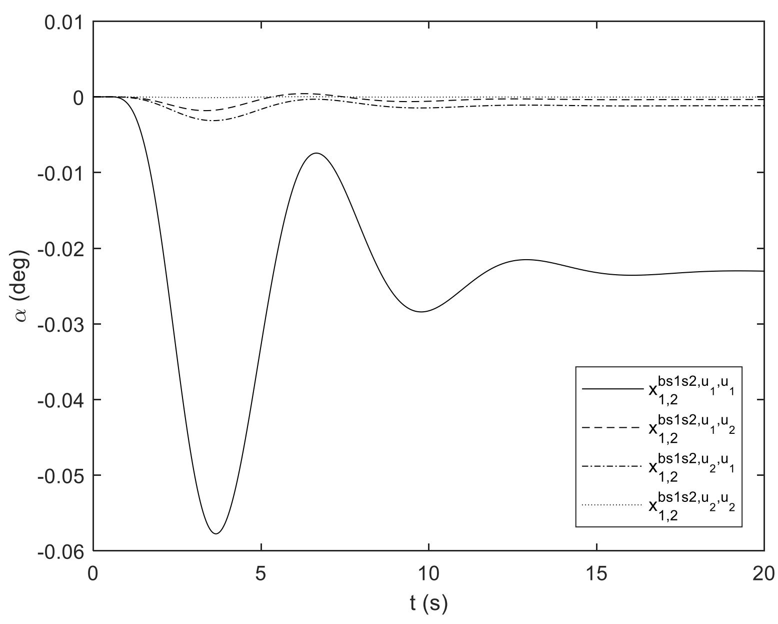

Figure 5.24 Bilinear State 1 - State 2 Subcomponents of Angle of Attack α Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

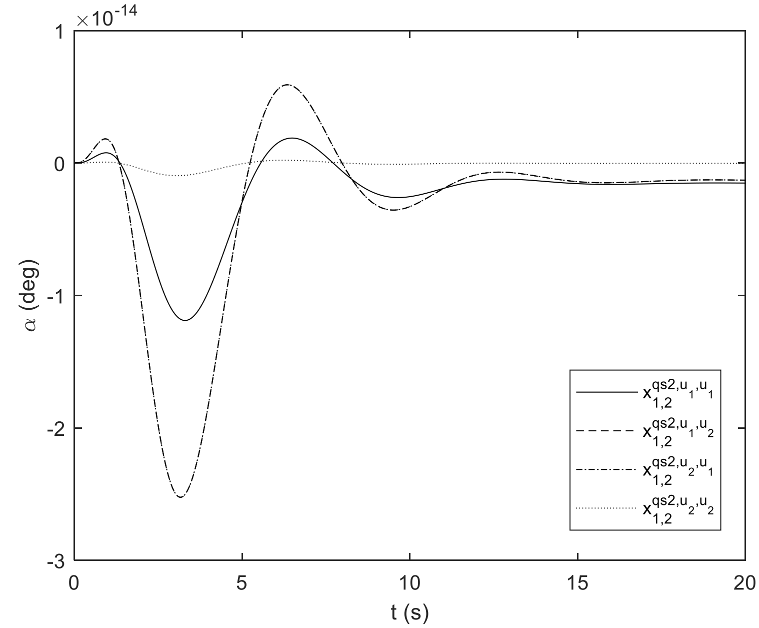

Figure 5.25 Quadratic State 2 Subcomponents of Angle of Attack α Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

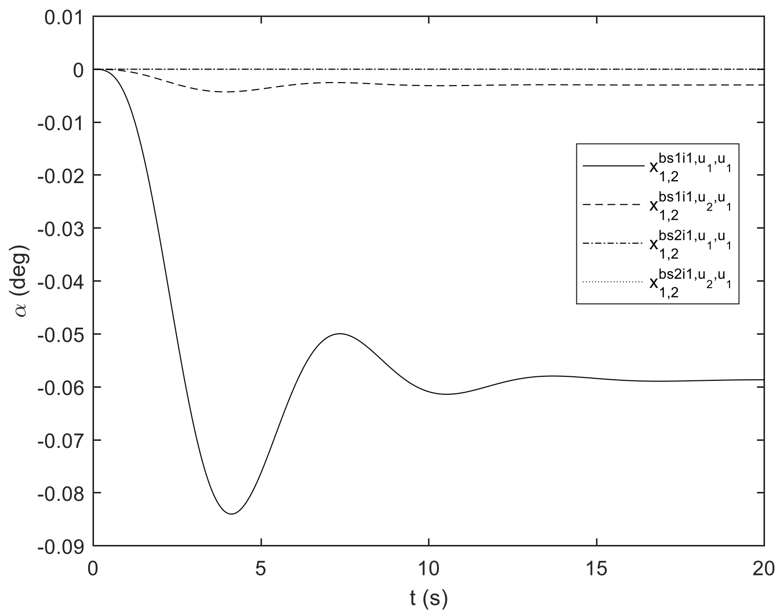

Figure 5.26 Bilinear State – Input 1 Subcomponents of Angle of Attack α Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

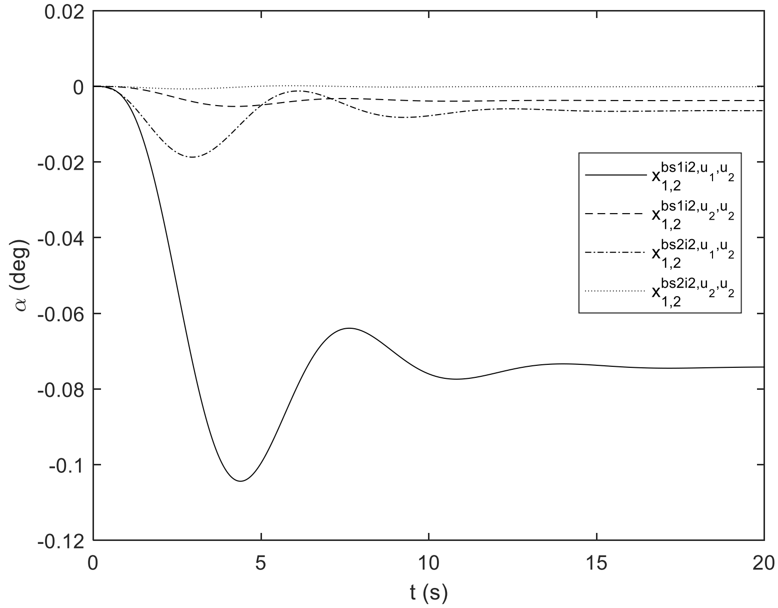

Figure 5.27 Bilinear State – Input 2 Subcomponents of Angle of Attack α Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

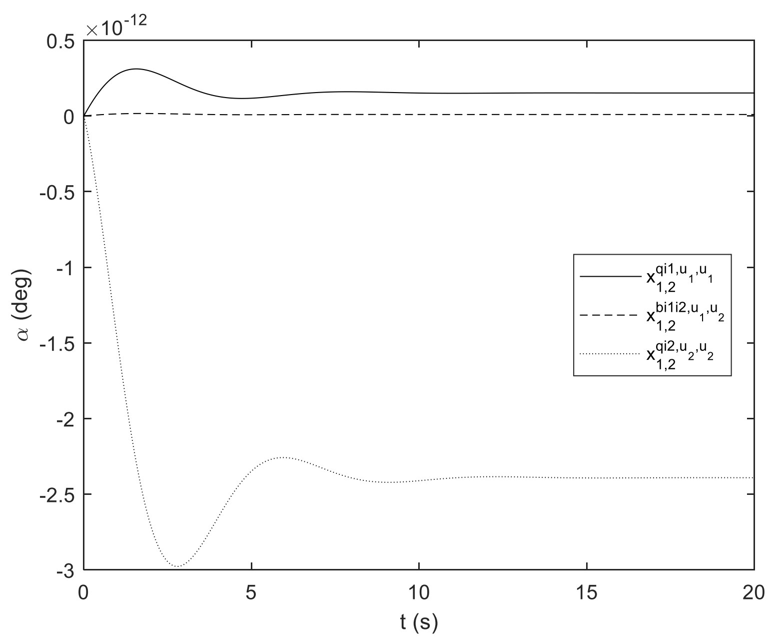

Figure 5.28 Quadratic Input and Bilinear Input Subcomponents of Angle of Attack α Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

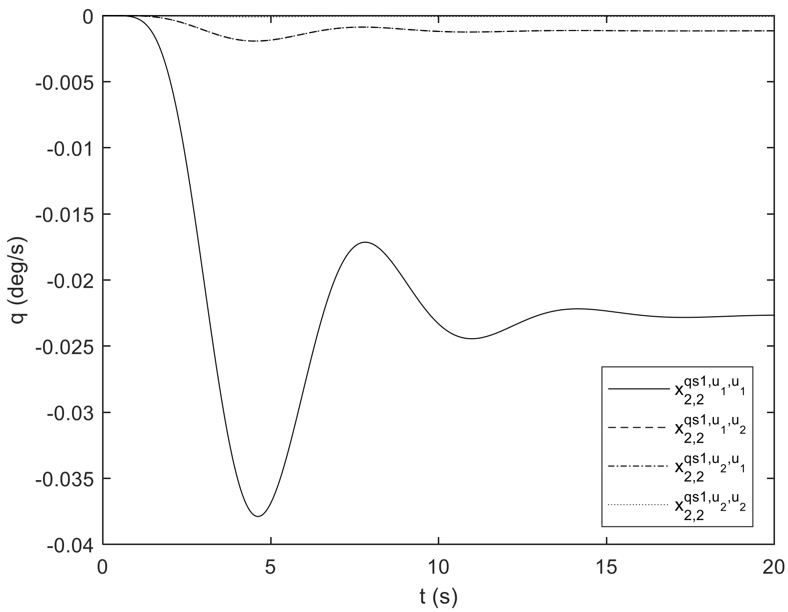

Figure 5.29 Quadratic State 1 Subcomponents of Pitch Rate q Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

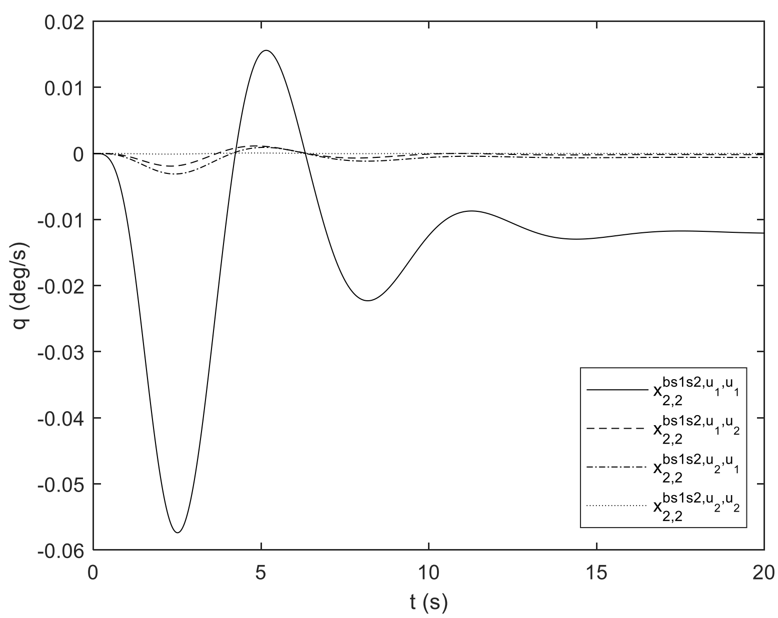

Figure 5.30 Bilinear State 1 – State 2 Subcomponents of Pitch Rate q Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

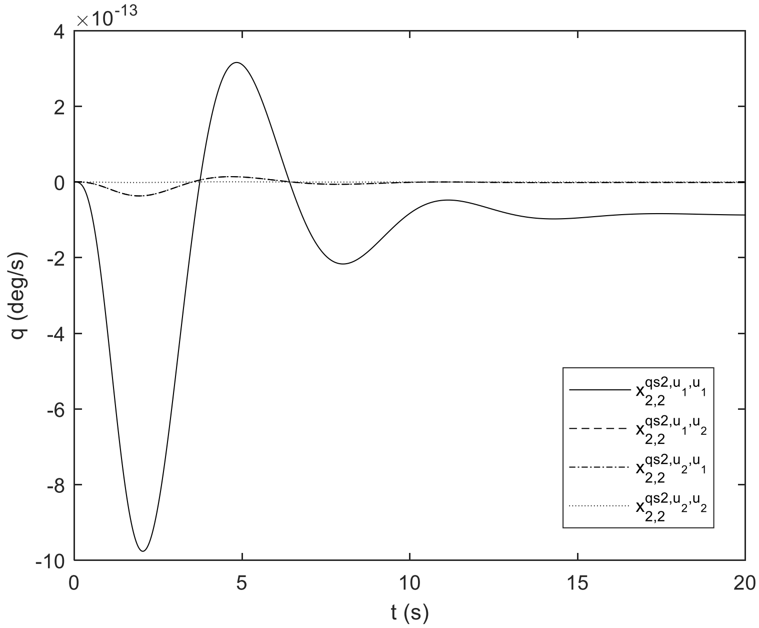

Figure 5.31 Quadratic State 2 Subcomponents of Pitch Rate q Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

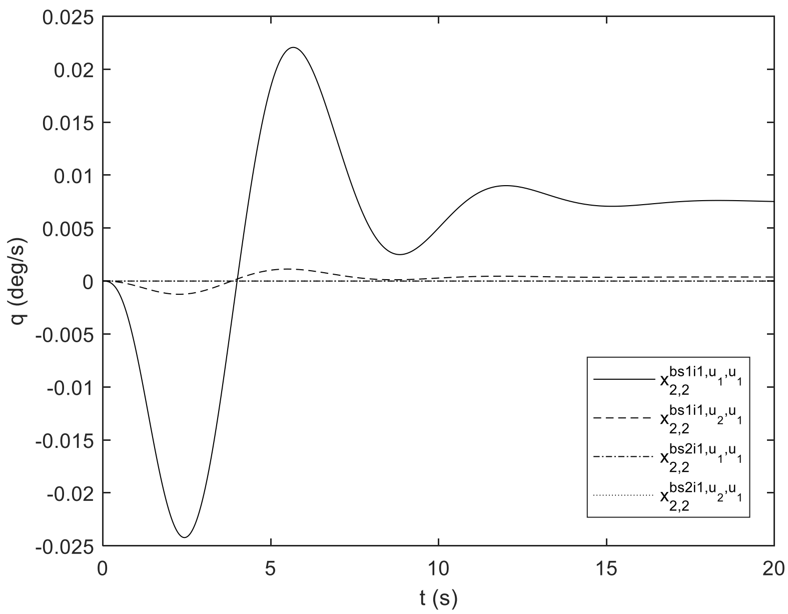

Figure 5.32 Bilinear State - Input 1 Subcomponents of Pitch Rate q Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

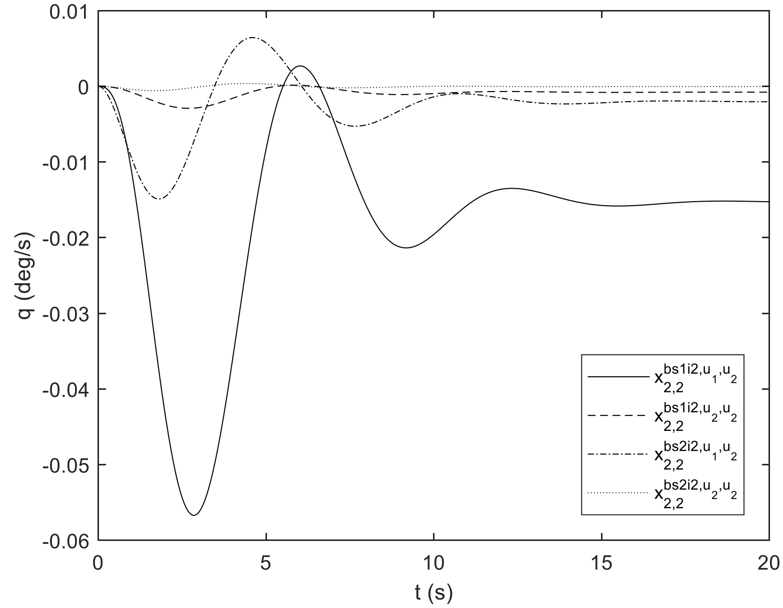

Figure 5.33 Bilinear State - Input 2 Subcomponents of Pitch Rate q Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

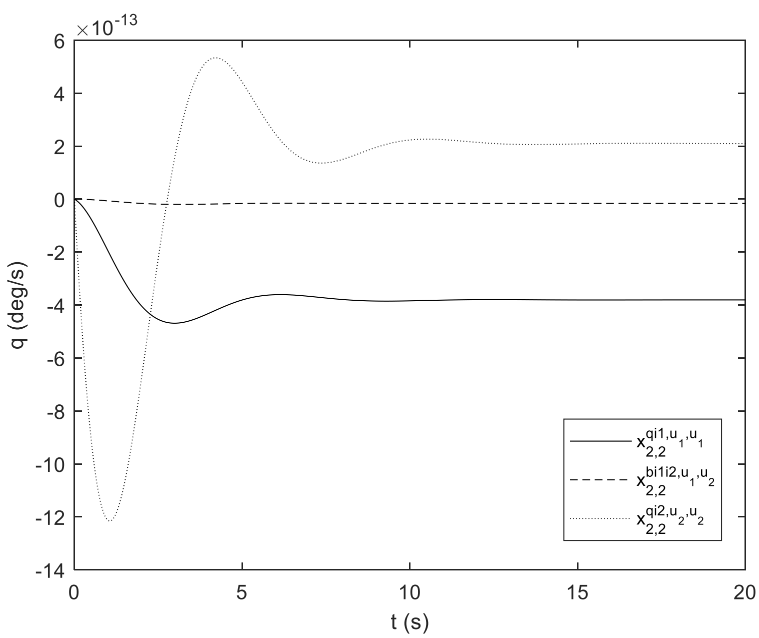

Figure 5.34 Quadratic Input and Bilinear Input Subcomponents of Pitch Rate q Step Response of δ_{hrzt}=+2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

An interesting comparison of the nonlinear terms can be made when an opposite step input is made to the horizontal tail. Using the same initial conditions and Volterra gains from Table 5.2, the step input of the leading edge flap is kept at +2 degrees; however, the opposite input to the horizontal tail is taken at -2 degrees from equilibrium. The angle of attack and pitch rate outputs are shown in Figures 5.35 and 5.36.

As can be seen, the total response and the linear-only response overlap and are nearly identical. The responses track the reduced order model well, and the same phugoid behavior mentioned before is seen in the full order model. The phugoid mode may be the cause of the divergence from the full order model.

Figures 5.37 and 5.38 show the breakdown of the linear and nonlinear total components to the responses. Here, the nonlinear component is small, with almost no contributing effects on the total response. This is shown in the near overlap of the total Volterra response and the linear-only response in Figures 5.35 and 5.36.

However, a further breakdown to the individual nonlinear components shows a different story. Figures 5.39 and 5.40 show the individual nonlinear component responses and show the significant nonlinearities present. The $x_{1,2}^{qs1}$, $x_{1,2}^{bs1s2}$, $x_{1,2}^{bs1i1}$, $x_{1,2}^{bs1i2}$, and $x_{1,2}^{bs2i2}$ components for the angle of attack output and the $x_{2,2}^{qs1}$, $x_{2,2}^{bs1s2}$, $x_{2,2}^{bs1i1}$, $x_{2,2}^{bs1i2}$, and $x_{2,2}^{bs2i2}$ components for the pitch rate output are the largest nonlinearities involved in each. These are the same components as with the positive horizontal tail deflection. When compared to Figures 5.21 and 5.22 for the positive horizontal tail deflection, the bilinear state 1 – input 2 and bilinear state 2 – input 2 terms (that is, the $x_{1,2}^{bs1i2}$, $x_{1,2}^{bs2i2}$, $x_{2,2}^{bs1i2}$ and $x_{2,2}^{bs2i2}$) have a near opposite phase. These components are seen to add to the total nonlinear response in the positive horizontal tail case. In the negative horizontal tail case, the components are now opposite phase and cancel out the effects of the other components. This results in a small overall nonlinear effect. Looking back to Figures 5.35 and 5.36, the nonlinear effects are present but hidden, and the canceling nonlinear components lead to the linear response to track the reduced order model closely.

As with the two positive inputs scenario, the nonlinear components are further investigated by the subcomponents responses with respect to each input. Figures 5.41 through 5.46 show the subcomponents for the angle of attack nonlinear response while Figures 5.47 through 5.52 show the subcomponents for the pitch rate nonlinear response.

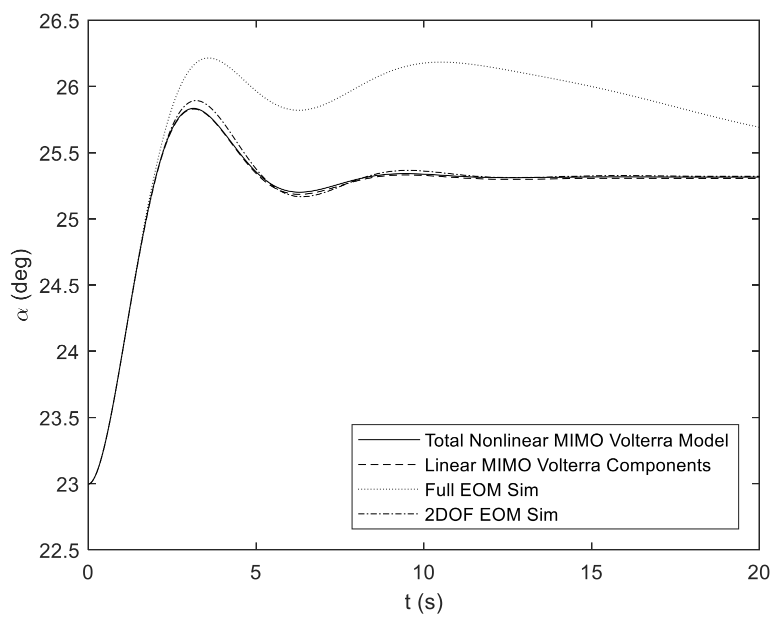

Figure 5.35 Angle of Attack α Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

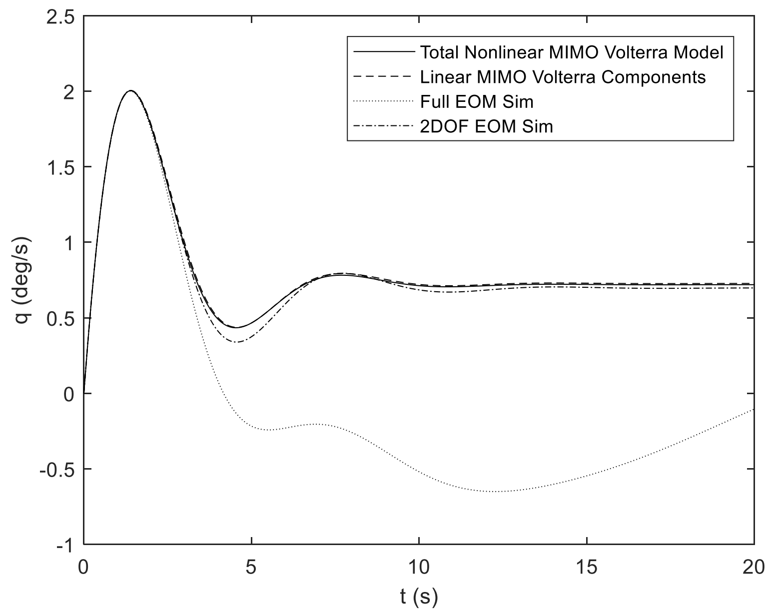

Figure 5.36 Pitch Rate q Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

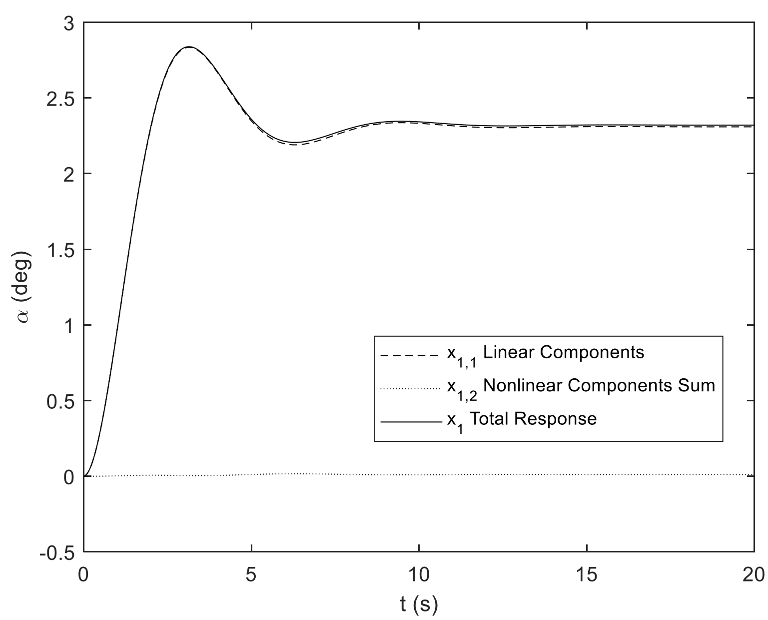

Figure 5.37 MIMO Volterra Model Angle of Attack α Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

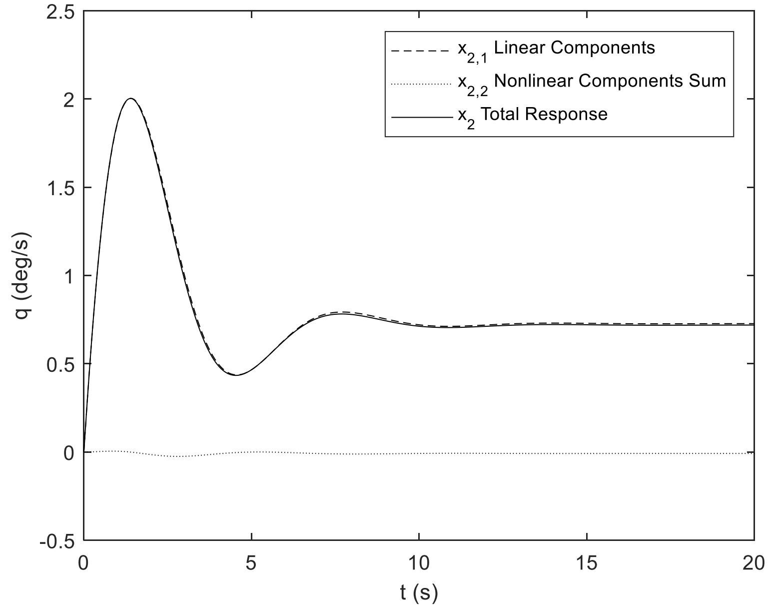

Figure 5.38 MIMO Volterra Model Pitch Rate q Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

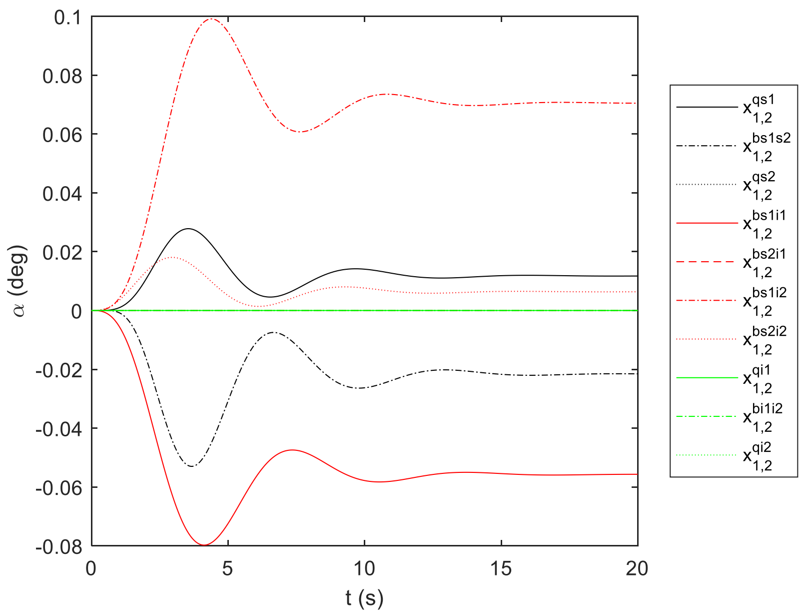

Figure 5.39 Nonlinear Components of Angle of Attack α Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

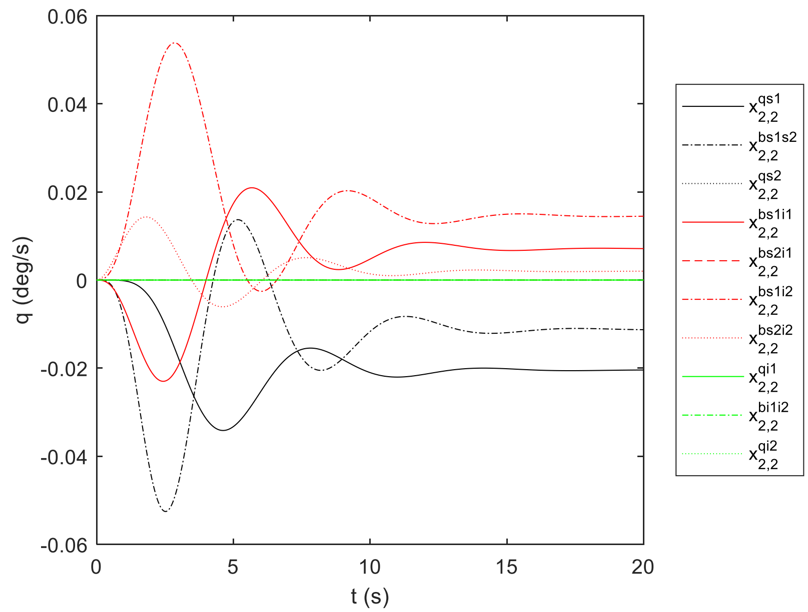

Figure 5.40 Nonlinear Components of Pitch Rate q Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

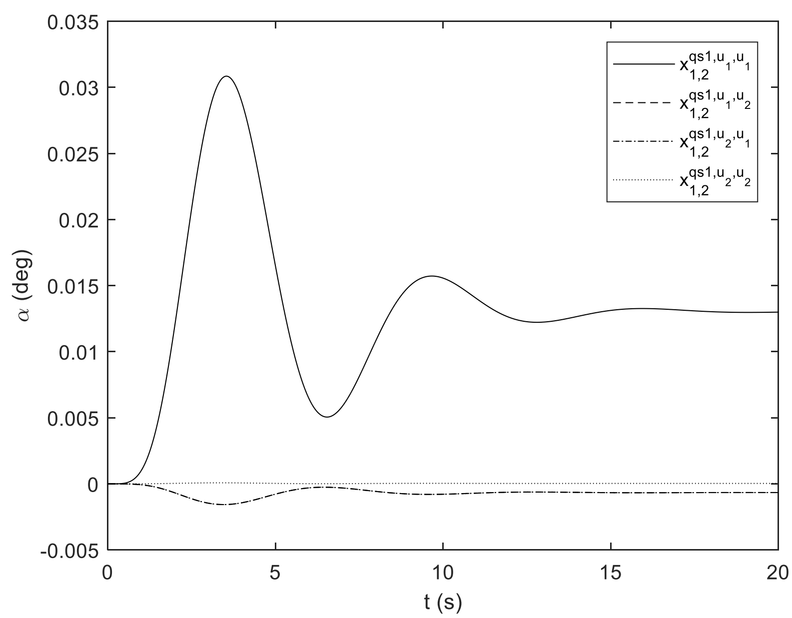

Figure 5.41 Quadratic State 1 Subcomponents of Angle of Attack α Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

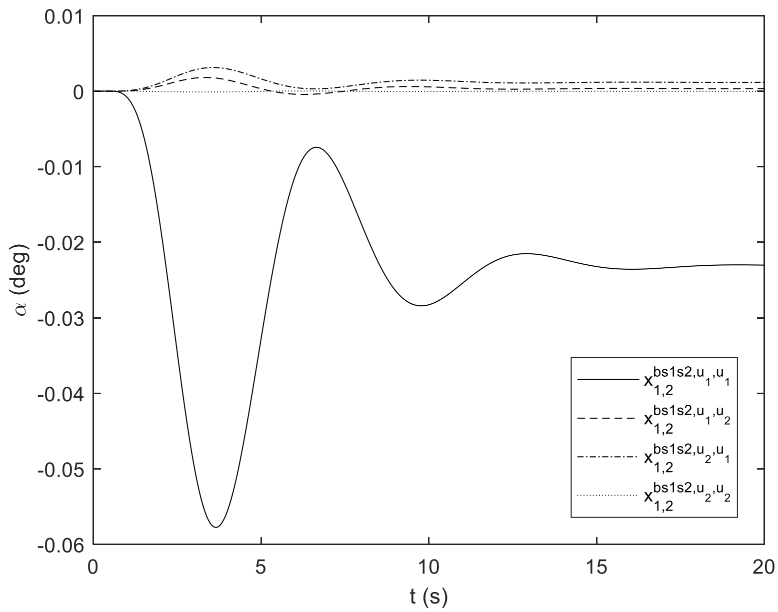

Figure 5.42 Bilinear State 1 – State 2 Subcomponents of Angle of Attack α Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

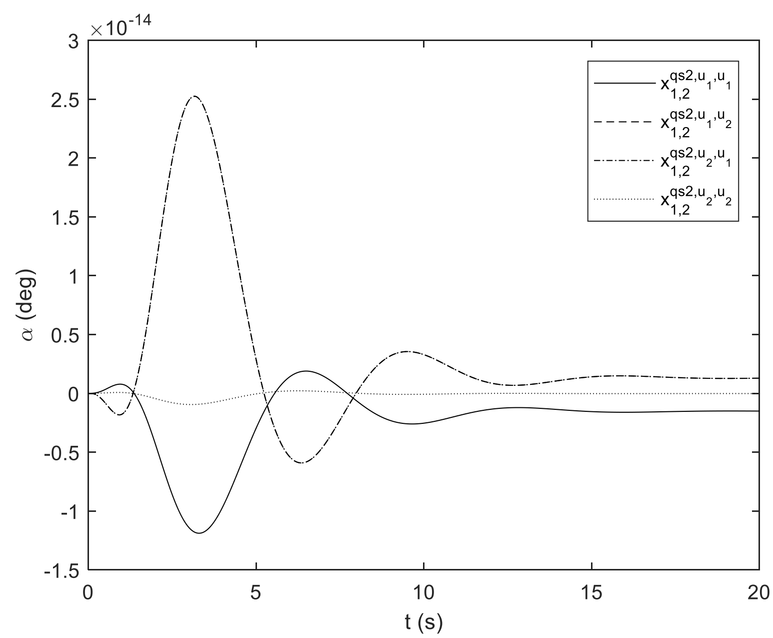

Figure 5.43 Quadratic State 2 Subcomponents of Angle of Attack α Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

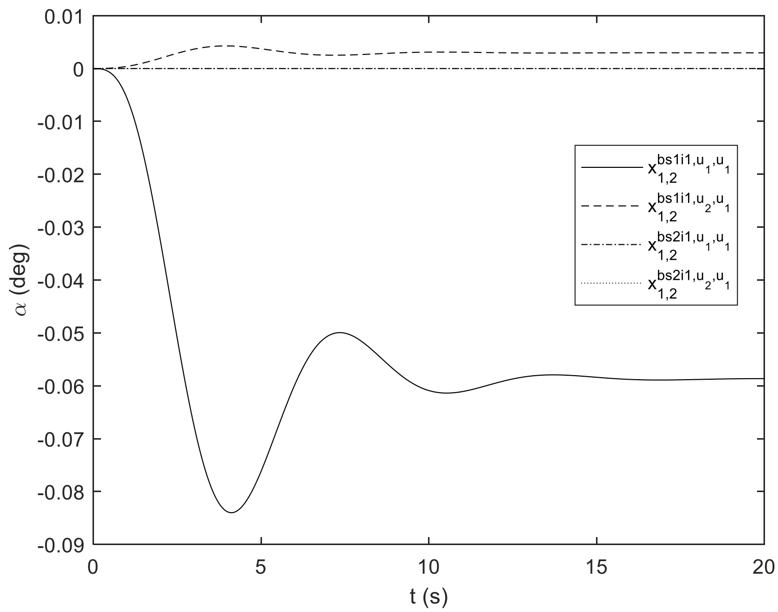

Figure 5.44 Bilinear State – Input 1 Subcomponents of Angle of Attack α Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

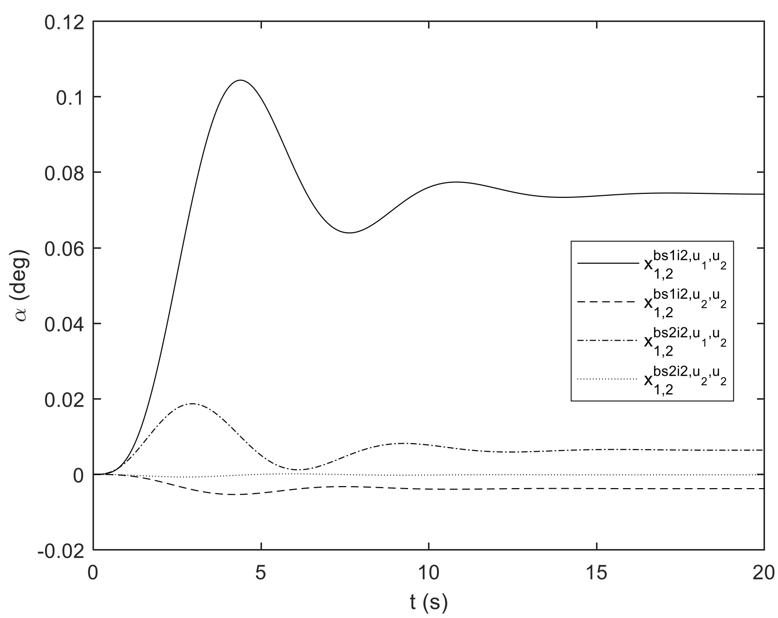

Figure 5.45 Bilinear State – Input 2 Subcomponents of Angle of Attack α Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

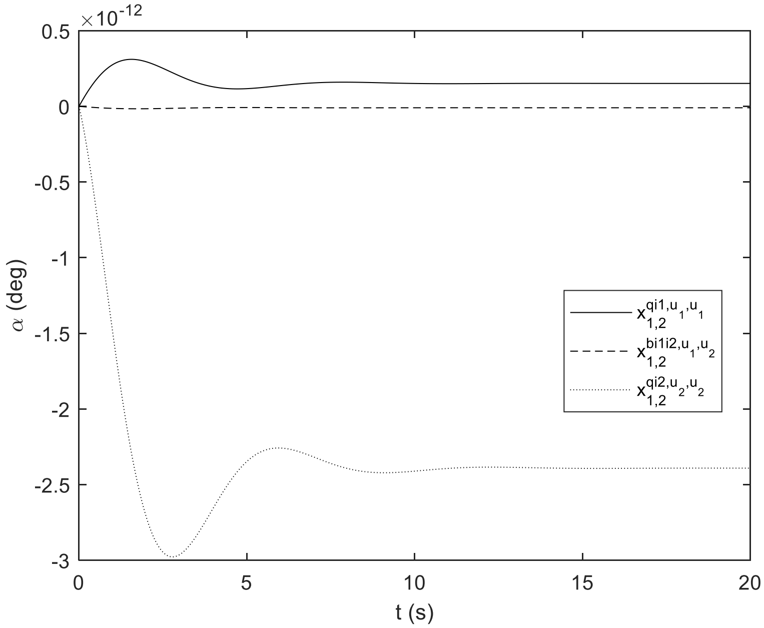

Figure 5.46 Quadratic Input and Bilinear Input Subcomponents of Angle of Attack α Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

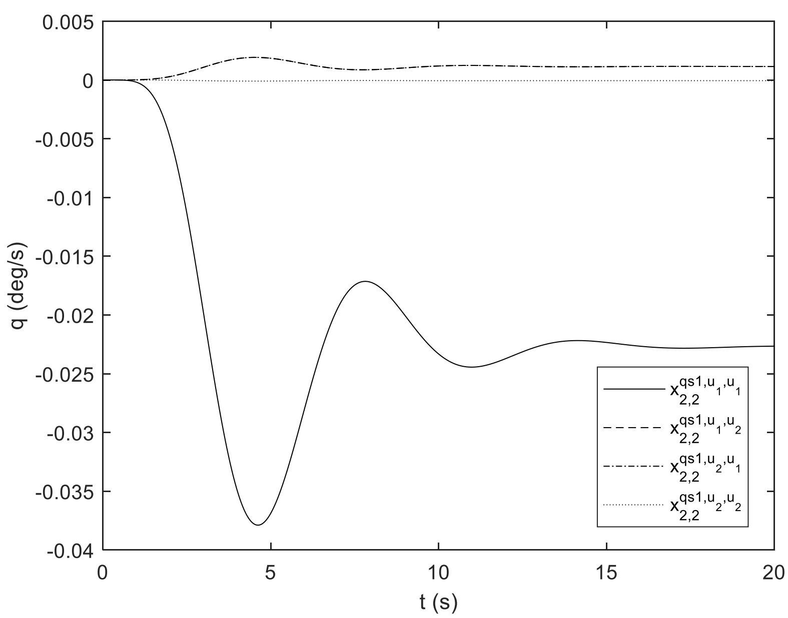

Figure 5.47 Quadratic State 1 Subcomponents of Pitch Rate q Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

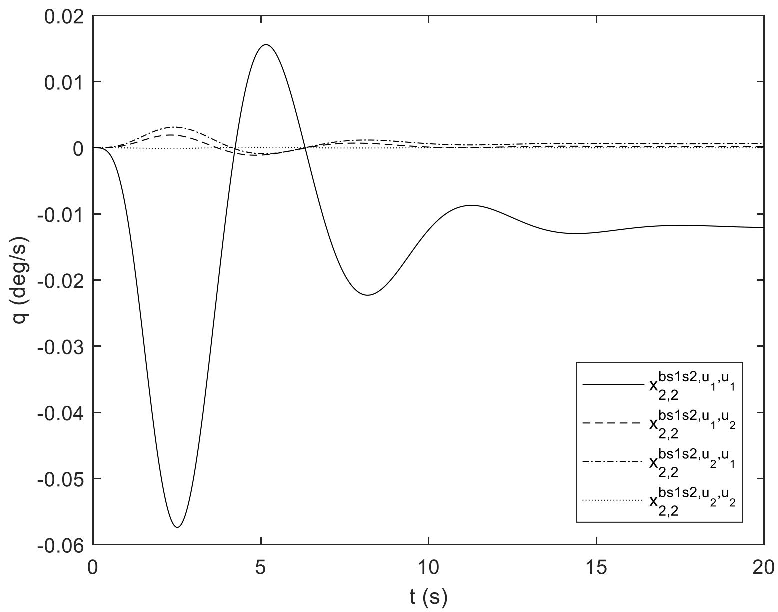

Figure 5.48 Bilinear State 1 – State 2 Subcomponents of Pitch Rate q Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

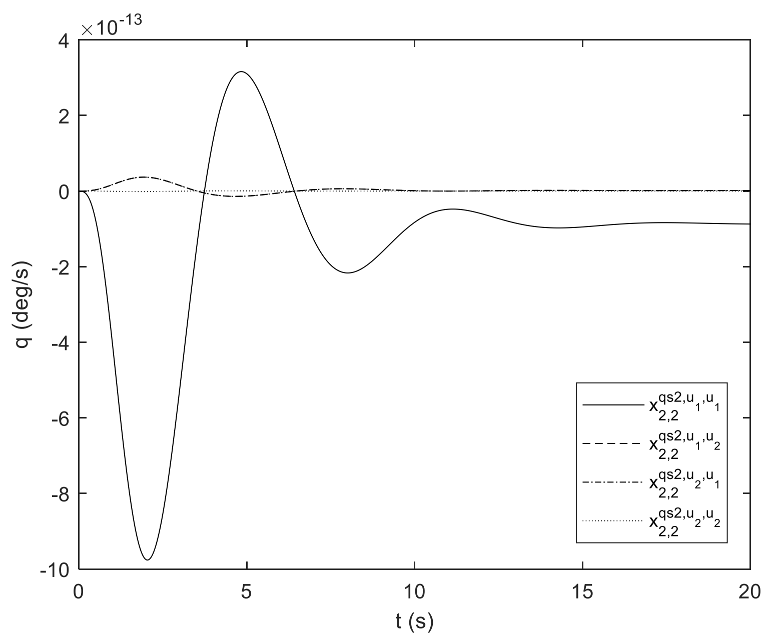

Figure 5.49 Quadratic State 2 Subcomponents of Pitch Rate q Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

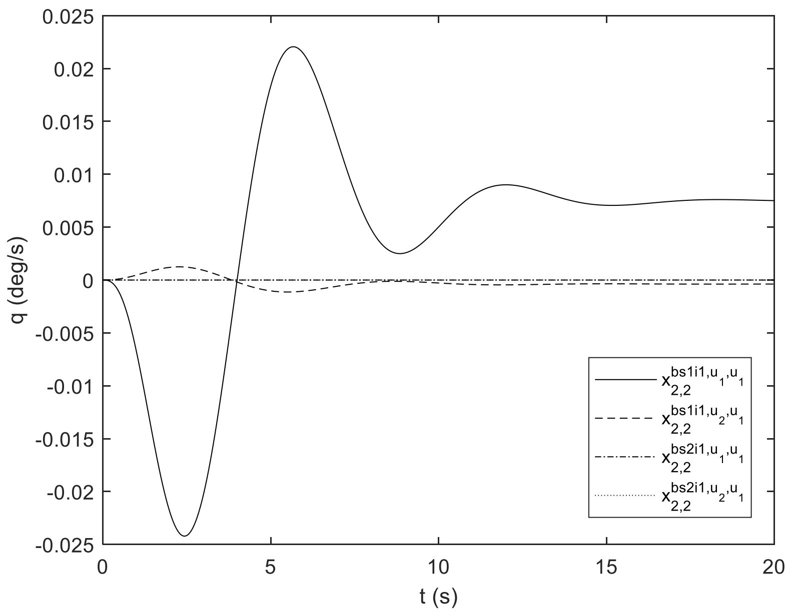

Figure 5.50 Bilinear State – Input 1 Subcomponents of Pitch Rate q Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

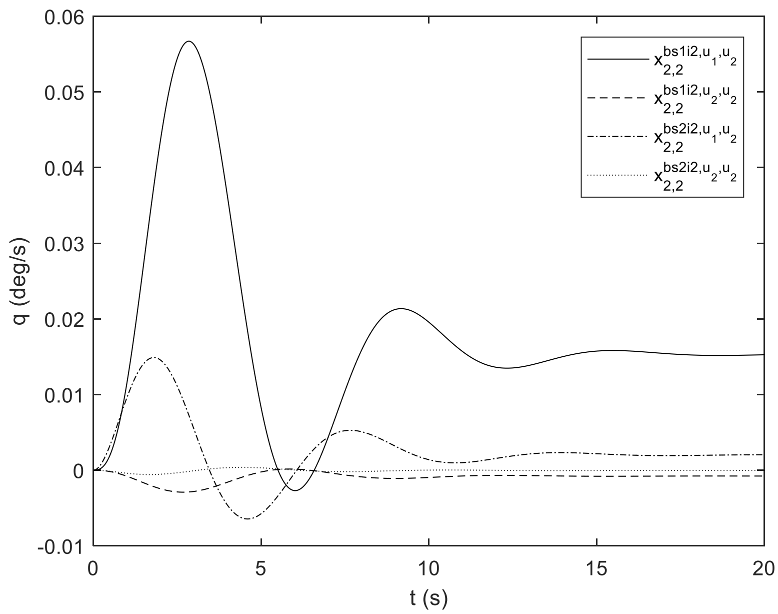

Figure 5.51 Bilinear State – Input 2 Subcomponents of Pitch Rate q Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

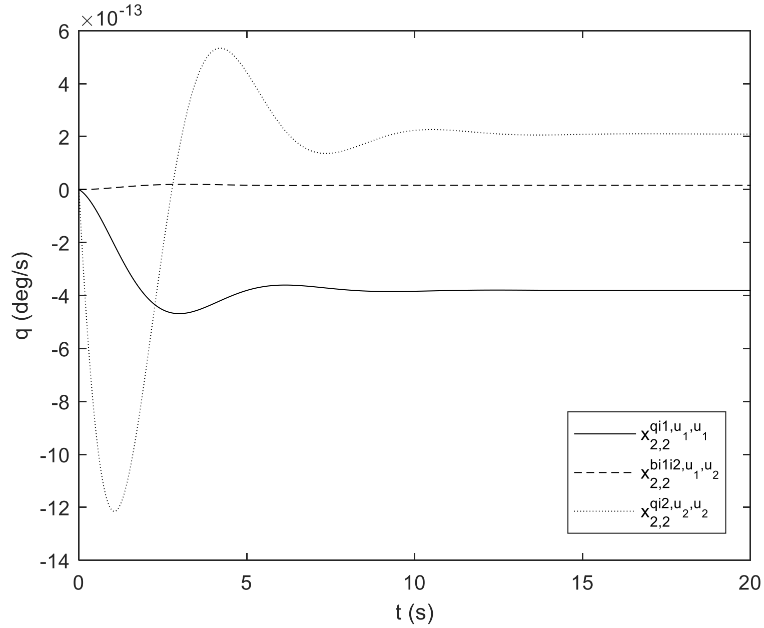

Figure 5.52 Quadratic Input and Bilinear Input Subcomponents of Pitch Rate q Step Response of δ_{hrzt}=-2 deg and δ_{lef}=+2 deg from Equilibrium at V_T=220 ft/s

5.4 Dutch Roll MIMO Volterra Scenario

The dutch roll motion has been described by the reduced order model in Equation (4.19). The MIMO Volterra gains $K_{1000} \cdots L_{0002}$, are initialized around an equilibrium point with δ_{rdr} and δ_{alrn} as inputs. This is described by the multivariable Taylor series expansion as follows

$$\begin{aligned}\Delta\dot{\beta} \approx & \frac{\partial f}{\partial \beta}\Delta\beta + \frac{\partial f}{\partial r}\Delta r + \frac{\partial f}{\partial \delta_{alrn}}\Delta\delta_{alrn} + \frac{\partial f}{\partial \delta_{rdr}}\Delta\delta_{rdr} + \frac{1}{2!}\frac{\partial^2 f}{\partial \beta^2}\Delta\beta^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial r^2}\Delta r^2 \\ & + \frac{1}{2!}\frac{\partial^2 f}{\partial {\delta_{alrn}}^2}\Delta{\delta_{alrn}}^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial {\delta_{rdr}}^2}\Delta{\delta_{rdr}}^2 + \frac{\partial^2 f}{\partial \beta \partial r}\Delta\beta\Delta r + \frac{\partial^2 f}{\partial \beta \partial \delta_{alrn}}\Delta\beta\Delta\delta_{alrn} \\ & + \frac{\partial^2 f}{\partial \beta \partial \delta_{rdr}}\Delta\beta\Delta\delta_{rdr} + \frac{\partial^2 f}{\partial r \partial \delta_{alrn}}\Delta r\Delta\delta_{alrn} + \frac{\partial^2 f}{\partial r \partial \delta_{rdr}}\Delta r\Delta\delta_{rdr} \\ & + \frac{\partial^2 f}{\partial \delta_{rdr} \partial \delta_{alrn}}\Delta\delta_{rdr}\Delta\delta_{alrn} \\ & + \cdots \end{aligned} \tag{5.5}$$

$$\Delta\dot{r} \approx \frac{\partial f}{\partial \beta}\Delta\beta + \frac{\partial f}{\partial r}\Delta r + \frac{\partial f}{\partial \delta_{alrn}}\Delta\delta_{alrn} + \frac{\partial f}{\partial \delta_{rdr}}\Delta\delta_{rdr} + \frac{1}{2!}\frac{\partial^2 f}{\partial \beta^2}\Delta\beta^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial r^2}\Delta r^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial {\delta_{alrn}}^2}\Delta{\delta_{alrn}}^2 + \frac{1}{2!}\frac{\partial^2 f}{\partial {\delta_{rdr}}^2}\Delta{\delta_{rdr}}^2 + \frac{\partial^2 f}{\partial \beta \partial r}\Delta\beta\Delta r + \frac{\partial^2 f}{\partial \beta \partial \delta_{alrn}}\Delta\beta\Delta\delta_{alrn} + \frac{\partial^2 f}{\partial \beta \partial \delta_{rdr}}\Delta\beta\Delta\delta_{rdr} + \frac{\partial^2 f}{\partial r \partial \delta_{alrn}}\Delta r\Delta\delta_{alrn} + \frac{\partial^2 f}{\partial r \partial \delta_{rdr}}\Delta r\Delta\delta_{rdr} + \frac{\partial^2 f}{\partial \delta_{rdr} \partial \delta_{alrn}}\Delta\delta_{rdr}\Delta\delta_{alrn} + \cdots \tag{5.6}$$

The partial derivatives correspond to the Volterra model gains $K_{1000} \cdots L_{0002}$ based on the state and/or input of each of the terms (e.g. $\Delta\beta\Delta\delta_{rdr}$ term for $\dot{\beta}$ corresponds to K_{1001}, for state 1 – input 2). Again, like the short period motion, they were calculated by the central finite difference method utilizing the experimental data and the reduced order model equations.

Various scenarios were run in order to discover significant nonlinearities; however, none could be found. The F-16 data was found to be highly linear. Figures 5.53 and 5.54 show a plot of the aerodynamic C_y data and aerodynamic coefficient C_n data across the angle of attack and sideslip angle range. The coefficient values do not contain significant variations along the beta equals zero centerline; they are roughly flat. There are significant curvatures seen in the C_n data when the sideslip angle is near thirty degrees. However, the rectilinear, steady, wings-level, equilibrium flight condition is not satisfied when β is greater than zero.

An attempt was made to find another equilibrium flight condition of steady turning flight with the wings banked. This condition has the aircraft banked ($\phi \neq 0$) in a steady turn. It allows the sideslip angle, the aileron deflection, and the rudder deflection to be solved so the necessary state derivatives are zero. Investigating this equilibrium condition led to another issue; that is, a rudder deflection greater than the maximum thirty degrees was necessary to reach the nonlinearities seen in the aerodynamic data. These nonlinearities are reached naturally when aircraft maneuvers are performed; however, an equilibrium condition for which the MIMO Volterra model could be used for perturbation study proved elusive.

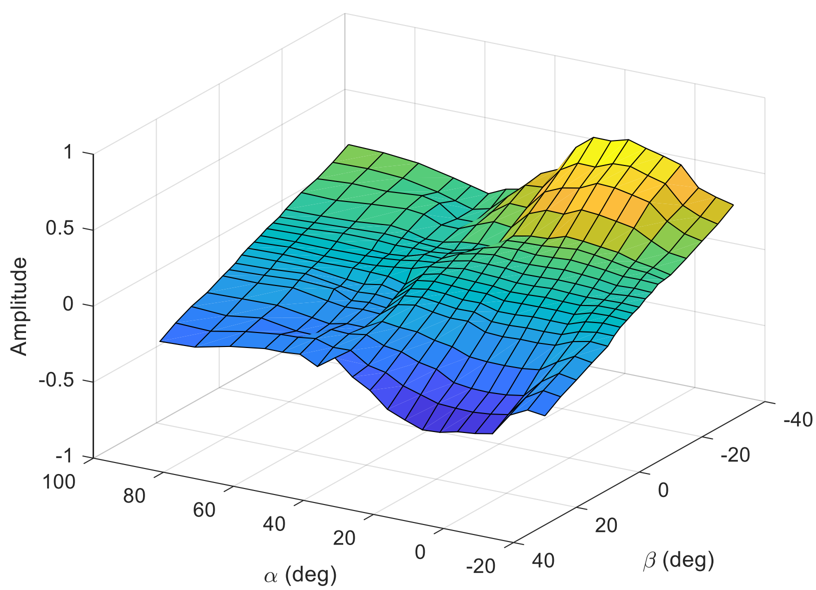

Figure 5.53 Aerodynamic Data for $C_y(\alpha, \beta, \delta_{rdr})$

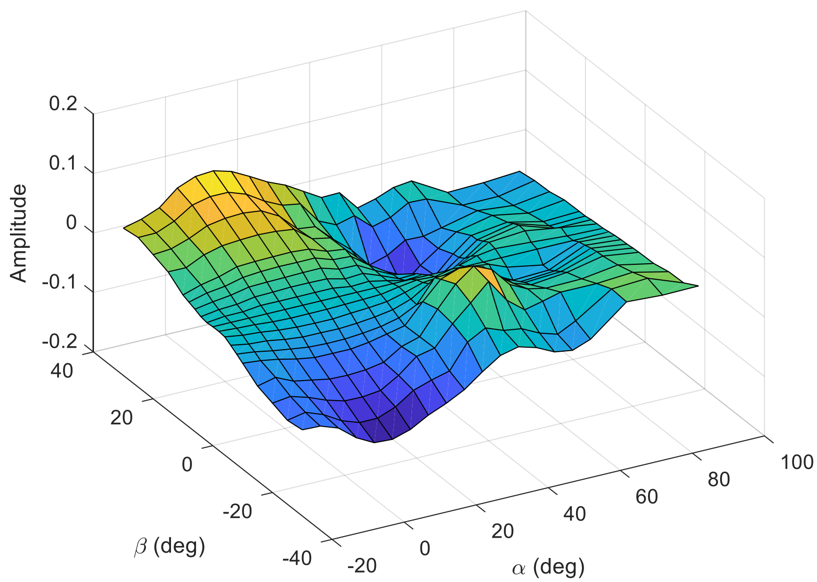

Figure 5.54 Aerodynamic Data for $C_n(\alpha, \beta, \delta_{hrzt}=0)$

5.4.1 One Input Dutch Roll Analysis

The gains in Table 5.3 were calculated based on rectilinear, wings-level flight at an altitude of 50,000 feet, a total velocity of 1,673.7 feet per second, an angle of attack and pitch angle of 0.68 degrees, a horizontal tail deflection of -1.22 degrees, a thrust power of 0.99, and a center of mass at the nominal placement of 0.3. Note, the parameters K_{abcd} and L_{abcd} have units dependent on the two states, sideslip angle (radians) and yaw rate (radians per second), and the two inputs, aileron deflection (radians) and rudder deflection (radians). For example, K_{1001} has the units of 1/rad.

Figures 5.55 and 5.56 show the calculated responses for the sideslip angle and yaw rate given the gains in Table 5.3. The reduced order model provides a decent approximation of the full order model. The sideslip angle response follows closely, while the yaw rate response follows closely for about seven seconds. At that point, the full order model yaw rate response starts to follow an oscillatory exponential drift, which the reduced order model does not capture. The reduced order approximation is accurate due to the low angle of attack and high total velocity of the initial conditions. The reduced order model deviates significantly when the total velocity is reduced. At slower speeds and higher angle of attack, the bank angle and roll rate become more of a significant driver to the overall latitudinal response. Looking at the deviation of the full order model yaw rate response in Figure 5.56, the deviation is due to the increasing angle of attack, bank angle, and roll rate coupling inherent to the full order state equations.

The MIMO Volterra model follows the reduced order simulation well. It appears to match phasing better than the linear response, although the amplitude of the oscillations remains slightly more accurate in the linear response. However, looking at the sideslip angle responses, the MIMO Volterra response appears to settle to the steady state of the reduced order simulation more accurately. The linear response appears to settle slightly higher than the reduced order simulation.

Table 5.3 Dutch Roll MIMO Volterra Parameter Values for β, r, δ_{alrn}, and δ_{rdr} at V_T=1,673.7 ft/s

Gain Parameter	**Value**	**Corresponding Matrix**
K_{1000}	-1.54e-01	A
K_{0100}	-9.99e-01	A
L_{1000}	1.40e+01	A
L_{0100}	-2.52e-01	A
K_{0010}	8.85e-03	B
K_{0001}	2.38e-02	B
L_{0010}	-3.19e+00	B
L_{0001}	-6.29e+00	B
K_{2000}	1.57e+00	B_{qs}
L_{2000}	5.58e+02	B_{qs}
K_{0200}	-1.17e-08	B_{qs}
L_{0200}	8.90e-15	B_{qs}
K_{1100}	0	B_{bsbi}
L_{1100}	0	B_{bsbi}
K_{0011}	0	B_{bsbi}
L_{0011}	-2.85e-13	B_{bsbi}
K_{1010}	2.27e-03	B_{bsi1}
L_{1010}	4.44e+00	B_{bsi1}
K_{0110}	0	B_{bsi1}
L_{0110}	0	B_{bsi1}
K_{1001}	1.99e-02	B_{bsi2}
L_{1001}	1.75e+00	B_{bsi2}
K_{0101}	0	B_{bsi2}
L_{0101}	-1.42e-13	B_{bsi2}
K_{0020}	-1.17e-08	B_{qi1}
L_{0020}	1.90e-13	B_{qi1}
K_{0002}	-1.17e-08	B_{qi2}
L_{0002}	1.90e-13	B_{qi2}

*Units for K_{abcd} and L_{abcd} are dependent on β, r, δ_{alrn}, and δ_{rdr}

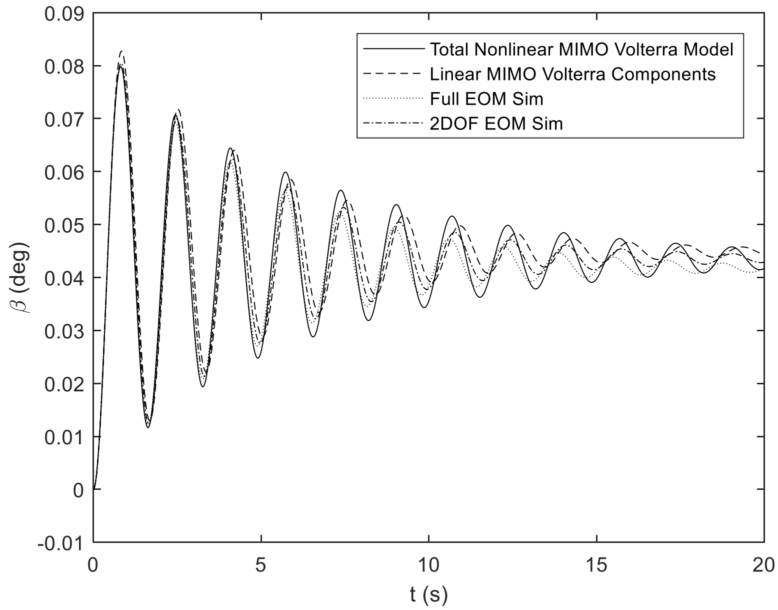

Figure 5.55 Sideslip Angle β Step Response of δ_{rdr}=+0.1 deg from Equilibrium at V_T=1,673.7 ft/s

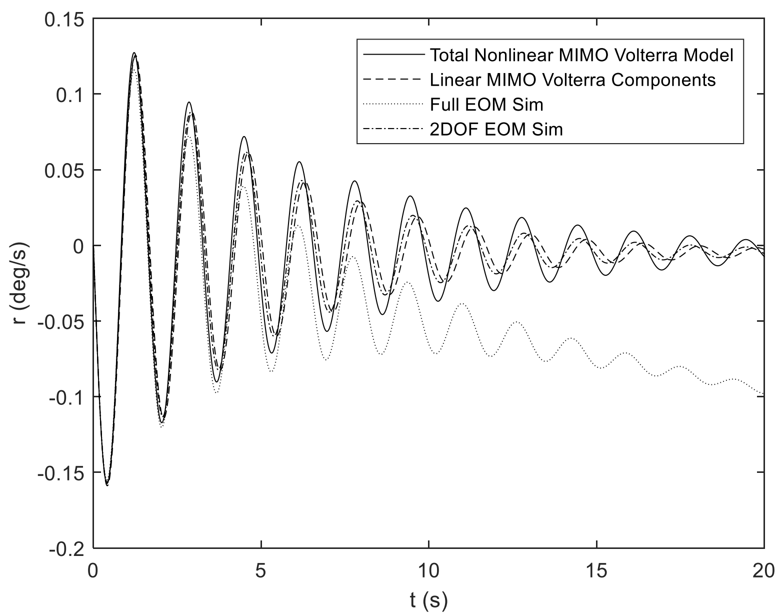

Figure 5.56 Yaw Rate r Step Response of δ_{rdr}=+0.1 deg from Equilibrium at V_T=1,673.7 ft/s

As provided previously with the short period study, Figures 5.57 and 5.58 break down the sideslip angle and yaw rate into their linear and total nonlinear components, respectively. Here, the total summation of the nonlinear components shows the overall impact to the total response. The strength of the nonlinearity can now be measured, which is a capability that cannot be extracted from numerical simulation.

Figures 5.59 and 5.60 break down the nonlinear components further, into the quadratic state components, bilinear state-input components, etc., for the sideslip angle and yaw rate respectively. It is shown here that the strongest nonlinear component is the quadratic state 1 component, or the second derivative effect of the change in angle of attack. The quadratic state 1 component can be further broken down into its subcomponents, representing the terms with respect to each of the inputs. These are shown in Figures 5.61 and 5.62 for the sideslip angle and yaw rate respectively. These plots show the capability of the MIMO Volterra model to capture the responses of individual nonlinear components, and their overall effects on the total system response. This is a capability that is absent from numerical simulation.

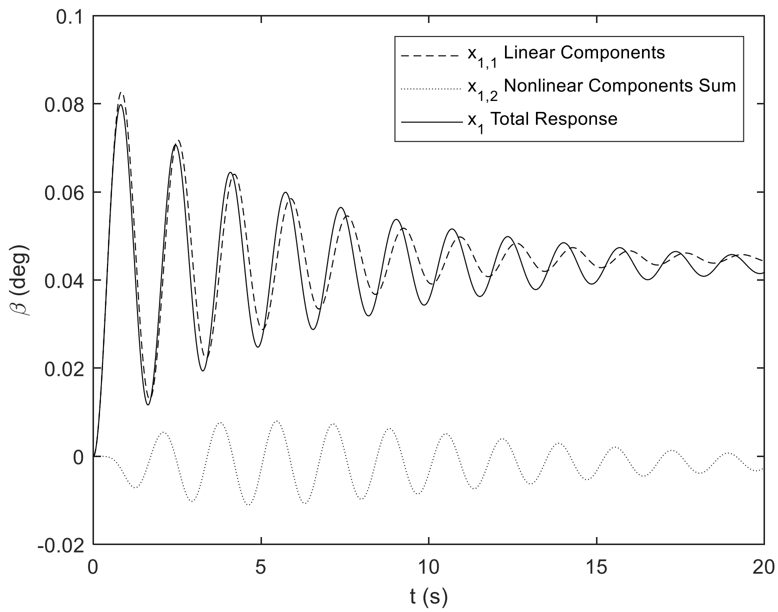

Figure 5.57 Linear, Nonlinear, and Total Sideslip Angle β Step Response of δ_{rdr}=+0.1 deg from Equilibrium at V_T=1,673.7 ft/s

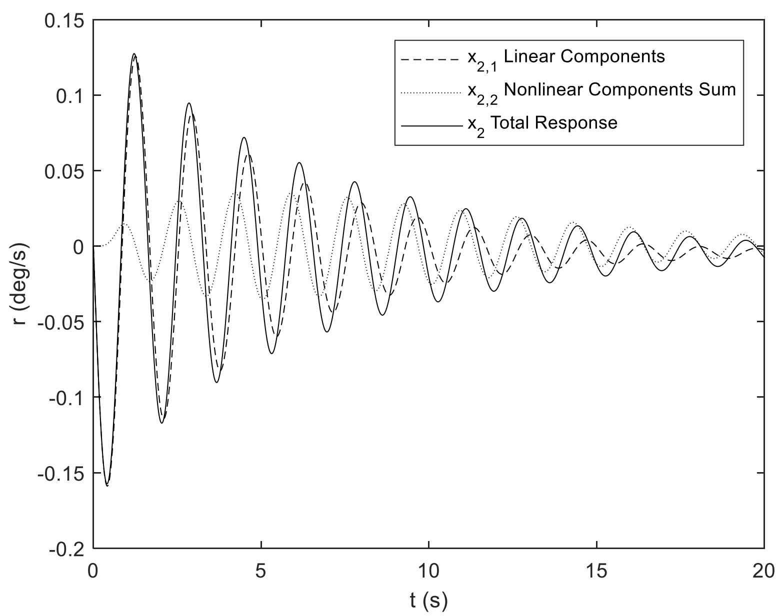

Figure 5.58 Linear, Nonlinear, and Total Yaw Rate r Step Response of δ_{rdr}=+0.1 deg from Equilibrium at V_T=1,673.7 ft/s

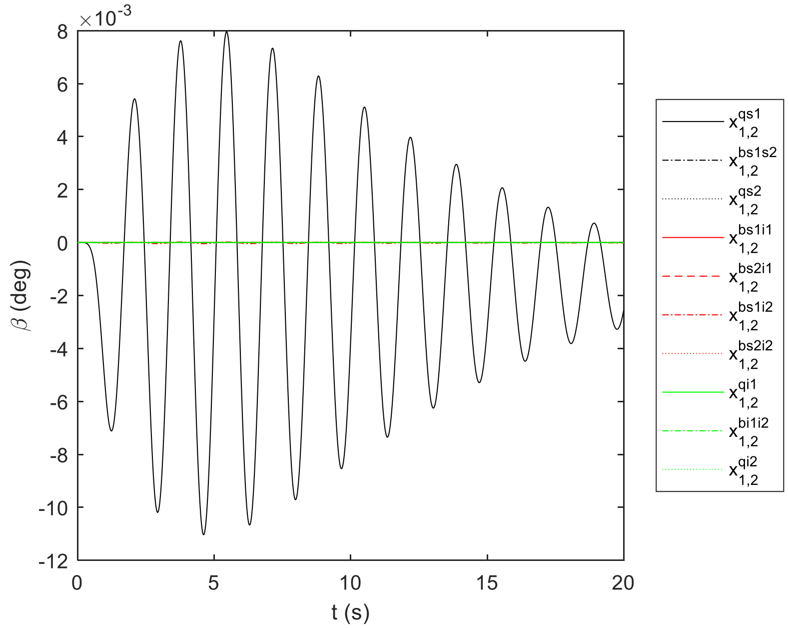

Figure 5.59 Nonlinear Components of Sideslip Angle β Step Response of δ_{rdr}=+0.1 deg from Equilibrium at V_T=1,673.7 ft/s

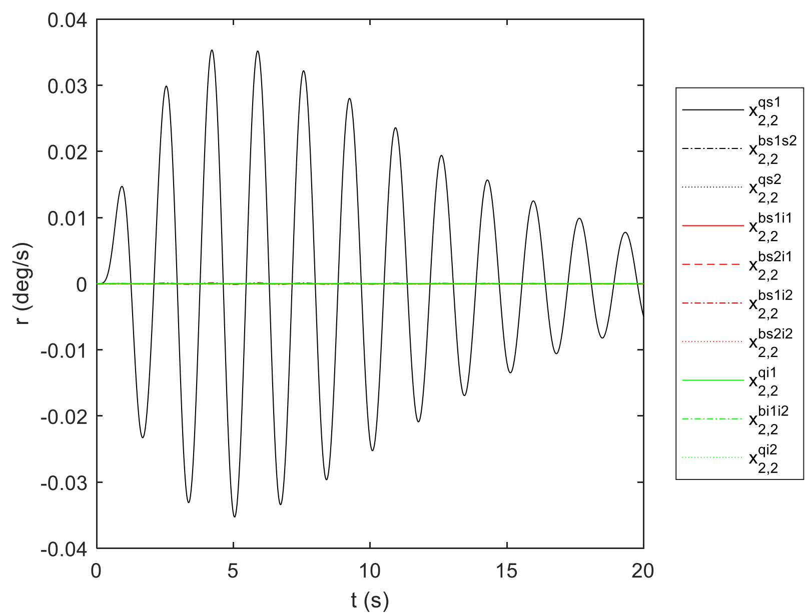

Figure 5.60 Nonlinear Components of Yaw Rate r Step Response of δ_{rdr}=+0.1 deg from Equilibrium at V_T=1,673.7 ft/s

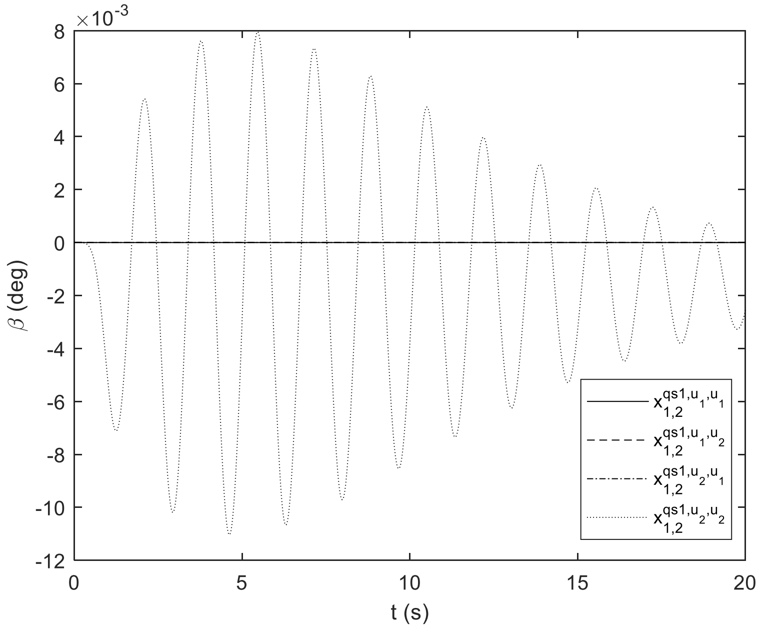

Figure 5.61 Quadratic State 1 Subcomponents of Sideslip Angle β Step Response of δ_{rdr}=+0.1 deg from Equilibrium at V_T=1,673.7 ft/s

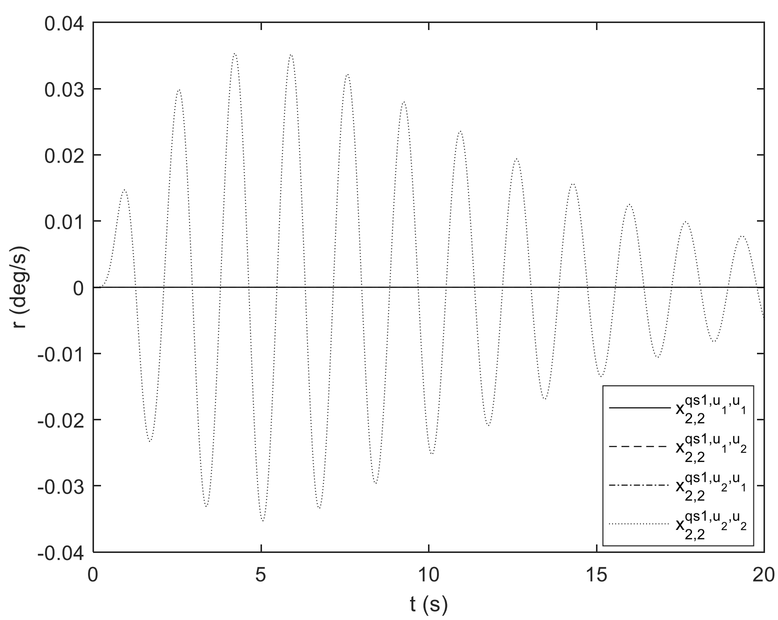

Figure 5.62 Quadratic State 1 Subcomponents of Yaw Rate r Step Response of δ_{rdr}=+0.1 deg from Equilibrium at V_T=1,673.7 ft/s

5.4.2 Two Input Dutch Roll Analysis

As previously shown for the short period motion, assumptions made in the derivation of the dutch roll reduced order model lead to inaccuracies when trying to incorporate the second input. Results in this section explain the constraints when incorporating two inputs into the system response: the rudder deflection δ_{rdr} and the aileron deflection, δ_{alrn}.

Upon inspection of Equation (4.19), both the aileron and rudder inputs are captured in the C_y, C_l, and C_n aerodynamic coefficients present in the $\dot{\beta}$ and $\dot{r}$ equations; therefore, changes to both of these inputs are incorporated into the reduced order model. However, the approximations of constraining both the bank angle derivative and the roll rate derivative to zero means that the coupling of the change in aileron to these additional degrees of freedom are not captured. Thus, this limits the accuracy of the reduced order model and MIMO Volterra model.

While these issues are known, Figures 5.63 and 5.64 show the sideslip angle and yaw rate responses to a change in both the aileron input δ_{alrn} and the rudder input δ_{rdr}. The system is again initialized using the initial conditions and Volterra gains from Table 5.3.

The MIMO Volterra model is able to handle the two-input, two-output scenario. The responses deviate from the full order model due to the constraints noted earlier. Note, that this scenario used the gains provided previously. Further investigation will be necessary in order to set up a latitudinal two-input, two-output reduced order model to better handle a change in both latitudinal inputs, δ_{alrn} and δ_{rdr}.

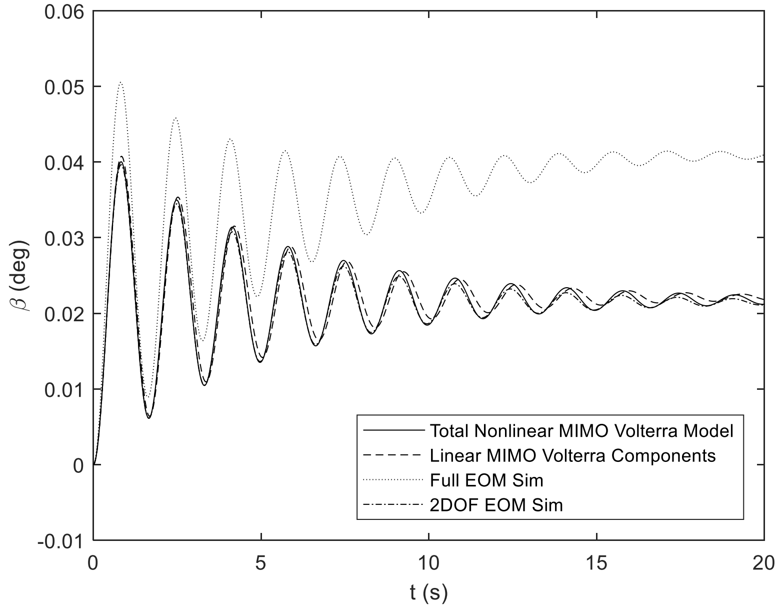

Figure 5.63 Sideslip Angle β Step Response of δ_{rdr}=+0.1 deg and δ_{alrn}=-0.1 deg from Equilibrium at V_T=1,673.7 ft/s

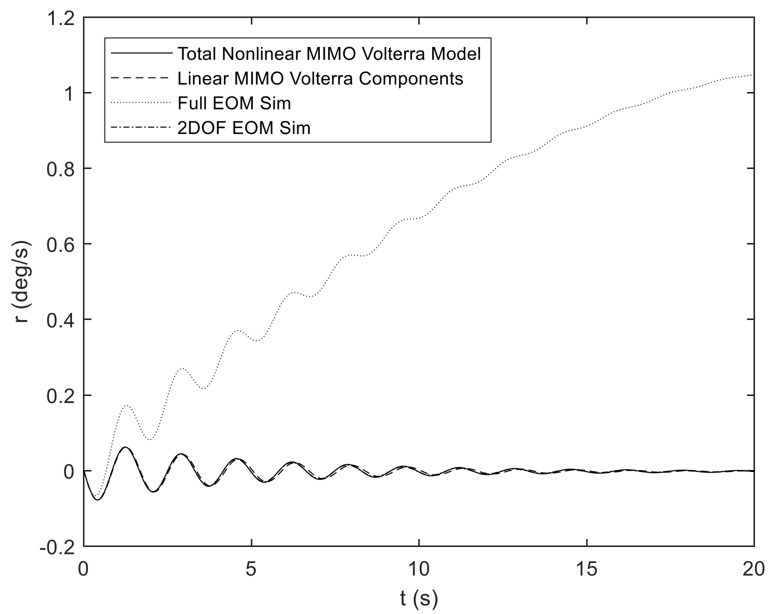

Figure 5.64 Yaw Rate r Step Response of δ_{rdr}=+0.1 deg and δ_{alrn}=-0.1 deg from Equilibrium at V_T=1,673.7 ft/s

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The purpose of this thesis was to develop an analytical, mathematical model for a nonlinear, multiple-degree of freedom system. Upon reviewing previous literature, Volterra theory was seen to have the most potential to describe the nonlinear dynamics sought. Specifically, the variational expansion method was found to formulate the input-output representations in a way that would be advantageous for study. Previous analytical research explored single-axis systems only; however, a framework to expand Volterra theory to multiple inputs and multiple outputs was described by Worden. This MIMO system description was a natural extension to previous research performed by Omran and Newman[2-11].

Due to the added complexity driven by the number of degrees of freedom, a second order system description with two states and two inputs was pursued. It was determined that adding additional degrees of freedom would require exhaustive computation and the kernels would be left at an unwieldy length. Along with this simplification, it was determined that the Volterra series would also be truncated to order two kernels, again to limit an unwieldy length. A result of these simplifications is the ability for the resulting model to only model weak nonlinearities. In order to extend the model to describe more significant nonlinearities, an additional third order kernel would need to be formulated. Other methods could also be applied, such as defining sub-regions for parameterization, to extend the model to describe the more significant nonlinearities.

The MIMO Volterra analytical model for the two degree of freedom, two-input, two-output, system was generated, and the steps to formulate it were outlined. An organization of the system kernels was then shown to group kernels into digestible components. This allows both easier communication and a method for analysis to break down which kernels are significant to the overall response. After the system kernels were investigated, integration was performed to compute an analytical, step response solution for the system. Responses were generated for a generic set of system parameters to show the kind of response expected from the system description.

The formulated analytical MIMO Volterra model was then applied to flight dynamics. Both the general aircraft equations of motion and the F-16 experimental wind tunnel data were outlined. Assumptions were made to simplify the full twelve equations of motion to a reduced order of two equations describing the longitudinal short period natural mode. Then, another set of assumptions were made to formulate two equations describing the latitudinal dutch roll natural mode.

The experimental data was then used to calculate the Volterra system parameters via the finite difference method. Responses to step inputs were then generated and compared to a linear solution response, a numerical simulation response of the reduced order model, and a numerical simulation response of the full order model.

For the longitudinal short period case, nonlinearities were discovered at high angles of attack. The results show that the MIMO Volterra model provides less error than the linear solution at high angles of attack. A breakdown of the nonlinear components of the MIMO Volterra model also showed that the nonlinearities were mainly attributed to the quadratic state 1 Volterra kernels. This was reinforced by plotting the experimental data for the angle of attack versus the z body-axis aerodynamic coefficient. This proves the capabilities of the MIMO Volterra model to capture significant components of nonlinearities, to provide the individual responses of those components, and to observe the effect each nonlinearity has on the total system response.

The latitudinal dutch roll investigation yielded similar insights; however, the nonlinearities present in the experimental data were found to be very minor. Significant nonlinearities were found, but outside the region where an equilibrium solution could be established. Further investigation will be needed to model these nonlinearities.

The above insights were gathered utilizing one step input with two output responses. Two inputs were also investigated for both the short period and dutch roll modes. For the longitudinal short period mode, two scenarios were analyzed. One scenario used the horizontal tail and the thrust power as inputs. This scenario proved insightful; however, there was significant error due to the constraints made with the reduced order model. An alternative input combination was analyzed, using the horizontal tail and the leading edge flap as the two inputs. This scenario was both accurate to the reduced order model and gave considerable insight to the nonlinear components. The key contributing nonlinear components were the quadratic state 1, bilinear state 1 – state 2, bilinear state 1 – input 1, bilinear state 1 – input 2, and bilinear state 2 – input 2 components. It was also shown that in certain conditions, these components could be present and significant in magnitude, but could cancel each other out and lead to a mostly linear system response.

For the latitudinal two-input analysis, it was foreseen that the assumptions made to establish the two-degree models would limit the accuracy of the two-input case compared to numerical simulation. Responses to two step inputs to the ailerons and rudder were generated and analyzed anyway to show the two-input capability of the MIMO Volterra model. Further study will be needed to develop a reduced order model fully suitable for the two-input analysis.

6.2 Recommendations

During the process of producing this thesis, future work was identified which could provide further insight into nonlinear flight dynamics. An obvious extension would be to formulate the third order Volterra kernel for the two degree of freedom system. Prior literature has shown that the third order provides a description of most of the nonlinear phenomenon encountered. However, significant obstacles would be present in such a formulation. It would be expected to take significant computation time to not only calculate the kernels, but to simplify and organize them in such a way as to be useable. Future work could also extend the system to three degrees of freedom; however, like the third order kernel extension, extensive computational time is foreseen.

Instead of these extensions, investigation can be performed to apply the analytical model in this thesis to the parameter varying sub-domain model developed by Omran and Newman[7]. This would expand the capability of the model from describing weak nonlinearities to more global ones. Omran has successfully modeled nonlinear phenomenon such as limit cycles by applying only a second order Volterra description to this method.

Another area of future work is to develop a reduced order, two degree of freedom, latitudinal model of flight dynamics that would fully incorporate two inputs. As was discussed, the assumptions made limited the accuracy of the two-input aileron and rudder case in this thesis. Perhaps a different reduced order model could further show the use of the MIMO Volterra model for latitudinal two-input responses.

Another area of future work would be to transform the current time-domain formulation of the MIMO Volterra model to the s-domain. The book by Rugh[20] outlines a method to do so. Further characteristics may be gathered from this form, as it should be much simpler and easier to describe using classical frequency domain theory. This would be an expansive theoretical undertaking, but could lead to further methods of nonlinear analysis for developing flight control systems.

Finally, future work could simply be to investigate further nonlinear flight dynamics using the MIMO Volterra model developed in this thesis. The F-16 experimental data used in this research proved to be highly linear for much of the flight envelope. Either further exploration of the F-16 data at the extreme ends of the flight envelope, or utilizing other aircraft data, may provide insights into nonlinear phenomenon that has not been captured before.

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Appendix A

Second Order 2DOF Volterra Kernels

$$x_{1,2} = x_{1,2}^{qs1} + x_{1,2}^{bs1s2} + x_{1,2}^{qs2} + x_{1,2}^{bs1i1} + x_{1,2}^{bs2i1} + x_{1,2}^{bs1i2} + x_{1,2}^{bs2i2} + x_{1,2}^{qi1} + x_{1,2}^{bi1i2} + x_{1,2}^{qi2} \quad (A.1)$$

$$\begin{aligned} x_{1,2}^{qs1} = &\int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs1},u_1,u_1\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_1(\tau_2)\, d\tau_1 d\tau_2 \\ &+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs1},u_1,u_2\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 \\ &+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs1},u_2,u_1\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_1(\tau_2)\, d\tau_1 d\tau_2 \\ &+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs1},u_2,u_2\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 \end{aligned} \quad (A.2)$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{qs1},u_1,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_1,\boldsymbol{t}-\boldsymbol{\tau}_2)=\frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{-\frac{2\sigma\left(-4A_2^3L_{0010}^2L_{2000}\omega_d^2+A_1^2A_2K_{0010}K_{2000}L_{0010}(\sigma^2+\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$
$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_1\sigma\left(A_1^2K_{0010}^2K_{2000}(\sigma^2+\omega_d^2)-A_2^2L_{0010}\left(16K_{0010}L_{2000}\omega_d^2+K_{2000}L_{0010}(\sigma^2+9\omega_d^2)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$
$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{2A_1^2A_2K_{0010}\sigma\left(4K_{0010}L_{2000}\omega_d^2+K_{2000}L_{0010}(\sigma^2+9\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$
$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$+\frac{A_1^3K_{0010}^2K_{2000}\sigma}{\sigma^2+\omega_d^2}$$
$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+3\varphi_1\right)$$

$$-\frac{A_1\sigma(A_1^2K_{0010}^2K_{2000}+A_2^2K_{2000}L_{0010}^2)}{\sigma^2+\omega_d^2}$$
$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_2K_{0010}K_{2000}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$
$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+2\varphi_1\right)$$

$$-\frac{A_1\sigma(A_1^2K_{0010}^2K_{2000}+A_2^2K_{2000}L_{0010}^2)}{\sigma^2+\omega_d^2}$$
$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_2K_{0010}K_{2000}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$
$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1\right)$$

$$+\frac{A_1A_2^2K_{2000}L_{0010}^2\sigma}{\sigma^2+9\omega_d^2}$$
$$\times\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{-2\omega_d\left(A_2^3L_{0010}^2L_{2000}(\sigma^2-3\omega_d^2)+3A_1^2A_2K_{0010}K_{2000}L_{0010}(\sigma^2+\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$
$$\times\cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\right)$$

$$+\frac{2\omega_d\left(A_2^3L_{0010}^2L_{2000}+A_1^2A_2K_{0010}(K_{2000}L_{0010}+K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}$$
$$\times\cos\left(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)\right)$$

$$-\frac{A_1\omega_d\left(3A_1^2K_{0010}^2K_{2000}(\sigma^2+\omega_d^2)-A_2^2L_{0010}\left(-4K_{0010}L_{2000}(\sigma^2-3\omega_d^2)+K_{2000}L_{0010}(\sigma^2+9\omega_d^2)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$
$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{2A_1^2A_2K_{0010}\omega_d\left(-K_{0010}L_{2000}(\sigma^2-3\omega_d^2)+K_{2000}L_{0010}(\sigma^2+9\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$
$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1\big)$$

$$+\frac{A_1^3K_{0010}^2K_{2000}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+3\varphi_1\big)$$

$$+\frac{A_1\omega_d\left(A_1^2K_{0010}^2K_{2000}+A_2^2L_{0010}(K_{2000}L_{0010}+2K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}$$
$$\times\cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_1\big)$$

$$+\frac{A_1^2A_2K_{0010}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times\cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+2\varphi_1\big)$$

$$+\frac{A_1\omega_d\left(A_1^2K_{0010}^2K_{2000}+A_2^2L_{0010}(K_{2000}L_{0010}+2K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}$$
$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1^2A_2K_{0010}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+2\varphi_1\big)$$

$$-\frac{3A_1A_2^2K_{2000}L_{0010}^2\omega_d}{\sigma^2+9\omega_d^2}$$
$$\times\cos\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{A_2^3 L_{0010}^2 L_{2000}\sigma}{\sigma^2+\omega_d^2} \times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)\Big)$$

$$+\frac{\sigma\left(A_2^3 L_{0010}^2 L_{2000}+A_1^2 A_2 K_{0010}(K_{2000}L_{0010}+K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2} \times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\Big(\omega_d\Big(\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)+(t-\tau_2)\Big)\Big)$$

$$+\frac{\sigma\left(A_2^3 L_{0010}^2 L_{2000}+A_1^2 A_2 K_{0010}(K_{2000}L_{0010}+K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2} \times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\Big(\omega_d\Big((t-\tau_1)-(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\Big)\Big)$$

$$-\frac{\sigma(2A_1^2 A_2 K_{0010}K_{2000}L_{0010}+A_2^3 L_{0010}^2 L_{2000})}{\sigma^2+9\omega_d^2} \times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\Big(\omega_d\Big(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\Big)\Big)$$

$$+\frac{A_1 K_{0010}\sigma(A_1^2 K_{0010}K_{2000}+2A_2^2 L_{0010}L_{2000})}{\sigma^2+9\omega_d^2} \times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1\Big)$$

$$+\frac{A_1^2 A_2 K_{0010}^2 L_{2000}\sigma}{\sigma^2+9\omega_d^2} \times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\Big(-\omega_d\Big(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+2\varphi_1\Big)$$

$$-\frac{A_1 A_2^2 L_{0010}\sigma(K_{2000}L_{0010}+2K_{0010}L_{2000})}{\sigma^2+\omega_d^2} \times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1\Big)$$

$$-\frac{A_1^2 A_2 K_{0010}\sigma(2K_{2000}L_{0010}+K_{0010}L_{2000})}{\sigma^2+\omega_d^2} \times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+2\varphi_1\Big)$$

$$-\frac{A_1^3 K_{0010}^2 K_{2000}\sigma}{\sigma^2+\omega_d^2} \times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+3\varphi_1\Big)$$

$$-\frac{A_1 A_2^2 K_{0010}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2^2K_{0010}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1\sigma\left(A_1^2K_{0010}^2K_{2000}+A_2^2L_{0010}(K_{2000}L_{0010}+K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_2K_{0010}K_{2000}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$+\frac{A_1\sigma\left(A_1^2K_{0010}^2K_{2000}+A_2^2L_{0010}(K_{2000}L_{0010}+K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_2K_{0010}K_{2000}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2^2K_{2000}L_{0010}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2^3L_{0010}^2L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\omega_d\left(A_2^3L_{0010}^2L_{2000}+A_1^2A_2K_{0010}(K_{2000}L_{0010}+K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d\left(A_2^3L_{0010}^2L_{2000}+A_1^2A_2K_{0010}(K_{2000}L_{0010}+K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d(2A_1^2A_2K_{0010}K_{2000}L_{0010}+A_2^3L_{0010}^2L_{2000})}{\sigma^2+9\omega_d^2}\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\Big)\Big)$$

$$+\frac{3A_1K_{0010}\omega_d(A_1^2K_{0010}K_{2000}+2A_2^2L_{0010}L_{2000})}{\sigma^2+9\omega_d^2}\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1\Big)$$

$$+\frac{3A_1^2A_2K_{0010}^2L_{2000}\omega_d}{\sigma^2+9\omega_d^2}\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+2\varphi_1\Big)$$

$$-\frac{A_1A_2^2L_{0010}\omega_d(K_{2000}L_{0010}+2K_{0010}L_{2000})}{\sigma^2+\omega_d^2}\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1\Big)$$

$$-\frac{A_1^2A_2K_{0010}\omega_d(2K_{2000}L_{0010}+K_{0010}L_{2000})}{\sigma^2+\omega_d^2}\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+2\varphi_1\Big)$$

$$-\frac{A_1^3K_{0010}^2K_{2000}\omega_d}{\sigma^2+\omega_d^2}\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+3\varphi_1\Big)$$

$$-\frac{A_1A_2^2K_{0010}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1\Big)$$

$$-\frac{A_1A_2^2K_{0010}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big(-(t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1\Big)$$

$$-\frac{A_1\omega_d\left(A_1^2K_{0010}^2K_{2000}+A_2^2L_{0010}(K_{2000}L_{0010}+K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big(-(t-\tau_1)+(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1\Big)$$

$$-\frac{A_1^2A_2K_{0010}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big(-(t-\tau_1)+(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+2\varphi_1\Big)$$

$$-\frac{A_1\omega_d\left(A_1^2K_{0010}^2K_{2000}+A_2^2L_{0010}(K_{2000}L_{0010}+K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2} \times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_2K_{0010}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2} \times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$+\frac{3A_1A_2^2K_{2000}L_{0010}^2\omega_d}{\sigma^2+9\omega_d^2} \times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)\Big\} \quad (A.3)$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(\boldsymbol{x}_{\boldsymbol{1,2}}^{\boldsymbol{qs1}},\boldsymbol{u}_{\boldsymbol{1}},\boldsymbol{u}_{\boldsymbol{2}}\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{-\frac{\sigma\left(-8A_2^3L_{0001}L_{0010}L_{2000}\omega_d^2 + A_1^2A_2K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010})\left(\sigma^2+\omega_d^2\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_1^2A_2K_{2000}(K_{0010}L_{0001} - K_{0001}L_{0010})\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$-\frac{A_1\sigma\left(-A_2^2K_{2000}L_{0001}L_{0010}\sigma^2 - 9A_2^2K_{2000}L_{0001}L_{0010}\omega_d^2 - 8A_2^2K_{0010}L_{0001}L_{2000}\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_1\sigma\left(-8A_2^2K_{0001}L_{0010}L_{2000}\omega_d^2 + A_1^2K_{0001}K_{0010}K_{2000}\left(\sigma^2+\omega_d^2\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_2\sigma\left(K_{0001}K_{2000}L_{0010}\left(\sigma^2+9\omega_d^2\right) + K_{0010}\left(8K_{0001}L_{2000}\omega_d^2 + K_{2000}L_{0001}\left(\sigma^2+9\omega_d^2\right)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$+\frac{A_1^3K_{0001}K_{0010}K_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+3\varphi_1\right)$$

$$-\frac{A_1\sigma(A_1^2K_{0001}K_{0010}K_{2000} + A_2^2K_{2000}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_2K_{0001}K_{2000}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+2\varphi_1\right)$$

$$-\frac{A_1\sigma(A_1^2K_{0001}K_{0010}K_{2000}+A_2^2K_{2000}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{A_1^2A_2K_{0010}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+2\varphi_1\big)$$

$$+\frac{A_1A_2^2K_{2000}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{\omega_d\left(2A_2^3L_{0001}L_{0010}L_{2000}(\sigma^2-3\omega_d^2)+3A_1^2A_2K_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^2+\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\right)$$

$$+\frac{\omega_d\left(2A_2^3L_{0001}L_{0010}L_{2000}+A_1^2A_2(K_{0010}K_{2000}L_{0001}+K_{0001}K_{2000}L_{0010}+2K_{0001}K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)\right)$$

$$-\frac{A_1\omega_d(-A_2^2K_{2000}L_{0001}L_{0010}\sigma^2+2A_2^2K_{0010}L_{0001}L_{2000}\sigma^2+2A_2^2K_{0001}L_{0010}L_{2000}\sigma^2)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{A_1\omega_d(-9A_2^2K_{2000}L_{0001}L_{0010}\omega_d^2-6A_2^2K_{0010}L_{0001}L_{2000}\omega_d^2-6A_2^2K_{0001}L_{0010}L_{2000}\omega_d^2)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{A_1\omega_d\left(3A_1^2K_{0001}K_{0010}K_{2000}(\sigma^2+\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1^2A_2\omega_d\left(K_{0001}K_{2000}L_{0010}(\sigma^2+9\omega_d^2)+K_{0010}\left(-2K_{0001}L_{2000}(\sigma^2-3\omega_d^2)+K_{2000}L_{0001}(\sigma^2+9\omega_d^2)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1\big)$$

$$+\frac{A_1^3 K_{0001} K_{0010} K_{2000} \omega_d}{\sigma^2 + \omega_d^2} \times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+3\varphi_1\right)$$

$$+\frac{A_1 \omega_d \left(A_1^2 K_{0001} K_{0010} K_{2000} + A_2^2 L_{0010}(K_{2000} L_{0001} + 2K_{0001} L_{2000})\right)}{\sigma^2 + \omega_d^2} \times \cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$+\frac{A_1^2 A_2 K_{0001} K_{2000} L_{0010} \omega_d}{\sigma^2 + \omega_d^2} \times \cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+2\varphi_1\right)$$

$$+\frac{A_1 \omega_d \left(A_1^2 K_{0001} K_{0010} K_{2000} + A_2^2 L_{0001}(K_{2000} L_{0010} + 2K_{0010} L_{2000})\right)}{\sigma^2 + \omega_d^2} \times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1^2 A_2 K_{0010} K_{2000} L_{0001} \omega_d}{\sigma^2 + \omega_d^2} \times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1\right)$$

$$-\frac{3A_1 A_2^2 K_{2000} L_{0001} L_{0010} \omega_d}{\sigma^2 + 9\omega_d^2} \times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_2^3 L_{0001} L_{0010} L_{2000} \sigma}{\sigma^2 + \omega_d^2} \times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma\left(A_2^3 L_{0001} L_{0010} L_{2000} + A_1^2 A_2 K_{0010}(K_{2000} L_{0001} + K_{0001} L_{2000})\right)}{\sigma^2 + \omega_d^2} \times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma\left(A_2^3 L_{0001} L_{0010} L_{2000} + A_1^2 A_2 K_{0001}(K_{2000} L_{0010} + K_{0010} L_{2000})\right)}{\sigma^2 + \omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\sigma(A_1^2A_2K_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})+A_2^3L_{0001}L_{0010}L_{2000})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_1\sigma\left(A_1^2K_{0001}K_{0010}K_{2000}+A_2^2L_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})\right)}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_2K_{0001}K_{0010}L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2^2\sigma(K_{2000}L_{0001}L_{0010}+K_{0010}L_{0001}L_{2000}+K_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_2\sigma(K_{0010}K_{2000}L_{0001}+K_{0001}K_{2000}L_{0010}+K_{0001}K_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1^3K_{0001}K_{0010}K_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+3\varphi_1\right)$$

$$-\frac{A_1A_2^2K_{0010}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2^2K_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1\sigma\left(A_1^2K_{0001}K_{0010}K_{2000}+A_2^2L_{0010}(K_{2000}L_{0001}+K_{0001}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_2K_{0001}K_{2000}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$+\frac{A_1\sigma\left(A_1^2K_{0001}K_{0010}K_{2000}+A_2^2L_{0001}(K_{2000}L_{0010}+K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_2K_{0010}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2^2K_{2000}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2^3L_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\omega_d\left(A_2^3L_{0001}L_{0010}L_{2000}+A_1^2A_2K_{0010}(K_{2000}L_{0001}+K_{0001}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d\left(A_2^3L_{0001}L_{0010}L_{2000}+A_1^2A_2K_{0001}(K_{2000}L_{0010}+K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d(A_1^2A_2K_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})+A_2^3L_{0001}L_{0010}L_{2000})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_1\omega_d\left(A_1^2K_{0001}K_{0010}K_{2000}+A_2^2L_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})\right)}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$+\frac{3A_1^2A_2K_{0001}K_{0010}L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2^2\omega_d(K_{2000}L_{0001}L_{0010}+K_{0010}L_{0001}L_{2000}+K_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_2\omega_d(K_{0010}K_{2000}L_{0001}+K_{0001}K_{2000}L_{0010}+K_{0001}K_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_1^3K_{0001}K_{0010}K_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+3\varphi_1\right)$$

$$-\frac{A_1A_2^2K_{0010}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2^2K_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1\omega_d\left(A_1^2K_{0001}K_{0010}K_{2000}+A_2^2L_{0010}(K_{2000}L_{0001}+K_{0001}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2 A_2 K_{0001} K_{2000} L_{0010} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1\omega_d\left(A_1^2 K_{0001} K_{0010} K_{2000}+A_2^2 L_{0001}(K_{2000} L_{0010}+K_{0010} L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2 A_2 K_{0010} K_{2000} L_{0001} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$+\frac{3A_1 A_2^2 K_{2000} L_{0001} L_{0010} \omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)\Big\} \quad (A.4)$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{qs1},u_2,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_1,\boldsymbol{t}-\boldsymbol{\tau}_2)=\frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{-\frac{\sigma\left(-8A_2^3L_{0001}L_{0010}L_{2000}\omega_d^2+A_1^2A_2K_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})\left(\sigma^2+\omega_d^2\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{A_1^2A_2K_{2000}\sigma(K_{0010}L_{0001}-K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$+\frac{A_1\sigma\left(A_2^2K_{2000}L_{0001}L_{0010}\sigma^2+9A_2^2K_{2000}L_{0001}L_{0010}\omega_d^2+8A_2^2K_{0010}L_{0001}L_{2000}\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_1\sigma\left(-8A_2^2K_{0001}L_{0010}L_{2000}\omega_d^2+A_1^2K_{0001}K_{0010}K_{2000}\left(\sigma^2+\omega_d^2\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_2\sigma\left(K_{0001}K_{2000}L_{0010}\left(\sigma^2+9\omega_d^2\right)+K_{0010}\left(8K_{0001}L_{2000}\omega_d^2+K_{2000}L_{0001}\left(\sigma^2+9\omega_d^2\right)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$+\frac{A_1^3K_{0001}K_{0010}K_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+3\varphi_1\right)$$

$$-\frac{A_1\sigma(A_1^2K_{0001}K_{0010}K_{2000}+A_2^2K_{2000}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_2K_{0010}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+2\varphi_1\right)$$

$$-\frac{A_1\sigma(A_1^2K_{0001}K_{0010}K_{2000}+A_2^2K_{2000}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{A_1^2A_2K_{0001}K_{2000}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+2\varphi_1\big)$$

$$+\frac{A_1A_2^2K_{2000}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{\omega_d\left(2A_2^3L_{0001}L_{0010}L_{2000}(\sigma^2-3\omega_d^2)+3A_1^2A_2K_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^2+\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\right)$$

$$+\frac{\omega_d\left(2A_2^3L_{0001}L_{0010}L_{2000}+A_1^2A_2(K_{0010}K_{2000}L_{0001}+K_{0001}K_{2000}L_{0010}+2K_{0001}K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)\right)$$

$$-\frac{A_1A_2^2\omega_d\sigma^2(-K_{2000}L_{0001}L_{0010}+2K_{0010}L_{0001}L_{2000}+2K_{0001}L_{0010}L_{2000})}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1A_2^2\omega_d^3(9K_{2000}L_{0001}L_{0010}+6K_{0010}L_{0001}L_{2000}+6K_{0001}L_{0010}L_{2000})}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{3A_1^3K_{0001}K_{0010}K_{2000}\omega_d(\sigma^2+\omega_d^2)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1^2A_2\omega_d\left(K_{0001}K_{2000}L_{0010}(\sigma^2+9\omega_d^2)+K_{0010}\left(-2K_{0001}L_{2000}(\sigma^2-3\omega_d^2)+K_{2000}L_{0001}(\sigma^2+9\omega_d^2)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1\big)$$

$$+\frac{A_1^3 K_{0001} K_{0010} K_{2000} \omega_d}{\sigma^2 + \omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+3\varphi_1\right)$$

$$+\frac{A_1 \omega_d \left(A_1^2 K_{0001} K_{0010} K_{2000} + A_2^2 L_{0001}(K_{2000} L_{0010} + 2K_{0010} L_{2000})\right)}{\sigma^2 + \omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$+\frac{A_1^2 A_2 K_{0010} K_{2000} L_{0001} \omega_d}{\sigma^2 + \omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+2\varphi_1\right)$$

$$+\frac{A_1 \omega_d \left(A_1^2 K_{0001} K_{0010} K_{2000} + A_2^2 L_{0010}(K_{2000} L_{0001} + 2K_{0001} L_{2000})\right)}{\sigma^2 + \omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1^2 A_2 K_{0001} K_{2000} L_{0010} \omega_d}{\sigma^2 + \omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1\right)$$

$$-\frac{3A_1 A_2^2 K_{2000} L_{0001} L_{0010} \omega_d}{\sigma^2 + 9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_2^3 L_{0001} L_{0010} L_{2000} \sigma}{\sigma^2 + \omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma\left(A_2^3 L_{0001} L_{0010} L_{2000} + A_1^2 A_2 K_{0001}(K_{2000} L_{0010} + K_{0010} L_{2000})\right)}{\sigma^2 + \omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma\left(A_2^3 L_{0001} L_{0010} L_{2000} + A_1^2 A_2 K_{0010}(K_{2000} L_{0001} + K_{0001} L_{2000})\right)}{\sigma^2 + \omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\sigma(A_1^2A_2K_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})+A_2^3L_{0001}L_{0010}L_{2000})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_1\sigma\left(A_1^2K_{0001}K_{0010}K_{2000}+A_2^2L_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})\right)}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_2K_{0001}K_{0010}L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2^2\sigma(K_{2000}L_{0001}L_{0010}+K_{0010}L_{0001}L_{2000}+K_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_2\sigma(K_{0010}K_{2000}L_{0001}+K_{0001}K_{2000}L_{0010}+K_{0001}K_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1^3K_{0001}K_{0010}K_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+3\varphi_1\right)$$

$$-\frac{A_1A_2^2K_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2^2K_{0010}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1\sigma\left(A_1^2K_{0001}K_{0010}K_{2000}+A_2^2L_{0001}(K_{2000}L_{0010}+K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_2K_{0010}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$+\frac{A_1\sigma\left(A_1^2K_{0001}K_{0010}K_{2000}+A_2^2L_{0010}(K_{2000}L_{0001}+K_{0001}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_2K_{0001}K_{2000}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2^2K_{2000}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2^3L_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\omega_d\left(A_2^3L_{0001}L_{0010}L_{2000}+A_1^2A_2K_{0001}(K_{2000}L_{0010}+K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d\left(A_2^3L_{0001}L_{0010}L_{2000}+A_1^2A_2K_{0010}(K_{2000}L_{0001}+K_{0001}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d(A_1^2A_2K_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})+A_2^3L_{0001}L_{0010}L_{2000})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_1\omega_d(A_1^2K_{0001}K_{0010}K_{2000}+A_2^2(K_{0010}L_{0001}+K_{0001}L_{0010})L_{2000})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$+\frac{3A_1^2A_2K_{0001}K_{0010}L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2^2\omega_d(K_{2000}L_{0001}L_{0010}+K_{0010}L_{0001}L_{2000}+K_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_2\omega_d(K_{0010}K_{2000}L_{0001}+K_{0001}K_{2000}L_{0010}+K_{0001}K_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_1^3K_{0001}K_{0010}K_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+3\varphi_1\right)$$

$$-\frac{A_1A_2^2K_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2^2K_{0010}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1\omega_d\left(A_1^2K_{0001}K_{0010}K_{2000}+A_2^2L_{0001}(K_{2000}L_{0010}+K_{0010}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2 A_2 K_{0010} K_{2000} L_{0001} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1 \omega_d \left(A_1^2 K_{0001} K_{0010} K_{2000}+A_2^2 L_{0010}(K_{2000} L_{0001}+K_{0001} L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2 A_2 K_{0001} K_{2000} L_{0010} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$+\frac{3A_1 A_2^2 K_{2000} L_{0001} L_{0010} \omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)\Big\}$$

$$(A.5)$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{qs1},u_2,u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\Bigg\{-\frac{2\sigma\left(-4A_2^3L_{0001}^2L_{2000}\omega_d^2 + A_1^2A_2K_{0001}K_{2000}L_{0001}\left(\sigma^2+\omega_d^2\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_1\sigma\left(A_1^2K_{0001}^2K_{2000}\left(\sigma^2+\omega_d^2\right) - A_2^2L_{0001}\left(16K_{0001}L_{2000}\omega_d^2 + K_{2000}L_{0001}\left(\sigma^2+9\omega_d^2\right)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{2A_1^2A_2K_{0001}\sigma\left(4K_{0001}L_{2000}\omega_d^2 + K_{2000}L_{0001}\left(\sigma^2+9\omega_d^2\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$+\frac{A_1^3K_{0001}^2K_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+3\varphi_1\right)$$

$$-\frac{A_1\sigma(A_1^2K_{0001}^2K_{2000} + A_2^2K_{2000}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_2K_{0001}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+2\varphi_1\right)$$

$$-\frac{A_1\sigma(A_1^2K_{0001}^2K_{2000} + A_2^2K_{2000}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_2K_{0001}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1\right)$$

$$+\frac{A_1A_2^2K_{2000}L_{0001}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{2\omega_d\left(A_2^3L_{0001}^2L_{2000}(\sigma^2-3\omega_d^2)+3A_1^2A_2K_{0001}K_{2000}L_{0001}(\sigma^2+\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{2\omega_d\left(A_2^3L_{0001}^2L_{2000}+A_1^2A_2K_{0001}(K_{2000}L_{0001}+K_{0001}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$-\frac{A_1\omega_d\left(3A_1^2K_{0001}^2K_{2000}(\sigma^2+\omega_d^2)-A_2^2L_{0001}\left(-4K_{0001}L_{2000}(\sigma^2-3\omega_d^2)+K_{2000}L_{0001}(\sigma^2+9\omega_d^2)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{2A_1^2A_2K_{0001}\omega_d\left(-K_{0001}L_{2000}(\sigma^2-3\omega_d^2)+K_{2000}L_{0001}(\sigma^2+9\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$+\frac{A_1^3K_{0001}^2K_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+3\varphi_1\right)$$

$$+\frac{A_1\omega_d\left(A_1^2K_{0001}^2K_{2000}+A_2^2L_{0001}(K_{2000}L_{0001}+2K_{0001}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_2K_{0001}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+2\varphi_1\right)$$

$$+\frac{A_1\omega_d\left(A_1^2K_{0001}^2K_{2000}+A_2^2L_{0001}(K_{2000}L_{0001}+2K_{0001}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1^2 A_2 K_{0001} K_{2000} L_{0001} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1\right)$$

$$-\frac{3A_1 A_2^2 K_{2000} L_{0001}^2 \omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_2^3 L_{0001}^2 L_{2000} \sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma\left(A_2^3 L_{0001}^2 L_{2000}+A_1^2 A_2 K_{0001}(K_{2000} L_{0001}+K_{0001} L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)} \sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma\left(A_2^3 L_{0001}^2 L_{2000}+A_1^2 A_2 K_{0001}(K_{2000} L_{0001}+K_{0001} L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\sigma(2A_1^2 A_2 K_{0001} K_{2000} L_{0001}+A_2^3 L_{0001}^2 L_{2000})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)} \sin\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_1 K_{0001} \sigma(A_1^2 K_{0001} K_{2000}+2A_2^2 L_{0001} L_{2000})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1^2 A_2 K_{0001}^2 L_{2000} \sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)} \sin\left(-\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2^2L_{0001}\sigma(K_{2000}L_{0001}+2K_{0001}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_2K_{0001}\sigma(2K_{2000}L_{0001}+K_{0001}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_1^3K_{0001}^2K_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+3\varphi_1\right)$$

$$-\frac{A_1A_2^2K_{0001}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2^2K_{0001}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$+\frac{A_1\sigma\left(A_1^2K_{0001}^2K_{2000}+A_2^2L_{0001}(K_{2000}L_{0001}+K_{0001}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_2K_{0001}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$+\frac{A_1\sigma\left(A_1^2K_{0001}^2K_{2000}+A_2^2L_{0001}(K_{2000}L_{0001}+K_{0001}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_2K_{0001}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1 A_2^2 K_{2000} L_{0001}^2 \sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2^3 L_{0001}^2 L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\omega_d\left(A_2^3 L_{0001}^2 L_{2000}+A_1^2 A_2 K_{0001}(K_{2000}L_{0001}+K_{0001}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d\left(A_2^3 L_{0001}^2 L_{2000}+A_1^2 A_2 K_{0001}(K_{2000}L_{0001}+K_{0001}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d(2A_1^2 A_2 K_{0001}K_{2000}L_{0001}+A_2^3 L_{0001}^2 L_{2000})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_1 K_{0001}\omega_d(A_1^2 K_{0001}K_{2000}+2A_2^2 L_{0001}L_{2000})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{3A_1^2 A_2 K_{0001}^2 L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1 A_2^2 L_{0001}\omega_d(K_{2000}L_{0001}+2K_{0001}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2 A_2 K_{0001}\omega_d(2K_{2000}L_{0001}+K_{0001}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1^3 K_{0001}^2 K_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+3\varphi_1\right)$$

$$-\frac{A_1 A_2^2 K_{0001}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1 A_2^2 K_{0001}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1\omega_d\left(A_1^2 K_{0001}^2 K_{2000}+A_2^2 L_{0001}(K_{2000}L_{0001}+K_{0001}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2 A_2 K_{0001}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1\omega_d\left(A_1^2 K_{0001}^2 K_{2000}+A_2^2 L_{0001}(K_{2000}L_{0001}+K_{0001}L_{2000})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2 A_2 K_{0001}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$+\frac{3A_1 A_2^2 K_{2000}L_{0001}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)\Big\}$$

$$(A.6)$$

$$x_{1,2}^{bs1s2} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1s2},u_1,u_1\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_1(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1s2},u_1,u_2\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1s2},u_2,u_1\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_1(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1s2},u_2,u_2\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2$$

(A. 7)

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{\boldsymbol{1,2}}^{\boldsymbol{bs1s2}},\boldsymbol{u_1},\boldsymbol{u_1}\right)}}(\boldsymbol{t}-\boldsymbol{\tau_1},\boldsymbol{t}-\boldsymbol{\tau_2}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{A_2^2A_3K_{0010}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{A_2^2A_3K_{0010}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$+\frac{\sigma(A_1^2A_3K_{0010}^2K_{1100}+A_2^2A_3K_{0010}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_1A_2A_3K_{0010}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_3K_{0010}^2K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$+\frac{A_2^2A_4L_{0010}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4L_{0010}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_4K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{1100}L_{0010}^2\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0010}^2K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2A_4K_{1100}L_{0010}^2\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1-\varphi_2\right)$$

$$-\frac{\sigma(A_1^2A_3K_{0010}^2K_{1100}+A_2^2A_3K_{0010}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_1A_2A_3K_{0010}^2L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_4L_{0010}\sigma(A_1^2K_{0010}K_{1100}+A_2^2L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1A_2A_3K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1A_2A_4K_{1100}L_{0010}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$+\frac{A_2^2A_3K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\right)$$

$$+\frac{A_2^2A_3K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)\right)$$

$$+\frac{\omega_d(A_1^2A_3K_{0010}^2K_{1100}+A_2^2A_3K_{0010}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)\right)$$

$$+\frac{A_1A_2A_3K_{0010}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1^2A_3K_{0010}^2K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1\big)$$

$$+\frac{A_2^2A_4L_{0010}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_1A_2A_4L_{0010}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1^2A_4K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1+\varphi_2\big)$$

$$+\frac{A_1A_2A_3K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_1\big)$$

$$+\frac{A_4\omega_d(A_1^2K_{0010}K_{1100}L_{0010}+2A_2^2L_{0010}^2L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_2\big)$$

$$+\frac{A_1A_2A_4K_{1100}L_{0010}^2\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1A_2A_3K_{0010}\omega_d(K_{1100}L_{0010}+2K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1^2A_3K_{0010}^2K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+2\varphi_1\big)$$

$$+\frac{A_1A_2A_4L_{0010}\omega_d(K_{1100}L_{0010}+2K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$+\frac{A_1^2A_4K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+2\varphi_1-\varphi_2\big)$$

$$-\frac{3\omega_d(A_1^2A_3K_{0010}^2K_{1100}+A_2^2A_3K_{0010}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{3A_1A_2A_3K_{0010}^2L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{3A_4L_{0010}\omega_d(A_1^2K_{0010}K_{1100}+A_2^2L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{3A_1A_2A_4K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3A_1A_2A_3K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{3A_1A_2A_4K_{1100}L_{0010}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0010}^2K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_2^2A_4L_{0010}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4L_{0010}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2 A_4 K_{0010} K_{1100} L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2 A_3 K_{0010} L_{0010} L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma(A_1^2 A_3 K_{0010}^2 K_{1100}+A_2^2 A_3 K_{0010} L_{0010} L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{A_2^2 A_3 K_{0010} L_{0010} L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\sigma(A_1^2 A_3 K_{0010}^2 K_{1100}+A_2^2 A_3 K_{0010} L_{0010} L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1 A_2 A_3 K_{0010}^2 L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1 A_2 A_4 K_{0010} L_{0010} L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1 A_2 A_3 K_{0010} K_{1100} L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4\sigma(A_1^2K_{0010}K_{1100}L_{0010}+A_2^2L_{0010}^2L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{1100}L_{0010}^2\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2A_4L_{0010}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0010}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_3K_{0010}^2K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$+\frac{A_1A_2A_4L_{0010}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0010}^2L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4L_{0010}\sigma(A_1^2K_{0010}K_{1100}+A_2^2L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_4K_{1100}L_{0010}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0010}^2K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_2^2A_4L_{0010}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4L_{0010}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2A_3K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\omega_d(A_1^2A_3K_{0010}^2K_{1100}+A_2^2A_3K_{0010}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_2^2A_3K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d(A_1^2A_3K_{0010}^2K_{1100}+A_2^2A_3K_{0010}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1A_2A_3K_{0010}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_4\omega_d(A_1^2K_{0010}K_{1100}L_{0010}+A_2^2L_{0010}^2L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{1100}L_{0010}^2\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2A_4L_{0010}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0010}^2K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2A_4L_{0010}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{3A_1A_2A_3K_{0010}^2L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{3A_4L_{0010}\omega_d(A_1^2K_{0010}K_{1100}+A_2^2L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_1A_2A_4K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{3A_1A_2A_3K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{3A_1A_2A_4K_{1100}L_{0010}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(-\omega_d\Big((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1-\varphi_2\Big)\Big\}$$

$$(A.8)$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{bs1s2},u_1,u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{A_2^2A_3K_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{A_2^2A_3K_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$+\frac{\sigma(A_1^2A_3K_{0001}K_{0010}K_{1100}+A_2^2A_3K_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_1A_2A_3K_{0001}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$+\frac{A_2^2A_4L_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4L_{0001}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0010}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$-\frac{A_1^2 A_4 K_{0010} K_{1100} L_{0001} \sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_2\right)$$

$$-\frac{A_1 A_2 A_4 K_{1100} L_{0001} L_{0010} \sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1 A_2 A_3 K_{0001} K_{1100} L_{0010} \sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_1^2 A_3 K_{0001} K_{0010} K_{1100} \sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1\right)$$

$$-\frac{A_1 A_2 A_4 K_{1100} L_{0001} L_{0010} \sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1^2 A_4 K_{0010} K_{1100} L_{0001} \sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1-\varphi_2\right)$$

$$-\frac{\sigma(A_1^2 A_3 K_{0001} K_{0010} K_{1100}+A_2^2 A_3 K_{0001} L_{0010} L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_1 A_2 A_3 K_{0001} K_{0010} L_{1100} \sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_4 \sigma(A_1^2 K_{0010} K_{1100} L_{0001}+A_2^2 L_{0001} L_{0010} L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0001}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0001}K_{1100}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_2A_4K_{1100}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2^2A_3K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{A_2^2A_3K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$+\frac{\omega_d\left(A_1^2A_3K_{0001}K_{0010}K_{1100}+A_2^2A_3K_{0001}L_{0010}L_{1100}\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_1A_2A_3K_{0001}\omega_d\left(K_{1100}L_{0010}+K_{0010}L_{1100}\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$+\frac{A_2^2A_4L_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4L_{0001}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1^2A_4K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1+\varphi_2\big)$$

$$+\frac{A_1A_2A_3K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_1\big)$$

$$+\frac{A_4\omega_d(A_1^2K_{0010}K_{1100}L_{0001}+2A_2^2L_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_2\big)$$

$$+\frac{A_1A_2A_4K_{1100}L_{0001}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1A_2A_3K_{0001}\omega_d(K_{1100}L_{0010}+2K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+2\varphi_1\big)$$

$$+\frac{A_1A_2A_4L_{0001}\omega_d(K_{1100}L_{0010}+2K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$+\frac{A_1^2A_4K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+2\varphi_1-\varphi_2\big)$$

$$-\frac{3\omega_d(A_1^2A_3K_{0001}K_{0010}K_{1100}+A_2^2A_3K_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{3A_1A_2A_3K_{0001}K_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{3A_4\omega_d(A_1^2K_{0010}K_{1100}L_{0001}+A_2^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{3A_1A_2A_4K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3A_1A_2A_3K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{3A_1A_2A_4K_{1100}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_2^2A_4L_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4L_{0001}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2 A_4 K_{0010} K_{1100} L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2 A_3 K_{0001} L_{0010} L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma(A_1^2 A_3 K_{0001} K_{0010} K_{1100}+A_2^2 A_3 K_{0001} L_{0010} L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{A_2^2 A_3 K_{0001} L_{0010} L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\sigma(A_1^2 A_3 K_{0001} K_{0010} K_{1100}+A_2^2 A_3 K_{0001} L_{0010} L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1 A_2 A_3 K_{0001} K_{0010} L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1 A_2 A_4 K_{0010} L_{0001} L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1 A_2 A_3 K_{0001} K_{1100} L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4\sigma(A_1^2K_{0010}K_{1100}L_{0001}+A_2^2L_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{1100}L_{0001}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2A_4L_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0001}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$+\frac{A_1A_2A_4L_{0001}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0010}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0001}K_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4\sigma(A_1^2K_{0010}K_{1100}L_{0001}+A_2^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0001}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{1100}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_4K_{1100}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_2^2A_4L_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4L_{0001}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2A_3K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)\right)$$

$$-\frac{\omega_d(A_1^2A_3K_{0001}K_{0010}K_{1100}+A_2^2A_3K_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t+\min(-\tau_1,-\tau_2))-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_2^2A_3K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t+\min(-\tau_1,-\tau_2))+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d(A_1^2A_3K_{0001}K_{0010}K_{1100}+A_2^2A_3K_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(3(t+\min(-\tau_1,-\tau_2))-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_4\omega_d(A_1^2K_{0010}K_{1100}L_{0001}+A_2^2L_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{1100}L_{0001}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2A_4L_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2A_4L_{0001}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{3A_1A_2A_3K_{0001}K_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{3A_4\omega_d(A_1^2K_{0010}K_{1100}L_{0001}+A_2^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_1A_2A_4K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{3A_1A_2A_3K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{3A_1A_2A_4K_{1100}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-\varphi_2\right)\Big\} \tag{A.9}$$

$$h_{2_{\left(x_{1,2}^{bs1s2},u_2,u_1\right)}}(t-\tau_1,t-\tau_2)=\frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{A_2^2A_3K_{0010}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{A_2^2A_3K_{0010}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$+\frac{\sigma(A_1^2A_3K_{0001}K_{0010}K_{1100}+A_2^2A_3K_{0010}L_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_1A_2A_3K_{0010}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$+\frac{A_2^2A_4L_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4L_{0010}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0001}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_4K_{0001}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{1100}L_{0001}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2A_4K_{1100}L_{0001}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0001}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1-\varphi_2\right)$$

$$-\frac{\sigma(A_1^2A_3K_{0001}K_{0010}K_{1100}+A_2^2A_3K_{0010}L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_4\sigma(A_1^2K_{0001}K_{1100}L_{0010}+A_2^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1A_2A_3K_{0010}K_{1100}L_{0001}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1A_2A_4K_{1100}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$+\frac{A_2^2A_3K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\Big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\Big)$$

$$+\frac{A_2^2A_3K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\Big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)\Big)$$

$$+\frac{\omega_d(A_1^2A_3K_{0001}K_{0010}K_{1100}+A_2^2A_3K_{0010}L_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\Big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)\Big)$$

$$+\frac{A_1A_2A_3K_{0010}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1\big)$$

$$+\frac{A_2^2A_4L_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_1A_2A_4L_{0010}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1^2A_4K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1+\varphi_2\big)$$

$$+\frac{A_1A_2A_3K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_1\big)$$

$$+\frac{A_4\omega_d(A_1^2K_{0001}K_{1100}L_{0010}+2A_2^2L_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_2\big)$$

$$+\frac{A_1A_2A_4K_{1100}L_{0001}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1A_2A_3K_{0010}\omega_d(K_{1100}L_{0001}+2K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+2\varphi_1\big)$$

$$+\frac{A_1A_2A_4L_{0010}\omega_d(K_{1100}L_{0001}+2K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$+\frac{A_1^2A_4K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+2\varphi_1-\varphi_2\big)$$

$$-\frac{3\omega_d(A_1^2A_3K_{0001}K_{0010}K_{1100}+A_2^2A_3K_{0010}L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\right)$$

$$-\frac{3A_1A_2A_3K_{0001}K_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{3A_4\omega_d(A_1^2K_{0001}K_{1100}L_{0010}+A_2^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$-\frac{3A_1A_2A_4K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$-\frac{3A_1A_2A_3K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{3A_1A_2A_4K_{1100}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$-\frac{A_1A_2A_3K_{0010}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_2^2A_4L_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4L_{0010}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2 A_4 K_{0001} K_{1100} L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2 A_3 K_{0010} L_{0001} L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma(A_1^2 A_3 K_{0001} K_{0010} K_{1100}+A_2^2 A_3 K_{0010} L_{0001} L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{A_2^2 A_3 K_{0010} L_{0001} L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\sigma(A_1^2 A_3 K_{0001} K_{0010} K_{1100}+A_2^2 A_3 K_{0010} L_{0001} L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1 A_2 A_3 K_{0001} K_{0010} L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1 A_2 A_4 K_{0001} L_{0010} L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1 A_2 A_3 K_{0010} K_{1100} L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4\sigma(A_1^2K_{0001}K_{1100}L_{0010}+A_2^2L_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{1100}L_{0001}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2A_4L_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0010}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$+\frac{A_1A_2A_4L_{0010}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0001}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0001}K_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4\sigma(A_1^2K_{0001}K_{1100}L_{0010}+A_2^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{1100}L_{0001}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_4K_{1100}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_2^2A_4L_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4L_{0010}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2A_3K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)\right)$$

$$-\frac{\omega_d(A_1^2A_3K_{0001}K_{0010}K_{1100}+A_2^2A_3K_{0010}L_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_2^2A_3K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d(A_1^2A_3K_{0001}K_{0010}K_{1100}+A_2^2A_3K_{0010}L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_4\omega_d(A_1^2K_{0001}K_{1100}L_{0010}+A_2^2L_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{1100}L_{0001}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2A_4L_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0001}K_{0010}K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2A_4L_{0010}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{3A_1A_2A_3K_{0001}K_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{3A_4\omega_d(A_1^2K_{0001}K_{1100}L_{0010}+A_2^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_1A_2A_4K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{3A_1A_2A_3K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{3A_1A_2A_4K_{1100}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\, \cos\left(-\omega_d\left(\left(t-\tau_1\right)+\left(t-\tau_2\right)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)\Big\} \tag{A.10}$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{bs1s2},u_2,u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{A_2^2A_3K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{A_2^2A_3K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$+\frac{\sigma(A_1^2A_3K_{0001}^2K_{1100}+A_2^2A_3K_{0001}L_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_1A_2A_3K_{0001}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_3K_{0001}^2K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$+\frac{A_2^2A_4L_{0001}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4L_{0001}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0001}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$-\frac{A_1^2 A_4 K_{0001} K_{1100} L_{0001} \sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_2\right)$$

$$-\frac{A_1 A_2 A_4 K_{1100} L_{0001}^2 \sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1 A_2 A_3 K_{0001} K_{1100} L_{0001} \sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_1^2 A_3 K_{0001}^2 K_{1100} \sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1\right)$$

$$-\frac{A_1 A_2 A_4 K_{1100} L_{0001}^2 \sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1^2 A_4 K_{0001} K_{1100} L_{0001} \sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1-\varphi_2\right)$$

$$-\frac{\sigma(A_1^2 A_3 K_{0001}^2 K_{1100}+A_2^2 A_3 K_{0001} L_{0001} L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_1 A_2 A_3 K_{0001}^2 L_{1100} \sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_4 L_{0001} \sigma(A_1^2 K_{0001} K_{1100}+A_2^2 L_{0001} L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1A_2A_3K_{0001}K_{1100}L_{0001}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1A_2A_4K_{1100}L_{0001}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$+\frac{A_2^2A_3K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\right)$$

$$+\frac{A_2^2A_3K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)\right)$$

$$+\frac{\omega_d(A_1^2A_3K_{0001}^2K_{1100}+A_2^2A_3K_{0001}L_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)\right)$$

$$+\frac{A_1A_2A_3K_{0001}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1^2A_3K_{0001}^2K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1\big)$$

$$+\frac{A_2^2A_4L_{0001}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_1A_2A_4L_{0001}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$+\frac{A_4\omega_d(A_1^2K_{0001}K_{1100}L_{0001}+2A_2^2L_{0001}^2L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{1100}L_{0001}^2\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0001}\omega_d(K_{1100}L_{0001}+2K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_3K_{0001}^2K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1\right)$$

$$+\frac{A_1A_2A_4L_{0001}\omega_d(K_{1100}L_{0001}+2K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_1-\varphi_2\right)$$

$$-\frac{3\omega_d(A_1^2A_3K_{0001}^2K_{1100}+A_2^2A_3K_{0001}L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{3A_1A_2A_3K_{0001}^2L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{3A_4L_{0001}\omega_d(A_1^2K_{0001}K_{1100}+A_2^2L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{3A_1A_2A_4K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3A_1A_2A_3K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{3A_1A_2A_4K_{1100}L_{0001}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0001}^2K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_2^2A_4L_{0001}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4L_{0001}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0001}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2A_3K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma(A_1^2A_3K_{0001}^2K_{1100}+A_2^2A_3K_{0001}L_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{A_2^2A_3K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\sigma(A_1^2A_3K_{0001}^2K_{1100}+A_2^2A_3K_{0001}L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1A_2A_3K_{0001}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0001}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4\sigma(A_1^2K_{0001}K_{1100}L_{0001}+A_2^2L_{0001}^2L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{1100}L_{0001}^2\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2A_4L_{0001}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0001}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_3K_{0001}^2K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$+\frac{A_1A_2A_4L_{0001}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0001}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0001}^2L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4L_{0001}\sigma(A_1^2K_{0001}K_{1100}+A_2^2L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{1100}L_{0001}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_4K_{1100}L_{0001}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0001}^2K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_2^2A_4L_{0001}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4L_{0001}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2A_3K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\omega_d(A_1^2A_3K_{0001}^2K_{1100}+A_2^2A_3K_{0001}L_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_2^2A_3K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d(A_1^2A_3K_{0001}^2K_{1100}+A_2^2A_3K_{0001}L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1A_2A_3K_{0001}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_4\omega_d(A_1^2K_{0001}K_{1100}L_{0001}+A_2^2L_{0001}^2L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{1100}L_{0001}^2\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2^2 A_4 L_{0001}^2 L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1 A_2 A_3 K_{0001}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2 A_3 K_{0001}^2 K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1 A_2 A_4 L_{0001}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1^2 A_4 K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{3A_1 A_2 A_3 K_{0001}^2 L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{3A_4 L_{0001}\omega_d(A_1^2 K_{0001}K_{1100}+A_2^2 L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_1 A_2 A_4 K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{3A_1 A_2 A_3 K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{3A_1A_2A_4K_{1100}L_{0001}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-\varphi_2\right)\Big\}$$

$$(A.11)$$

$$x_{1,2}^{qs2} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs2},u_1,u_1\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs2},u_1,u_2\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_2(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs2},u_2,u_1\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qs2},u_2,u_2\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_2(\tau_2)\, d\tau_1 d\tau_2$$

(A. 12)

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{\boldsymbol{1,2}}^{\boldsymbol{qs2}},\boldsymbol{u_1},\boldsymbol{u_1}\right)}}(\boldsymbol{t}-\boldsymbol{\tau_1},\boldsymbol{t}-\boldsymbol{\tau_2}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{8A_2A_3^2K_{0010}^2L_{0200}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{A_1A_3^2K_{0010}^2K_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{16A_2A_3A_4K_{0010}L_{0010}L_{0200}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{2A_1A_3A_4K_{0010}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{8A_2A_4^2L_{0010}^2L_{0200}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_1A_4^2K_{0200}L_{0010}^2\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_1\sigma(A_3^2K_{0010}^2K_{0200}+A_4^2K_{0200}L_{0010}^2)}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1\sigma(A_3^2K_{0010}^2K_{0200}+A_4^2K_{0200}L_{0010}^2)}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_3^2K_{0010}^2K_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_4^2K_{0200}L_{0010}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-2\varphi_2\right)$$

$$+\frac{2A_1A_3A_4K_{0010}K_{0200}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{2A_2A_3^2K_{0010}^2L_{0200}\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{2\omega_d(A_2A_3^2K_{0010}^2L_{0200}+A_2A_4^2L_{0010}^2L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$+\frac{A_1A_3^2K_{0010}^2K_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{4A_2A_3A_4K_{0010}L_{0010}L_{0200}\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{2A_1A_3A_4K_{0010}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{2A_2A_4^2L_{0010}^2L_{0200}\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_1A_4^2K_{0200}L_{0010}^2\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+2\varphi_2\right)$$

$$+\frac{A_1\omega_d(A_3^2K_{0010}^2K_{0200}+A_4^2K_{0200}L_{0010}^2)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{2A_2A_3A_4K_{0010}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1\omega_d(A_3^2K_{0010}^2K_{0200}+A_4^2K_{0200}L_{0010}^2)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{2A_2A_3A_4K_{0010}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3A_1A_3^2K_{0010}^2K_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{3A_1A_4^2K_{0200}L_{0010}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{6A_1A_3A_4K_{0010}K_{0200}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3^2K_{0010}^2L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0010}^2L_{0200}+A_2A_4^2L_{0010}^2L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0010}^2L_{0200}+A_2A_4^2L_{0010}^2L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{A_2A_3^2K_{0010}^2L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{2A_2A_3A_4K_{0010}L_{0010}L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0010}^2L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0010}^2K_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{2A_2A_3A_4K_{0010}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0010}^2L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{2A_1A_3A_4K_{0010}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0200}L_{0010}^2\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1\sigma(A_3^2K_{0010}^2K_{0200}+A_4^2K_{0200}L_{0010}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_2A_3A_4K_{0010}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1\sigma(A_3^2K_{0010}^2K_{0200}+A_4^2K_{0200}L_{0010}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_2A_3A_4K_{0010}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0200}L_{0010}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{2A_1A_3A_4K_{0010}K_{0200}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0010}^2K_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3^2K_{0010}^2L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)\right)$$

$$-\frac{\omega_d(A_2A_3^2K_{0010}^2L_{0200}+A_2A_4^2L_{0010}^2L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d(A_2A_3^2K_{0010}^2L_{0200}+A_2A_4^2L_{0010}^2L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\big(t+\min(-\tau_1,-\tau_2)\big)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_2A_3^2K_{0010}^2L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{6A_2A_3A_4K_{0010}L_{0010}L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{3A_2A_4^2L_{0010}^2L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_1 A_3^2 K_{0010}^2 K_{0200} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{2A_2 A_3 A_4 K_{0010} L_{0010} L_{0200} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_2 A_4^2 L_{0010}^2 L_{0200} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_2 A_3 A_4 K_{0010} L_{0010} L_{0200} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{2A_1 A_3 A_4 K_{0010} K_{0200} L_{0010} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1 A_4^2 K_{0200} L_{0010}^2 \omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2 A_3 A_4 K_{0010} L_{0010} L_{0200} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1 \omega_d (A_3^2 K_{0010}^2 K_{0200}+A_4^2 K_{0200} L_{0010}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2 A_3 A_4 K_{0010} L_{0010} L_{0200} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1\omega_d(A_3^2K_{0010}^2K_{0200}+A_4^2K_{0200}L_{0010}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_1A_4^2K_{0200}L_{0010}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-2\varphi_2\right)$$

$$+\frac{6A_1A_3A_4K_{0010}K_{0200}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{3A_1A_3^2K_{0010}^2K_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)\Big\}$$

(A. 13)

$$\boldsymbol{h_{2}}_{\left(x_{1,2}^{qs2},u_1,u_2\right)}(\boldsymbol{t}-\boldsymbol{\tau_1},\boldsymbol{t}-\boldsymbol{\tau_2}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{8A_2A_3^2K_{0001}K_{0010}L_{0200}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{A_1A_3^2K_{0001}K_{0010}K_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{8A_2A_3A_4L_{0200}\sigma\omega_d^2(K_{0010}L_{0001}+K_{0001}L_{0010})}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0200}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{8A_2A_4^2L_{0001}L_{0010}L_{0200}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_1A_4^2K_{0200}L_{0001}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_1\sigma(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1\sigma(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_3^2K_{0001}K_{0010}K_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_4^2K_{0200}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-2\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0200}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{2A_2A_3^2K_{0001}K_{0010}L_{0200}\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\omega_d(2A_2A_3^2K_{0001}K_{0010}L_{0200}+2A_2A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$+\frac{A_1A_3^2K_{0001}K_{0010}K_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{2A_2A_3A_4L_{0200}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^2-3\omega_d^2)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_1A_3A_4K_{0200}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$-\frac{2A_2A_4^2L_{0001}L_{0010}L_{0200}\omega_d(\sigma^2-3\omega_d^2)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_2\big)$$

$$+\frac{A_1A_4^2K_{0200}L_{0001}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+2\varphi_2\big)$$

$$+\frac{A_1\omega_d(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_1\big)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$+\frac{2A_2A_3A_4K_{0010}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_2\big)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1\omega_d(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{2A_2A_3A_4K_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3A_1A_3^2K_{0001}K_{0010}K_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{3A_1A_4^2K_{0200}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{3A_1A_3A_4K_{0200}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3^2K_{0001}K_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0001}K_{0010}L_{0200}+A_2A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0001}K_{0010}L_{0200}+A_2A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{A_2A_3^2K_{0001}K_{0010}L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_2A_3A_4L_{0200}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0001}K_{0010}K_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4L_{0200}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0200}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0200}L_{0001}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1\sigma(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_2A_3A_4K_{0010}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1\sigma(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_2A_3A_4K_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0200}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0200}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0001}K_{0010}K_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3^2K_{0001}K_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)\right)$$

$$-\frac{\omega_d(A_2A_3^2K_{0001}K_{0010}L_{0200}+A_2A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d(A_2A_3^2K_{0001}K_{0010}L_{0200}+A_2A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\big(t+\min(-\tau_1,-\tau_2)\big)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_2A_3^2K_{0001}K_{0010}L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_2A_3A_4L_{0200}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{3A_2A_4^2L_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0001}K_{0010}K_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4L_{0200}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0200}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0200}L_{0001}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1\omega_d(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1\omega_d(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_1A_4^2K_{0200}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-2\varphi_2\right)$$

$$+\frac{3A_1A_3A_4K_{0200}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{3A_1A_3^2K_{0001}K_{0010}K_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)\Big\} \tag{A.14}$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{qs2},u_2,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{8A_2A_3^2K_{0001}K_{0010}L_{0200}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$

$$\times\sin\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\right)$$

$$+\frac{A_1A_3^2K_{0001}K_{0010}K_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{8A_2A_3A_4L_{0200}\sigma\omega_d^2(K_{0010}L_{0001}+K_{0001}L_{0010})}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_1A_3A_4K_{0200}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{8A_2A_4^2L_{0001}L_{0010}L_{0200}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_2\big)$$

$$+\frac{A_1A_4^2K_{0200}L_{0001}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+2\varphi_2\big)$$

$$-\frac{A_1\sigma(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_1\big)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\big(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$-\frac{A_1\sigma(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times \sin\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1A_3^2K_{0001}K_{0010}K_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1A_4^2K_{0200}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1-2\varphi_2\big)$$

$$+\frac{A_1A_3A_4K_{0200}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$-\frac{2A_2A_3^2K_{0001}K_{0010}L_{0200}\omega_d(\sigma^2-3\omega_d^2)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times \cos\Big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\Big)$$

$$+\frac{\omega_d(2A_2A_3^2K_{0001}K_{0010}L_{0200}+2A_2A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times \cos\Big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)\Big)$$

$$+\frac{A_1A_3^2K_{0001}K_{0010}K_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{2A_2A_3A_4L_{0200}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^2-3\omega_d^2)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_1A_3A_4K_{0200}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$-\frac{2A_2A_4^2L_{0001}L_{0010}L_{0200}\omega_d(\sigma^2-3\omega_d^2)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_2\big)$$

$$+\frac{A_1A_4^2K_{0200}L_{0001}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+2\varphi_2\big)$$

$$+\frac{A_1\omega_d(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_1\big)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$+\frac{2A_2A_3A_4K_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_2\big)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1\omega_d(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{2A_2A_3A_4K_{0010}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3A_1A_3^2K_{0001}K_{0010}K_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{3A_1A_4^2K_{0200}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{3A_1A_3A_4K_{0200}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3^2K_{0001}K_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0001}K_{0010}L_{0200}+A_2A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0001}K_{0010}L_{0200}+A_2A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{A_2A_3^2K_{0001}K_{0010}L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_2A_3A_4L_{0200}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0001}K_{0010}K_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4L_{0200}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0200}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0200}L_{0001}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1\sigma(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_2A_3A_4K_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1\sigma(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_2A_3A_4K_{0010}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0200}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0200}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0001}K_{0010}K_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3^2K_{0001}K_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)\right)$$

$$-\frac{\omega_d(A_2A_3^2K_{0001}K_{0010}L_{0200}+A_2A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d(A_2A_3^2K_{0001}K_{0010}L_{0200}+A_2A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\big(t+\min(-\tau_1,-\tau_2)\big)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_2A_3^2K_{0001}K_{0010}L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_2A_3A_4L_{0200}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{3A_2A_4^2L_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0001}K_{0010}K_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4L_{0200}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0200}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0200}L_{0001}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1\omega_d(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1\omega_d(A_3^2K_{0001}K_{0010}K_{0200}+A_4^2K_{0200}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_1A_4^2K_{0200}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-2\varphi_2\right)$$

$$+\frac{3A_1A_3A_4K_{0200}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{3A_1A_3^2K_{0001}K_{0010}K_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)\Big\} \tag{A.15}$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{qs2},u_2,u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{8A_2A_3^2K_{0001}^2L_{0200}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{A_1A_3^2K_{0001}^2K_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{16A_2A_3A_4K_{0001}L_{0001}L_{0200}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{2A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{8A_2A_4^2L_{0001}^2L_{0200}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_1A_4^2K_{0200}L_{0001}^2\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_1\sigma(A_3^2K_{0001}^2K_{0200}+A_4^2K_{0200}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\big(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$-\frac{A_1\sigma(A_3^2K_{0001}^2K_{0200}+A_4^2K_{0200}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times \sin\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1A_3^2K_{0001}^2K_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1A_4^2K_{0200}L_{0001}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1-2\varphi_2\big)$$

$$+\frac{2A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$-\frac{2A_2A_3^2K_{0001}^2L_{0200}\omega_d\big(\sigma^2-3\omega_d^2\big)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times \cos\Big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\Big)$$

$$+\frac{2\omega_d(A_2A_3^2K_{0001}^2L_{0200}+A_2A_4^2L_{0001}^2L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times \cos\Big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)\Big)$$

$$+\frac{A_1A_3^2K_{0001}^2K_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{4A_2A_3A_4K_{0001}L_{0001}L_{0200}\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{2A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{2A_2A_4^2L_{0001}^2L_{0200}\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_1A_4^2K_{0200}L_{0001}^2\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+2\varphi_2\right)$$

$$+\frac{A_1\omega_d(A_3^2K_{0001}^2K_{0200}+A_4^2K_{0200}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_1\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{2A_2A_3A_4K_{0001}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1\omega_d(A_3^2K_{0001}^2K_{0200}+A_4^2K_{0200}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{2A_2A_3A_4K_{0001}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3A_1A_3^2K_{0001}^2K_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{3A_1A_4^2K_{0200}L_{0001}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{6A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3^2K_{0001}^2L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0001}^2L_{0200}+A_2A_4^2L_{0001}^2L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0001}^2L_{0200}+A_2A_4^2L_{0001}^2L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{A_2A_3^2K_{0001}^2L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{2A_2A_3A_4K_{0001}L_{0001}L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0001}^2L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0001}^2K_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{2A_2A_3A_4K_{0001}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}^2L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{2A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0200}L_{0001}^2\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1\sigma(A_3^2K_{0001}^2K_{0200}+A_4^2K_{0200}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_2A_3A_4K_{0001}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1\sigma(A_3^2K_{0001}^2K_{0200}+A_4^2K_{0200}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_2A_3A_4K_{0001}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0200}L_{0001}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{2A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0001}^2K_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3^2K_{0001}^2L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)\right)$$

$$-\frac{\omega_d(A_2A_3^2K_{0001}^2L_{0200}+A_2A_4^2L_{0001}^2L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d(A_2A_3^2K_{0001}^2L_{0200}+A_2A_4^2L_{0001}^2L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\big(t+\min(-\tau_1,-\tau_2)\big)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_2A_3^2K_{0001}^2L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{6A_2A_3A_4K_{0001}L_{0001}L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{3A_2A_4^2L_{0001}^2L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0001}^2K_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{2A_2A_3A_4K_{0001}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}^2L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{2A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0200}L_{0001}^2\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1\omega_d(A_3^2K_{0001}^2K_{0200}+A_4^2K_{0200}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1\omega_d(A_3^2K_{0001}^2K_{0200}+A_4^2K_{0200}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_1A_4^2K_{0200}L_{0001}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-2\varphi_2\right)$$

$$+\frac{6A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{3A_1A_3^2K_{0001}^2K_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)\Big\} \tag{A.16}$$

$$x_{1,2}^{bs1i1} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1i1},u_1,u_1\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_1(\tau_2)\,d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1i1},u_2,u_1\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_1(\tau_2)\,d\tau_1 d\tau_2$$

(A. 17)

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{bs1i1},u_1,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}})=$$

$$\frac{1}{2}\Big(\big(A_1K_{1010}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_1)$$

$$+A_2L_{1010}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)\big)$$

$$\times\big(A_1K_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+A_2L_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\Big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\Big)\Big)\Big)$$

(A. 18)

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{bs1i1},u_2,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_1,\boldsymbol{t}-\boldsymbol{\tau}_2) =$$

$$\frac{1}{2}\Big(\big(A_1K_{1010}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_1)$$

$$+A_2L_{1010}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)\big)$$

$$\times\big(A_1K_{0001}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+A_2L_{0001}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\Big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\Big)\Big)\Big)$$

(A. 19)

$$x_{1,2}^{bs2i1} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs2i1},u_1,u_1\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_1(\tau_2)\,d\tau_1 d\tau_2 + \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs2i1},u_2,u_1\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_1(\tau_2)\,d\tau_1 d\tau_2 \tag{A.20}$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{bs2i1},u_1,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) =$$

$$\frac{1}{2}\Big(\big(A_1K_{0110}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_1)$$

$$+A_2L_{0110}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)\big)$$

$$\times\Big(A_3K_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\Big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\Big)$$

$$+A_4L_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_2\big)\big)\Big)$$

$$\text{(A.21)}$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{bs2i1},u_2,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) =$$

$$\frac{1}{2}\Big(\big(A_1 K_{0110} e^{-\sigma \min(t-\tau_1, t-\tau_2)} \sin(\omega_d \min(t-\tau_1, t-\tau_2) + \varphi_1)$$

$$+A_2 L_{0110} e^{-\sigma \min(t-\tau_1, t-\tau_2)} \sin\big(\omega_d \min(t-\tau_1, t-\tau_2)\big)\Big)$$

$$\times \big(A_3 K_{0001} e^{-\sigma \max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times \sin\Big(\omega_d \max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\Big)$$

$$+A_4 L_{0001} e^{-\sigma \max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times \sin\big(\omega_d \max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big) + \varphi_2\big)\big)\Big) \tag{A.22}$$

$$x_{1,2}^{bs1i2} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1i2},u_1,u_2\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs1i2},u_2,u_2\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 \qquad (A.23)$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{bs1i2},u_1,u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) =$$

$$\frac{1}{2}\Big(\big(A_1K_{1001}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_1)$$

$$+A_2L_{1001}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)\big)$$

$$\times\big(A_1K_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+A_2L_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\Big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\big)\big)\Big) \quad \text{(A. 24)}$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{bs1i2},u_2,u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) =$$

$$\frac{1}{2}\Big(\big(A_1K_{1001}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_1)$$

$$+A_2L_{1001}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)\big)$$

$$\times\big(A_1K_{0001}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+A_2L_{0001}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\Big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\big)\big)\Big)$$

$$(A.25)$$

$$x_{1,2}^{bs2i2} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs2i2}, u_1, u_2\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 + \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bs2i2}, u_2, u_2\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 \tag{A. 26}$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{bs2i2},u_1,u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) =$$

$$\frac{1}{2}\Big(\big(A_1K_{0101}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_1)$$

$$+A_2L_{0101}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)\Big)$$

$$\times\big(A_3K_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\Big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\Big)$$

$$+A_4L_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_2)\big)\Big)$$

(A. 27)

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{1,2}^{bs2i2},u_2,u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) =$$

$$\frac{1}{2}\Big(\big(A_1K_{0101}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_1)$$

$$+A_2L_{0101}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)\Big)$$

$$\times\left(A_3K_{0001}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}\right.$$

$$\times\sin\left(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\right)$$

$$+A_4L_{0001}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\Big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_2\big)\Big)\Big)$$

$$(\text{A}.28)$$

$$x_{1,2}^{qi1} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qi1},u_1,u_1\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_1(\tau_2)\,d\tau_1 d\tau_2 \qquad \text{(A. 29)}$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(\boldsymbol{x}_{\boldsymbol{1,2}}^{\boldsymbol{qi1}},\boldsymbol{u_1},\boldsymbol{u_1}\right)}}(\boldsymbol{t}-\boldsymbol{\tau_1}, \boldsymbol{t}-\boldsymbol{\tau_2}) = A_1 K_{0020} e^{-\sigma(t-\tau_1)} \sin\big(\varphi_1 + \omega_d(t-\tau_1)\big)\,\delta\big((t-\tau_2)-(t-\tau_1)\big)$$

$$+A_2 L_{0020} e^{-\sigma(t-\tau_1)} \sin\big(\omega_d(t-\tau_1)\big)\,\delta\big((t-\tau_2)-(t-\tau_1)\big)$$

$$\text{(A. 30)}$$

$$x_{1,2}^{bi1i2} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{bi1i2},u_1,u_2\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 \tag{A.31}$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(\boldsymbol{x}_{\boldsymbol{1,2}}^{\boldsymbol{bi1i2}},\boldsymbol{u_1},\boldsymbol{u_2}\right)}}(\boldsymbol{t}-\boldsymbol{\tau_1}, \boldsymbol{t}-\boldsymbol{\tau_2}) = A_1 K_{0011} e^{-\sigma(t-\tau_1)} \sin\big(\varphi_1 + \omega_d (t-\tau_1)\big)\, \delta\big((t-\tau_2)-(t-\tau_1)\big)$$

$$+A_2 L_{0011} e^{-\sigma(t-\tau_1)} \sin\big(\omega_d (t-\tau_1)\big)\, \delta\big((t-\tau_2)-(t-\tau_1)\big)$$

(A.32)

$$x_{1,2}^{qi2} = \int_0^t \int_0^t h_{2_{\left(x_{1,2}^{qi2},u_2,u_2\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 \tag{A.33}$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(\boldsymbol{x}_{\boldsymbol{1,2}}^{\boldsymbol{qi2}},\boldsymbol{u_2},\boldsymbol{u_2}\right)}}(\boldsymbol{t}-\boldsymbol{\tau_1}, \boldsymbol{t}-\boldsymbol{\tau_2}) = A_1 K_{0002} e^{-\sigma(t-\tau_1)} \sin\left(\varphi_1 + \omega_d(t-\tau_1)\right) \delta\left((t-\tau_2)-(t-\tau_1)\right)$$

$$+A_2 L_{0002} e^{-\sigma(t-\tau_1)} \sin\left(\omega_d(t-\tau_1)\right) \delta\left((t-\tau_2)-(t-\tau_1)\right)$$

(A. 34)

$$x_{2,2} = x_{2,2}^{qs1} + x_{2,2}^{bs1s2} + x_{2,2}^{qs2} + x_{2,2}^{bs1i1} + x_{2,2}^{bs2i1} + x_{2,2}^{bs1i2} + x_{2,2}^{bs2i2} + x_{2,2}^{qi1} + x_{2,2}^{bi1i2} + x_{2,2}^{qi2} \qquad (A.35)$$

$$\begin{aligned} x_{2,2}^{qs1} = & \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{qs1},u_1,u_1\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_1(\tau_2)\, d\tau_1 d\tau_2 \\ & + \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{qs1},u_1,u_2\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 \\ & + \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{qs1},u_2,u_1\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_1(\tau_2)\, d\tau_1 d\tau_2 \\ & + \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{qs1},u_2,u_2\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 \end{aligned} \qquad (A.36)$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{2,2}^{qs1},u_1,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau_1},\boldsymbol{t}-\boldsymbol{\tau_2}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{8A_2^2A_3K_{2000}L_{0010}^2\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$

$$\times\sin\Big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\Big)$$

$$+\frac{16A_1A_2A_3K_{0010}K_{2000}L_{0010}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{8A_1^2A_3K_{0010}^2K_{2000}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1\big)$$

$$-\frac{2A_1A_2A_4K_{0010}L_{0010}L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$-\frac{A_1^2A_4K_{0010}^2L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1-\varphi_2\big)$$

$$+\frac{A_2^2A_4L_{0010}^2L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{2A_1A_2A_4K_{0010}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1^2A_4K_{0010}^2L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1+\varphi_2\big)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_4\sigma(A_1^2K_{0010}^2L_{2000}+A_2^2L_{0010}^2L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_4\sigma(A_1^2K_{0010}^2L_{2000}+A_2^2L_{0010}^2L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_2^2A_4L_{0010}^2L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{2A_2^2A_3K_{2000}L_{0010}^2\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{2\omega_d(A_1^2A_3K_{0010}^2K_{2000}+A_2^2A_3K_{2000}L_{0010}^2)}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$-\frac{4A_1A_2A_3K_{0010}K_{2000}L_{0010}\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{2A_1^2A_3K_{0010}^2K_{2000}\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$-\frac{6A_1A_2A_4K_{0010}L_{0010}L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{3A_1^2A_4K_{0010}^2L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{A_2^2A_4L_{0010}^2L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{2A_1A_2A_4K_{0010}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0010}^2L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1+\varphi_2\right)$$

$$+\frac{2A_1A_2A_3K_{0010}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_4\omega_d(A_1^2K_{0010}^2L_{2000}+A_2^2L_{0010}^2L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{2A_1A_2A_3K_{0010}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_4\omega_d(A_1^2K_{0010}^2L_{2000}+A_2^2L_{0010}^2L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3A_2^2A_4L_{0010}^2L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_2^2A_3K_{2000}L_{0010}^2\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma(A_1^2A_3K_{0010}^2K_{2000}+A_2^2A_3K_{2000}L_{0010}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma(A_1^2A_3K_{0010}^2K_{2000}+A_2^2A_3K_{2000}L_{0010}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{A_2^2A_3K_{2000}L_{0010}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{2A_1A_2A_4K_{0010}L_{0010}L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0010}^2L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{2A_1A_2A_3K_{0010}K_{2000}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_3K_{0010}^2K_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{2A_1A_2A_3K_{0010}K_{2000}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0010}^2K_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_2^2A_4L_{0010}^2L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{2000}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{2A_1A_2A_4K_{0010}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2 A_4 K_{0010}^2 L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_1 A_2 A_4 K_{0010} L_{0010} L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1 A_2 A_4 K_{0010} L_{0010} L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1 A_2 A_3 K_{0010} K_{2000} L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1 A_2 A_3 K_{0010} K_{2000} L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4\sigma(A_1^2 K_{0010}^2 L_{2000}+A_2^2 L_{0010}^2 L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1 A_2 A_4 K_{0010} L_{0010} L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1 A_2 A_4 K_{0010} L_{0010} L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1 A_2 A_3 K_{0010} K_{2000} L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4\sigma(A_1^2K_{0010}^2L_{2000}+A_2^2L_{0010}^2L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_2^2A_4L_{0010}^2L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_2^2A_3K_{2000}L_{0010}^2\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\omega_d(A_1^2A_3K_{0010}^2K_{2000}+A_2^2A_3K_{2000}L_{0010}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d(A_1^2A_3K_{0010}^2K_{2000}+A_2^2A_3K_{2000}L_{0010}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_2^2A_3K_{2000}L_{0010}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{6A_1A_2A_4K_{0010}L_{0010}L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{3A_1^2A_4K_{0010}^2L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{6A_1A_2A_3K_{0010}K_{2000}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1\Big)$$

$$+\frac{3A_1^2A_3K_{0010}^2K_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+2\varphi_1\Big)$$

$$-\frac{2A_1A_2A_3K_{0010}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1\Big)$$

$$-\frac{A_1^2A_3K_{0010}^2K_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+2\varphi_1\Big)$$

$$-\frac{A_2^2A_4L_{0010}^2L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_2\Big)$$

$$-\frac{A_1A_2A_3K_{0010}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1\Big)$$

$$-\frac{2A_1A_2A_4K_{0010}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1+\varphi_2\Big)$$

$$-\frac{A_1^2A_4K_{0010}^2L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+2\varphi_1+\varphi_2\Big)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big(-(t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1-\varphi_2\Big)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_4\omega_d(A_1^2K_{0010}^2L_{2000}+A_2^2L_{0010}^2L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_4\omega_d(A_1^2K_{0010}^2L_{2000}+A_2^2L_{0010}^2L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_2^2A_4L_{0010}^2L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)\Big\} \tag{A.37}$$

$$\boldsymbol{h_{2}}_{\left(x_{2,2}^{qs1},u_1,u_2\right)}(\boldsymbol{t-\tau_1,t-\tau_2}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{8A_2^2A_3K_{2000}L_{0001}L_{0010}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{8A_1A_2A_3K_{2000}\sigma\omega_d^2(K_{0010}L_{0001}+K_{0001}L_{0010})}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{8A_1^2A_3K_{0001}K_{0010}K_{2000}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2A_4L_{2000}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0001}K_{0010}L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{A_2^2A_4L_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4L_{2000}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0001}K_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_4\sigma(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_4\sigma(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_2^2A_4L_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{2A_2^2A_3K_{2000}L_{0001}L_{0010}\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\omega_d(2A_1^2A_3K_{0001}K_{0010}K_{2000}+2A_2^2A_3K_{2000}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$-\frac{2A_1A_2A_3K_{2000}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{2A_1^2A_3K_{0001}K_{0010}K_{2000}\omega_d(\sigma^2-3\omega_d^2)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1\big)$$

$$-\frac{3A_1A_2A_4L_{2000}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$-\frac{3A_1^2A_4K_{0001}K_{0010}L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1-\varphi_2\big)$$

$$+\frac{A_2^2A_4L_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_1A_2A_4L_{2000}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_1^2A_4K_{0001}K_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_1+\varphi_2\big)$$

$$+\frac{2A_1A_2A_3K_{0001}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$+\frac{A_4\omega_d(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{2A_1A_2A_3K_{0010}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_4\omega_d(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3A_2^2A_4L_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_2^2A_3K_{2000}L_{0001}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma(A_1^2A_3K_{0001}K_{0010}K_{2000}+A_2^2A_3K_{2000}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma(A_1^2A_3K_{0001}K_{0010}K_{2000}+A_2^2A_3K_{2000}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{A_2^2A_3K_{2000}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d \big(3(t+\min(-\tau_1,-\tau_2)) - (t-\tau_1) - (t-\tau_2)\big)\Big)$$

$$+\frac{A_1 A_2 A_4 L_{2000}\sigma(K_{0010}L_{0001} + K_{0001}L_{0010})}{\sigma^2 + 9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d \big((t-\tau_1) + (t-\tau_2) - 3(t+\min(-\tau_1,-\tau_2))\big) + \varphi_1 - \varphi_2\Big)$$

$$+\frac{A_1^2 A_4 K_{0001}K_{0010}L_{2000}\sigma}{\sigma^2 + 9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d \big((t-\tau_1) + (t-\tau_2) - 3(t+\min(-\tau_1,-\tau_2))\big) + 2\varphi_1 - \varphi_2\Big)$$

$$+\frac{A_1 A_2 A_3 K_{2000}\sigma(K_{0010}L_{0001} + K_{0001}L_{0010})}{\sigma^2 + 9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d \big((t-\tau_1) + (t-\tau_2) - 3(t+\min(-\tau_1,-\tau_2))\big) + \varphi_1\Big)$$

$$+\frac{A_1^2 A_3 K_{0001}K_{0010}K_{2000}\sigma}{\sigma^2 + 9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(-\omega_d \big(-(t-\tau_1) - (t-\tau_2) + 3(t+\min(-\tau_1,-\tau_2))\big) + 2\varphi_1\Big)$$

$$-\frac{A_1 A_2 A_3 K_{2000}\sigma(K_{0010}L_{0001} + K_{0001}L_{0010})}{\sigma^2 + \omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d \big((t-\tau_1) + (t-\tau_2) - (t+\min(-\tau_1,-\tau_2))\big) + \varphi_1\Big)$$

$$-\frac{A_1^2 A_3 K_{0001}K_{0010}K_{2000}\sigma}{\sigma^2 + \omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d \big((t-\tau_1) + (t-\tau_2) - (t+\min(-\tau_1,-\tau_2))\big) + 2\varphi_1\Big)$$

$$-\frac{A_2^2 A_4 L_{0001}L_{0010}L_{2000}\sigma}{\sigma^2 + \omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d \big((t-\tau_1) + (t-\tau_2) - (t+\min(-\tau_1,-\tau_2))\big) + \varphi_2\Big)$$

$$-\frac{A_1 A_2 A_3 K_{0010}K_{2000}L_{0001}\sigma}{\sigma^2 + \omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d \big((t-\tau_1) - (t-\tau_2) - (t+\min(-\tau_1,-\tau_2))\big) + \varphi_1\Big)$$

$$-\frac{A_1A_2A_4L_{2000}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0001}K_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{2000}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1A_2A_3K_{0001}K_{2000}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4\sigma(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0010}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4\sigma(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_2^2A_4L_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_2^2A_3K_{2000}L_{0001}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\omega_d(A_1^2A_3K_{0001}K_{0010}K_{2000}+A_2^2A_3K_{2000}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d(A_1^2A_3K_{0001}K_{0010}K_{2000}+A_2^2A_3K_{2000}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_2^2A_3K_{2000}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_1A_2A_4L_{2000}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{3A_1^2A_4K_{0001}K_{0010}L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{3A_1A_2A_3K_{2000}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$+\frac{3A_1^2A_3K_{0001}K_{0010}K_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2A_3K_{2000}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0001}K_{0010}K_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_2^2A_4L_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_4L_{2000}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0001}K_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_4\omega_d(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_4\omega_d(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_2^2A_4L_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)\Big\} \quad (A.38)$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{2,2}^{qs1},u_2,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_1,\boldsymbol{t}-\boldsymbol{\tau}_2)=\frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{8A_2^2A_3K_{2000}L_{0001}L_{0010}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{8A_1A_2A_3K_{2000}\sigma\omega_d^2(K_{0010}L_{0001}+K_{0001}L_{0010})}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{8A_1^2A_3K_{0001}K_{0010}K_{2000}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2A_4L_{2000}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0001}K_{0010}L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{A_2^2A_4L_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4L_{2000}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0001}K_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_4\sigma(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_4\sigma(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_2^2A_4L_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{2A_2^2A_3K_{2000}L_{0001}L_{0010}\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\omega_d(2A_1^2A_3K_{0001}K_{0010}K_{2000}+2A_2^2A_3K_{2000}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$-\frac{2A_1A_2A_3K_{2000}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{2A_1^2A_3K_{0001}K_{0010}K_{2000}\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$-\frac{3A_1A_2A_4L_{2000}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{3A_1^2A_4K_{0001}K_{0010}L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{A_2^2A_4L_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4L_{2000}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0001}K_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1+\varphi_2\right)$$

$$+\frac{2A_1A_2A_3K_{0010}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_4\omega_d(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{2A_1A_2A_3K_{0001}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_4\omega_d(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3A_2^2A_4L_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_2^2A_3K_{2000}L_{0001}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma(A_1^2A_3K_{0001}K_{0010}K_{2000}+A_2^2A_3K_{2000}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma(A_1^2A_3K_{0001}K_{0010}K_{2000}+A_2^2A_3K_{2000}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{A_2^2A_3K_{2000}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_1A_2A_4L_{2000}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0001}K_{0010}L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{2000}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_3K_{0001}K_{0010}K_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2A_3K_{2000}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0001}K_{0010}K_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1\right)$$

$$-\frac{A_2^2A_4L_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{2000}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_4L_{2000}\sigma(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0001}K_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1A_2A_3K_{0010}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4\sigma(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0001}K_{2000}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4\sigma(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_2^2A_4L_{0001}L_{0010}L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_2^2A_3K_{2000}L_{0001}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\omega_d(A_1^2A_3K_{0001}K_{0010}K_{2000}+A_2^2A_3K_{2000}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d(A_1^2A_3K_{0001}K_{0010}K_{2000}+A_2^2A_3K_{2000}L_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_2^2A_3K_{2000}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_1A_2A_4L_{2000}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{3A_1^2A_4K_{0001}K_{0010}L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{3A_1A_2A_3K_{2000}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$+\frac{3A_1^2A_3K_{0001}K_{0010}K_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_1A_2A_3K_{2000}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0001}K_{0010}K_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_2^2A_4L_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_4L_{2000}\omega_d(K_{0010}L_{0001}+K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0001}K_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_2A_3K_{0010}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_4\omega_d(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0010}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{2000}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_4\omega_d(A_1^2K_{0001}K_{0010}L_{2000}+A_2^2L_{0001}L_{0010}L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_2^2A_4L_{0001}L_{0010}L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)} \cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)\Big\}$$

(A. 39)

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{2,2}^{qs1},u_2,u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_1,\boldsymbol{t}-\boldsymbol{\tau}_2)=\frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{8A_2^2A_3K_{2000}L_{0001}^2\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{16A_1A_2A_3K_{0001}K_{2000}L_{0001}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{8A_1^2A_3K_{0001}^2K_{2000}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$-\frac{2A_1A_2A_4K_{0001}L_{0001}L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0001}^2L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{A_2^2A_4L_{0001}^2L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{2A_1A_2A_4K_{0001}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0001}^2L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_4\sigma(A_1^2K_{0001}^2L_{2000}+A_2^2L_{0001}^2L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_4\sigma(A_1^2K_{0001}^2L_{2000}+A_2^2L_{0001}^2L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_2^2A_4L_{0001}^2L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{2A_2^2A_3K_{2000}L_{0001}^2\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{2\omega_d(A_1^2A_3K_{0001}^2K_{2000}+A_2^2A_3K_{2000}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$-\frac{4A_1A_2A_3K_{0001}K_{2000}L_{0001}\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{2A_1^2A_3K_{0001}^2K_{2000}\omega_d\left(\sigma^2-3\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1\right)$$

$$-\frac{6A_1A_2A_4K_{0001}L_{0001}L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{3A_1^2A_4K_{0001}^2L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{A_2^2A_4L_{0001}^2L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{2A_1A_2A_4K_{0001}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0001}^2L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_1+\varphi_2\right)$$

$$+\frac{2A_1A_2A_3K_{0001}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_4\omega_d(A_1^2K_{0001}^2L_{2000}+A_2^2L_{0001}^2L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{2A_1A_2A_3K_{0001}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_4\omega_d(A_1^2K_{0001}^2L_{2000}+A_2^2L_{0001}^2L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3A_2^2A_4L_{0001}^2L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_2^2A_3K_{2000}L_{0001}^2\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma(A_1^2A_3K_{0001}^2K_{2000}+A_2^2A_3K_{2000}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma(A_1^2A_3K_{0001}^2K_{2000}+A_2^2A_3K_{2000}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{A_2^2A_3K_{2000}L_{0001}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{2A_1A_2A_4K_{0001}L_{0001}L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_1^2A_4K_{0001}^2L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{2A_1A_2A_3K_{0001}K_{2000}L_{0001}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$+\frac{A_1^2A_3K_{0001}^2K_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{2A_1A_2A_3K_{0001}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0001}^2K_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_2^2A_4L_{0001}^2L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{2A_1A_2A_4K_{0001}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0001}^2L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1A_2A_3K_{0001}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_4\sigma(A_1^2K_{0001}^2L_{2000}+A_2^2L_{0001}^2L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1A_2A_3K_{0001}K_{2000}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$+\frac{A_4\sigma(A_1^2K_{0001}^2L_{2000}+A_2^2L_{0001}^2L_{2000})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_2^2A_4L_{0001}^2L_{2000}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_2^2A_3K_{2000}L_{0001}^2\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)\right)$$

$$-\frac{\omega_d(A_1^2A_3K_{0001}^2K_{2000}+A_2^2A_3K_{2000}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\cos\left(\omega_d\left(\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d(A_1^2A_3K_{0001}^2K_{2000}+A_2^2A_3K_{2000}L_{0001}^2)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\cos\left(\omega_d\left(\big(t+\min(-\tau_1,-\tau_2)\big)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_2^2A_3K_{2000}L_{0001}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\cos\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{6A_1A_2A_4K_{0001}L_{0001}L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{3A_1^2A_4K_{0001}^2L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1-\varphi_2\right)$$

$$+\frac{6A_1A_2A_3K_{0001}K_{2000}L_{0001}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$+\frac{3A_1^2A_3K_{0001}^2K_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{2A_1A_2A_3K_{0001}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_1^2A_3K_{0001}^2K_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1\right)$$

$$-\frac{A_2^2A_4L_{0001}^2L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_2A_3K_{0001}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{2A_1A_2A_4K_{0001}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_1^2A_4K_{0001}^2L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_1+\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$
$$-\frac{A_1A_2A_3K_{0001}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$
$$-\frac{A_1A_2A_3K_{0001}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$
$$-\frac{A_4\omega_d(A_1^2K_{0001}^2L_{2000}+A_2^2L_{0001}^2L_{2000})}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$
$$-\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$
$$-\frac{A_1A_2A_4K_{0001}L_{0001}L_{2000}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$
$$-\frac{A_1A_2A_3K_{0001}K_{2000}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$
$$-\frac{A_4\omega_d(A_1^2K_{0001}^2L_{2000}+A_2^2L_{0001}^2L_{2000})}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$
$$+\frac{3A_2^2A_4L_{0001}^2L_{2000}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)\Big\} \tag{A.40}$$

$$x_{2,2}^{bs1s2} = \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{bs1s2}, u_1, u_1\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_1(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{bs1s2}, u_1, u_2\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{bs1s2}, u_2, u_1\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_1(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{bs1s2}, u_2, u_2\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2$$

(A. 41)

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{\boldsymbol{2,2}}^{\boldsymbol{bs1s2}},\boldsymbol{u_1},\boldsymbol{u_1}\right)}}(\boldsymbol{t}-\boldsymbol{\tau_1},\boldsymbol{t}-\boldsymbol{\tau_2}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{A_2A_3^2K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0010}K_{1100}L_{0010}+A_2A_4^2L_{0010}^2L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$+\frac{A_2A_3^2K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_1A_3^2K_{0010}^2K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_2A_3A_4L_{0010}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0010}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_1A_4^2K_{0010}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_2\right)$$

$$-\frac{A_2 A_4^2 L_{0010}^2 L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+2\varphi_2\right)$$

$$-\frac{A_1 A_4^2 K_{0010} L_{0010} L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1 A_4^2 K_{0010} L_{0010} L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-2\varphi_2\right)$$

$$+\frac{A_1 A_3 A_4 K_{0010}^2 L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2 A_3 A_4 K_{0010} L_{0010} L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1 A_3 A_4 K_{0010}^2 L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{\sigma(A_2 A_3^2 K_{0010} K_{1100} L_{0010}+A_2 A_4^2 L_{0010}^2 L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_1 K_{0010}\sigma(A_3^2 K_{0010} K_{1100}+A_4^2 L_{0010} L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_1 A_3 A_4 K_{0010}^2 L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{1100}L_{0010}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$-\frac{A_1A_3A_4K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_2A_3A_4K_{0010}L_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_2A_3^2K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\Big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\Big)$$

$$+\frac{\omega_d(A_2A_3^2K_{0010}K_{1100}L_{0010}+A_2A_4^2L_{0010}^2L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\Big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)\Big)$$

$$+\frac{A_2A_3^2K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\Big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)\Big)$$

$$+\frac{A_1A_3^2K_{0010}^2K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_2A_3A_4L_{0010}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_1A_3A_4K_{0010}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_2A_4^2L_{0010}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_2\big)$$

$$+\frac{A_1A_4^2K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+2\varphi_2\big)$$

$$+\frac{A_2A_3A_4L_{0010}\omega_d(2K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_2\big)$$

$$+\frac{A_2A_4^2L_{0010}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+2\varphi_2\big)$$

$$+\frac{A_1\omega_d(2A_3^2K_{0010}^2K_{1100}+A_4^2K_{0010}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_1A_4^2K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1-2\varphi_2\big)$$

$$+\frac{A_1A_3A_4K_{0010}\omega_d(2K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$+\frac{A_2A_3A_4K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_1A_3A_4K_{0010}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$-\frac{3\omega_d(A_2A_3^2K_{0010}K_{1100}L_{0010}+A_2A_4^2L_{0010}^2L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{3A_1K_{0010}\omega_d(A_3^2K_{0010}K_{1100}+A_4^2L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{3A_1A_3A_4K_{0010}^2L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{3A_2A_3A_4K_{1100}L_{0010}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{3A_1A_3A_4K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3A_2A_3A_4K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0010}^2K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4L_{0010}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0010}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0010}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3^2K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{A_2A_3^2K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0010}K_{1100}L_{0010}+A_2A_4^2L_{0010}^2L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\sigma(A_2A_3^2K_{0010}K_{1100}L_{0010}+A_2A_4^2L_{0010}^2L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1A_3^2K_{0010}^2K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_4^2K_{0010}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2A_3A_4L_{0010}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0010}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{1100}L_{0010}^2\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1\sigma(A_3^2K_{0010}^2K_{1100}+A_4^2K_{0010}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2A_3A_4K_{0010}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1K_{0010}\sigma(A_3^2K_{0010}K_{1100}+A_4^2L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1A_3A_4K_{0010}^2L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2A_3A_4K_{1100}L_{0010}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_2\Big)$$

$$+\frac{A_1A_3A_4K_{0010}K_{1100}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1+\varphi_2\Big)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\Big(-\omega_d\Big((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_2\Big)$$

$$-\frac{A_1A_3^2K_{0010}^2K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1\Big)$$

$$-\frac{A_2A_3A_4L_{0010}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_2\Big)$$

$$-\frac{A_1A_3A_4K_{0010}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1+\varphi_2\Big)$$

$$-\frac{A_2A_4^2L_{0010}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+2\varphi_2\Big)$$

$$-\frac{A_1A_4^2K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1+2\varphi_2\Big)$$

$$-\frac{A_2A_3^2K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{A_2A_3^2K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d(A_2A_3^2K_{0010}K_{1100}L_{0010}+A_2A_4^2L_{0010}^2L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d(A_2A_3^2K_{0010}K_{1100}L_{0010}+A_2A_4^2L_{0010}^2L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1A_3^2K_{0010}^2K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_4^2K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3A_4L_{0010}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0010}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{1100}L_{0010}^2\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$
$$-\frac{A_1\omega_d(A_3^2K_{0010}^2K_{1100}+A_4^2K_{0010}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$
$$-\frac{A_1A_3A_4K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$
$$-\frac{A_2A_3A_4K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$
$$-\frac{A_1A_3A_4K_{0010}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$
$$+\frac{3A_1K_{0010}\omega_d(A_3^2K_{0010}K_{1100}+A_4^2L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$
$$+\frac{3A_1A_3A_4K_{0010}^2L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$
$$+\frac{3A_2A_3A_4K_{1100}L_{0010}^2\omega_d}{\sigma^2+9\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$
$$+\frac{3A_1A_3A_4K_{0010}K_{1100}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{3A_2A_3A_4K_{0010}L_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\Big(-\omega_d\Big((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_2\Big)\Big\}$$

$$(\mathrm{A}.42)$$

$$h_{2_{\left(x_{2,2}^{bs1s2},u_1,u_2\right)}}(t-\tau_1,t-\tau_2)=\frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{A_2A_3^2K_{0001}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0001}K_{1100}L_{0010}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$+\frac{A_2A_3^2K_{0001}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_1A_3^2K_{0001}K_{0010}K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_2A_3A_4L_{0010}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_1A_4^2K_{0010}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0010}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_4^2K_{0010}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-2\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{\sigma(A_2A_3^2K_{0001}K_{1100}L_{0010}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_1\sigma(A_3^2K_{0001}K_{0010}K_{1100}+A_4^2K_{0010}L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{1100}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$-\frac{A_1A_3A_4K_{0010}K_{1100}L_{0001}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_2A_3A_4K_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_2A_3^2K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos(\omega_d((t-\tau_1)+(t-\tau_2)))$$

$$+\frac{\omega_d(A_2A_3^2K_{0001}K_{1100}L_{0010}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\Big(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)\Big)$$

$$+\frac{A_2A_3^2K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\Big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)\Big)$$

$$+\frac{A_1A_3^2K_{0001}K_{0010}K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_2A_3A_4L_{0010}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_1A_3A_4K_{0010}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_1A_4^2K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+2\varphi_2\right)$$

$$+\frac{A_2A_3A_4L_{0010}\omega_d(2K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+2\varphi_2\right)$$

$$+\frac{A_1\omega_d(2A_3^2K_{0001}K_{0010}K_{1100}+A_4^2K_{0010}L_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_4^2K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-2\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}\omega_d(2K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2A_3A_4K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3\omega_d(A_2A_3^2K_{0001}K_{1100}L_{0010}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{3A_1\omega_d(A_3^2K_{0001}K_{0010}K_{1100}+A_4^2K_{0010}L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{3A_1A_3A_4K_{0001}K_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{3A_2A_3A_4K_{1100}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{3A_1A_3A_4K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3A_2A_3A_4K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0001}K_{0010}K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4L_{0010}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0010}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3^2K_{0001}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{A_2A_3^2K_{0001}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0001}K_{1100}L_{0010}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\sigma(A_2A_3^2K_{0001}K_{1100}L_{0010}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1A_3^2K_{0001}K_{0010}K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_4^2K_{0010}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2A_3A_4L_{0010}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{1100}L_{0001}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1\sigma(A_3^2K_{0001}K_{0010}K_{1100}+A_4^2K_{0010}L_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2A_3A_4K_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1\sigma(A_3^2K_{0001}K_{0010}K_{1100}+A_4^2K_{0010}L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2A_3A_4K_{1100}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0010}K_{1100}L_{0001}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0001}K_{0010}K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4L_{0010}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3^2K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)\right)$$

$$-\frac{A_2A_3^2K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d(A_2A_3^2K_{0001}K_{1100}L_{0010}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d(A_2A_3^2K_{0001}K_{1100}L_{0010}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1A_3^2K_{0001}K_{0010}K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_4^2K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0010}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3A_4L_{0010}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{1100}L_{0001}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1\omega_d(A_3^2K_{0001}K_{0010}K_{1100}+A_4^2K_{0010}L_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_3A_4K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{3A_1\omega_d(A_3^2K_{0001}K_{0010}K_{1100}+A_4^2K_{0010}L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{3A_1A_3A_4K_{0001}K_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{3A_2A_3A_4K_{1100}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_1A_3A_4K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{3A_2A_3A_4K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)\Big\} \quad (A.43)$$

$$\boldsymbol{h_{2_{\left(x_{2,2}^{bs1s2},u_2,u_1\right)}}(t-\tau_1,t-\tau_2)}=\frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{A_2A_3^2K_{0010}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0010}K_{1100}L_{0001}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$+\frac{A_2A_3^2K_{0010}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_1A_3^2K_{0001}K_{0010}K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_2A_3A_4L_{0001}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_1A_4^2K_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_4^2K_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-2\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{\sigma(A_2A_3^2K_{0010}K_{1100}L_{0001}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_1\sigma(A_3^2K_{0001}K_{0010}K_{1100}+A_4^2K_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{1100}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{1100}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_2A_3A_4K_{0010}L_{0001}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_2A_3^2K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\omega_d(A_2A_3^2K_{0010}K_{1100}L_{0001}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$+\frac{A_2A_3^2K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_1A_3^2K_{0001}K_{0010}K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_2A_3A_4L_{0001}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_1A_4^2K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+2\varphi_2\right)$$

$$+\frac{A_2A_3A_4L_{0001}\omega_d(2K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+2\varphi_2\right)$$

$$+\frac{A_1\omega_d(2A_3^2K_{0001}K_{0010}K_{1100}+A_4^2K_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_4^2K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-2\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}\omega_d(2K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2A_3A_4K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{3\omega_d(A_2A_3^2K_{0010}K_{1100}L_{0001}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\right)$$

$$-\frac{3A_1\omega_d(A_3^2K_{0001}K_{0010}K_{1100}+A_4^2K_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{3A_1A_3A_4K_{0001}K_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$-\frac{3A_2A_3A_4K_{1100}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$-\frac{3A_1A_3A_4K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$-\frac{3A_2A_3A_4K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$-\frac{A_1A_3^2K_{0001}K_{0010}K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4L_{0001}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3^2K_{0010}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{A_2A_3^2K_{0010}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0010}K_{1100}L_{0001}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\sigma(A_2A_3^2K_{0010}K_{1100}L_{0001}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1A_3^2K_{0001}K_{0010}K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_4^2K_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2A_3A_4L_{0001}\sigma(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{1100}L_{0001}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1\sigma(A_3^2K_{0001}K_{0010}K_{1100}+A_4^2K_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{1100}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2A_3A_4K_{0010}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0010}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1\sigma(A_3^2K_{0001}K_{0010}K_{1100}+A_4^2K_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{0010}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2A_3A_4K_{1100}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{1100}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0001}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0001}K_{0010}K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4L_{0001}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3^2K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)\right)$$

$$-\frac{A_2A_3^2K_{0010}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d(A_2A_3^2K_{0010}K_{1100}L_{0001}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d(A_2A_3^2K_{0010}K_{1100}L_{0001}+A_2A_4^2L_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1A_3^2K_{0001}K_{0010}K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_4^2K_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3A_4L_{0001}\omega_d(K_{1100}L_{0010}+K_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}L_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{1100}L_{0001}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1\omega_d(A_3^2K_{0001}K_{0010}K_{1100}+A_4^2K_{0001}L_{0010}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{0010}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{3A_1\omega_d(A_3^2K_{0001}K_{0010}K_{1100}+A_4^2K_{0001}L_{0010}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{3A_1A_3A_4K_{0001}K_{0010}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{3A_2A_3A_4K_{1100}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_1A_3A_4K_{0001}K_{1100}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{3A_2A_3A_4K_{0010}L_{0001}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)} \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)\Big\}$$

(A. 44)

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{2,2}^{bs1s2},u_2,u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{\frac{A_2A_3^2K_{0001}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}\right.$$

$$\times\sin\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\right)$$

$$+\frac{\sigma(A_2A_3^2K_{0001}K_{1100}L_{0001}+A_2A_4^2L_{0001}^2L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)\right)$$

$$+\frac{A_2A_3^2K_{0001}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)\right)$$

$$+\frac{A_1A_3^2K_{0001}^2K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$+\frac{A_2A_3A_4L_{0001}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_1A_3A_4K_{0001}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$+\frac{A_2A_4^2L_{0001}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_2\big)$$

$$+\frac{A_1A_4^2K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+2\varphi_2\big)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_1A_4^2K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-2\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{\sigma(A_2A_3^2K_{0001}K_{1100}L_{0001}+A_2A_4^2L_{0001}^2L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_1K_{0001}\sigma(A_3^2K_{0001}K_{1100}+A_4^2L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$-\frac{A_1A_3A_4K_{0001}^2L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{1100}L_{0001}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}K_{1100}L_{0001}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_2A_3A_4K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_2A_3^2K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\omega_d\left(A_2A_3^2K_{0001}K_{1100}L_{0001}+A_2A_4^2L_{0001}^2L_{1100}\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$+\frac{A_2A_3^2K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_1A_3^2K_{0001}^2K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1\right)$$

$$+\frac{A_2A_3A_4L_{0001}\omega_d\left(K_{1100}L_{0001}+K_{0001}L_{1100}\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}\omega_d\left(K_{1100}L_{0001}+K_{0001}L_{1100}\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0001}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_2\right)$$

$$+\frac{A_1A_4^2K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+2\varphi_2\right)$$

$$+\frac{A_2A_3A_4L_{0001}\omega_d(2K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0001}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)+2\varphi_2\right)$$

$$+\frac{A_1\omega_d(2A_3^2K_{0001}^2K_{1100}+A_4^2K_{0001}L_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1\right)$$

$$+\frac{A_1A_4^2K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1-2\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}\omega_d(2K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2A_3A_4K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_1+\varphi_2\right)$$

$$-\frac{3\omega_d(A_2A_3^2K_{0001}K_{1100}L_{0001}+A_2A_4^2L_{0001}^2L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\right)$$

$$-\frac{3A_1K_{0001}\omega_d(A_3^2K_{0001}K_{1100}+A_4^2L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1\big)$$

$$-\frac{3A_1A_3A_4K_{0001}^2L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1-\varphi_2\big)$$

$$-\frac{3A_2A_3A_4K_{1100}L_{0001}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$-\frac{3A_1A_3A_4K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_1+\varphi_2\big)$$

$$-\frac{3A_2A_3A_4K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$-\frac{A_1A_3^2K_{0001}^2K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4L_{0001}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+2\varphi_2\Big)$$

$$-\frac{A_1A_4^2K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1+2\varphi_2\Big)$$

$$-\frac{A_2A_3^2K_{0001}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d\Big((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)\Big)$$

$$+\frac{A_2A_3^2K_{0001}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d\Big(\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)+(t-\tau_2)\Big)\Big)$$

$$+\frac{\sigma(A_2A_3^2K_{0001}K_{1100}L_{0001}+A_2A_4^2L_{0001}^2L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d\Big((t-\tau_1)-(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\Big)\Big)$$

$$-\frac{\sigma(A_2A_3^2K_{0001}K_{1100}L_{0001}+A_2A_4^2L_{0001}^2L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d\Big(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\Big)\Big)$$

$$-\frac{A_1A_3^2K_{0001}^2K_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d\Big((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1\Big)$$

$$-\frac{A_1A_4^2K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d\Big((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1-2\varphi_2\Big)$$

$$-\frac{A_1A_3A_4K_{0001}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\Big(\omega_d\Big((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\Big)+\varphi_1-\varphi_2\Big)$$

$$+\frac{A_2A_3A_4L_{0001}\sigma(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_2A_4^2L_{0001}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{1100}L_{0001}^2\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1\sigma(A_3^2K_{0001}^2K_{1100}+A_4^2K_{0001}L_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{1100}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2A_3A_4K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}^2L_{1100}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{A_1K_{0001}\sigma(A_3^2K_{0001}K_{1100}+A_4^2L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$+\frac{A_1A_3A_4K_{0001}^2L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$+\frac{A_2A_3A_4K_{1100}L_{0001}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{A_1A_3A_4K_{0001}K_{1100}L_{0001}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{0001}L_{0001}L_{1100}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3^2K_{0001}^2K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1\right)$$

$$-\frac{A_2A_3A_4L_{0001}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_1A_4^2K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_1+2\varphi_2\right)$$

$$-\frac{A_2A_3^2K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{A_2A_3^2K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d(A_2A_3^2K_{0001}K_{1100}L_{0001}+A_2A_4^2L_{0001}^2L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d(A_2A_3^2K_{0001}K_{1100}L_{0001}+A_2A_4^2L_{0001}^2L_{1100})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$-\frac{A_1A_3^2K_{0001}^2K_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$

$$-\frac{A_1A_4^2K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-2\varphi_2\right)$$

$$-\frac{A_1A_3A_4K_{0001}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$

$$-\frac{A_2A_3A_4L_{0001}\omega_d(K_{1100}L_{0001}+K_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_2A_4^2L_{0001}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_2A_3A_4K_{1100}L_{0001}^2\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$
$$-\frac{A_1\omega_d(A_3^2K_{0001}^2K_{1100}+A_4^2K_{0001}L_{0001}L_{1100})}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$
$$-\frac{A_1A_3A_4K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$
$$-\frac{A_2A_3A_4K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$
$$-\frac{A_1A_3A_4K_{0001}^2L_{1100}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$
$$+\frac{3A_1K_{0001}\omega_d(A_3^2K_{0001}K_{1100}+A_4^2L_{0001}L_{1100})}{\sigma^2+9\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1\right)$$
$$+\frac{3A_1A_3A_4K_{0001}^2L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1-\varphi_2\right)$$
$$+\frac{3A_2A_3A_4K_{1100}L_{0001}^2\omega_d}{\sigma^2+9\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$
$$+\frac{3A_1A_3A_4K_{0001}K_{1100}L_{0001}\omega_d}{\sigma^2+9\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_1+\varphi_2\right)$$

$$+\frac{3A_2A_3A_4K_{0001}L_{0001}L_{1100}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)\Big\}$$

$$(A.45)$$

$$x_{2,2}^{qs2} = \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{qs2},u_1,u_1\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{qs2},u_1,u_2\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_2(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{qs2},u_2,u_1\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{qs2},u_2,u_2\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_2(\tau_2)\, d\tau_1 d\tau_2$$

(A. 46)

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{2,2}^{qs2},u_1,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau_1},\boldsymbol{t}-\boldsymbol{\tau_2}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{-\frac{2\sigma\left(-4A_3^3K_{0010}^2K_{0200}\omega_d^2 + A_3A_4^2K_{0010}L_{0010}L_{0200}\left(\sigma^2+\omega_d^2\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_4\sigma\left(A_4^2L_{0010}^2L_{0200}\left(\sigma^2+\omega_d^2\right) - A_3^2K_{0010}\left(16K_{0200}L_{0010}\omega_d^2 + K_{0010}L_{0200}\left(\sigma^2+9\omega_d^2\right)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{2A_3A_4^2L_{0010}\sigma\left(4K_{0200}L_{0010}\omega_d^2 + K_{0010}L_{0200}\left(\sigma^2+9\omega_d^2\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_4^3L_{0010}^2L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+3\varphi_2\right)$$

$$-\frac{A_4\sigma(A_3^2K_{0010}^2L_{0200} + A_4^2L_{0010}^2L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2K_{0010}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_2\right)$$

$$-\frac{A_4\sigma(A_3^2K_{0010}^2L_{0200} + A_4^2L_{0010}^2L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2K_{0010}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_3^2A_4K_{0010}^2L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\right)$$

$$-\frac{2\omega_d\left(A_3^3K_{0010}^2K_{0200}(\sigma^2-3\omega_d^2)+3A_3A_4^2K_{0010}L_{0010}L_{0200}(\sigma^2+\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\right)$$

$$+\frac{2\omega_d\left(A_3^3K_{0010}^2K_{0200}+A_3A_4^2L_{0010}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)\right)$$

$$-\frac{A_4\omega_d\left(3A_4^2L_{0010}^2L_{0200}(\sigma^2+\omega_d^2)-A_3^2K_{0010}\left(-4K_{0200}L_{0010}(\sigma^2-3\omega_d^2)+K_{0010}L_{0200}(\sigma^2+9\omega_d^2)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\right)$$

$$+\frac{2A_3A_4^2L_{0010}\omega_d\left(-K_{0200}L_{0010}(\sigma^2-3\omega_d^2)+K_{0010}L_{0200}(\sigma^2+9\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_2\right)$$

$$+\frac{A_4^3L_{0010}^2L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+3\varphi_2\right)$$

$$+\frac{A_4\omega_d\left(A_4^2L_{0010}^2L_{0200}+A_3^2K_{0010}(2K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0010}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+2\varphi_2\right)$$

$$+\frac{A_4\omega_d\left(A_4^2L_{0010}^2L_{0200}+A_3^2K_{0010}(2K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0010}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_2\right)$$

$$-\frac{3A_3^2A_4K_{0010}^2L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_3^3K_{0010}^2K_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma\left(A_3^3K_{0010}^2K_{0200}+A_3A_4^2L_{0010}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma\left(A_3^3K_{0010}^2K_{0200}+A_3A_4^2L_{0010}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\sigma(A_3^3K_{0010}^2K_{0200}+2A_3A_4^2K_{0010}L_{0010}L_{0200})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_4L_{0010}\sigma(2A_3^2K_{0010}K_{0200}+A_4^2L_{0010}L_{0200})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0200}L_{0010}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0010}\sigma(2K_{0200}L_{0010}+K_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2L_{0010}\sigma(K_{0200}L_{0010}+2K_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_4^3L_{0010}^2L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+3\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0010}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0010}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{A_4\sigma\left(A_4^2L_{0010}^2L_{0200}+A_3^2K_{0010}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0010}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$+\frac{A_4\sigma\left(A_4^2L_{0010}^2L_{0200}+A_3^2K_{0010}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0010}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0010}^2L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3^3K_{0010}^2K_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\omega_d\left(A_3^3K_{0010}^2K_{0200}+A_3A_4^2L_{0010}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d\left(A_3^3K_{0010}^2K_{0200}+A_3A_4^2L_{0010}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d(A_3^3K_{0010}^2K_{0200}+2A_3A_4^2K_{0010}L_{0010}L_{0200})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_4L_{0010}\omega_d(2A_3^2K_{0010}K_{0200}+A_4^2L_{0010}L_{0200})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_3A_4^2K_{0200}L_{0010}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0010}\omega_d(2K_{0200}L_{0010}+K_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2L_{0010}\omega_d(K_{0200}L_{0010}+2K_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_4^3L_{0010}^2L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+3\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0010}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0010}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_4\omega_d\left(A_4^2L_{0010}^2L_{0200}+A_3^2K_{0010}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2K_{0010}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_4\omega_d\left(A_4^2L_{0010}^2L_{0200}+A_3^2K_{0010}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2K_{0010}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$+\frac{3A_3^2A_4K_{0010}^2L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\, \cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)\Bigg\}$$

(A. 47)

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{2,2}^{qs2},u_1,u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{-\frac{\sigma\left(-8A_3^3K_{0001}K_{0010}K_{0200}\omega_d^2 + A_3A_4^2L_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})\left(\sigma^2+\omega_d^2\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{A_3A_4^2L_{0200}\sigma(K_{0010}L_{0001} - K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$-\frac{A_4^3L_{0001}L_{0010}L_{0200}\sigma\left(\sigma^2+\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{8A_3^2A_4K_{0010}K_{0200}L_{0001}\sigma\omega_d^2}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_3^2A_4K_{0001}\sigma\left(8K_{0200}L_{0010}\omega_d^2 + K_{0010}L_{0200}\left(\sigma^2+9\omega_d^2\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2\sigma\left(K_{0010}L_{0001}L_{0200}\left(\sigma^2+9\omega_d^2\right) + L_{0010}\left(8K_{0200}L_{0001}\omega_d^2 + K_{0001}L_{0200}\left(\sigma^2+9\omega_d^2\right)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_4^3L_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+3\varphi_2\right)$$

$$-\frac{A_4\sigma(A_3^2K_{0001}K_{0010}L_{0200} + A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2K_{0010}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_2\right)$$

$$-\frac{A_4\sigma(A_3^2K_{0001}K_{0010}L_{0200}+A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2K_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_3^2A_4K_{0001}K_{0010}L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{\omega_d\left(2A_3^3K_{0001}K_{0010}K_{0200}(\sigma^2-3\omega_d^2)+3A_3A_4^2L_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^2+\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\omega_d\left(2A_3^3K_{0001}K_{0010}K_{0200}+A_3A_4^2(2K_{0200}L_{0001}L_{0010}+K_{0010}L_{0001}L_{0200}+K_{0001}L_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$-\frac{3A_4^3L_{0001}L_{0010}L_{0200}\omega_d(\sigma^2+\omega_d^2)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{2A_3^2A_4K_{0001}K_{0200}L_{0010}\omega_d(\sigma^2-3\omega_d^2)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0010}\omega_d\left(2K_{0200}L_{0001}(\sigma^2-3\omega_d^2)-K_{0001}L_{0200}(\sigma^2+9\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2\omega_d\left(-2K_{0200}L_{0001}L_{0010}\left(\sigma^2-3\omega_d^2\right)+L_{0200}\left(K_{0010}L_{0001}+K_{0001}L_{0010}\right)\left(\sigma^2+9\omega_d^2\right)\right)}{\left(\sigma^2+\omega_d^2\right)\left(\sigma^2+9\omega_d^2\right)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_4^3L_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+3\varphi_2\right)$$

$$+\frac{A_4\omega_d\left(A_4^2L_{0001}L_{0010}L_{0200}+A_3^2K_{0010}\left(2K_{0200}L_{0001}+K_{0001}L_{0200}\right)\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0010}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_4\omega_d\left(A_4^2L_{0001}L_{0010}L_{0200}+A_3^2K_{0001}\left(2K_{0200}L_{0010}+K_{0010}L_{0200}\right)\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_2\right)$$

$$-\frac{3A_3^2A_4K_{0001}K_{0010}L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_3^3K_{0001}K_{0010}K_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\omega_d\left(A_3^3K_{0001}K_{0010}K_{0200}+A_3A_4^2L_{0010}\left(K_{0200}L_{0001}+K_{0001}L_{0200}\right)\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma\left(t+\min(-\tau_1,-\tau_2)\right)}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d\left(A_3^3K_{0001}K_{0010}K_{0200}+A_3A_4^2L_{0001}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d\left(A_3^3K_{0001}K_{0010}K_{0200}+A_3A_4^2L_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})\right)}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_4\omega_d(A_3^2K_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})+A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_3A_4^2K_{0200}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_3^2A_4\omega_d(K_{0010}K_{0200}L_{0001}+K_{0001}K_{0200}L_{0010}+K_{0001}K_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2\omega_d(K_{0200}L_{0001}L_{0010}+K_{0010}L_{0001}L_{0200}+K_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_4^3L_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+3\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0001}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0010}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_4\omega_d\left(A_4^2L_{0001}L_{0010}L_{0200}+A_3^2K_{0010}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2K_{0010}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_4\omega_d\left(A_4^2L_{0001}L_{0010}L_{0200}+A_3^2K_{0001}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2K_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$+\frac{3A_3^2A_4K_{0001}K_{0010}L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3^3K_{0001}K_{0010}K_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma\left(A_3^3K_{0001}K_{0010}K_{0200}+A_3A_4^2L_{0010}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma\left(A_3^3K_{0001}K_{0010}K_{0200}+A_3A_4^2L_{0001}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\sigma\left(A_3^3K_{0001}K_{0010}K_{0200}+A_3A_4^2L_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})\right)}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_4\sigma(A_3^2K_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})+A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0200}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_3^2A_4\sigma(K_{0010}K_{0200}L_{0001}+K_{0001}K_{0200}L_{0010}+K_{0001}K_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2\sigma(K_{0200}L_{0001}L_{0010}+K_{0010}L_{0001}L_{0200}+K_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_4^3L_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+3\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0001}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0010}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{A_4\sigma\left(A_4^2L_{0001}L_{0010}L_{0200}+A_3^2K_{0010}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0010}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$+\frac{A_4\sigma\left(A_4^2L_{0001}L_{0010}L_{0200}+A_3^2K_{0001}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0001}K_{0010}L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)\Big\} \quad (A.48)$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{2,2}^{qs2},u_2,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_1,\boldsymbol{t}-\boldsymbol{\tau}_2)=\frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{-\frac{\sigma\left(-8A_3^3K_{0001}K_{0010}K_{0200}\omega_d^2+A_3A_4^2L_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})\left(\sigma^2+\omega_d^2\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_3A_4^2L_{0200}\sigma(K_{0010}L_{0001}-K_{0001}L_{0010})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$-\frac{A_4^3L_{0001}L_{0010}L_{0200}\sigma\left(\sigma^2+\omega_d^2\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_3^2A_4\sigma\left(8K_{0010}K_{0200}L_{0001}\omega_d^2+K_{0001}\left(8K_{0200}L_{0010}\omega_d^2+K_{0010}L_{0200}\left(\sigma^2+9\omega_d^2\right)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2\sigma\left(K_{0010}L_{0001}L_{0200}\left(\sigma^2+9\omega_d^2\right)+L_{0010}\left(8K_{0200}L_{0001}\omega_d^2+K_{0001}L_{0200}\left(\sigma^2+9\omega_d^2\right)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_4^3L_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+3\varphi_2\right)$$

$$-\frac{A_4\sigma(A_3^2K_{0001}K_{0010}L_{0200}+A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2K_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_2\right)$$

$$-\frac{A_4\sigma(A_3^2K_{0001}K_{0010}L_{0200}+A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+\varphi_2\big)$$

$$-\frac{A_3A_4^2K_{0010}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\sin\big(\omega_d\big((t-\tau_1)-(t-\tau_2)\big)+2\varphi_2\big)$$

$$+\frac{A_3^2A_4K_{0001}K_{0010}L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\big(-\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$-\frac{\omega_d\left(2A_3^3K_{0001}K_{0010}K_{0200}(\sigma^2-3\omega_d^2)+3A_3A_4^2L_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^2+\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)\right)$$

$$+\frac{\omega_d\left(2A_3^3K_{0001}K_{0010}K_{0200}+A_3A_4^2(2K_{0200}L_{0001}L_{0010}+K_{0010}L_{0001}L_{0200}+K_{0001}L_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\big((t-\tau_2)-(t-\tau_1)\big)\right)$$

$$-\frac{3A_4^3L_{0001}L_{0010}L_{0200}\omega_d(\sigma^2+\omega_d^2)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$-\frac{A_3^2A_4\omega_d\left(2K_{0001}K_{0200}L_{0010}(\sigma^2-3\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$-\frac{A_3^2A_4K_{0010}\omega_d\left(2K_{0200}L_{0001}(\sigma^2-3\omega_d^2)-K_{0001}L_{0200}(\sigma^2+9\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+\varphi_2\big)$$

$$+\frac{A_3A_4^2\omega_d\left(-2K_{0200}L_{0001}L_{0010}(\sigma^2-3\omega_d^2)+L_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^2+9\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\big(\omega_d\big((t-\tau_1)+(t-\tau_2)\big)+2\varphi_2\big)$$

$$+\frac{A_4^3 L_{0001} L_{0010} L_{0200} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+3\varphi_2\right)$$

$$+\frac{A_4 \omega_d \left(A_4^2 L_{0001} L_{0010} L_{0200}+A_3^2 K_{0001}(2K_{0200} L_{0010}+K_{0010} L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_3 A_4^2 K_{0001} L_{0010} L_{0200} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_4 \omega_d \left(A_4^2 L_{0001} L_{0010} L_{0200}+A_3^2 K_{0010}(2K_{0200} L_{0001}+K_{0001} L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_3 A_4^2 K_{0010} L_{0001} L_{0200} \omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_2\right)$$

$$-\frac{3A_3^2 A_4 K_{0001} K_{0010} L_{0200} \omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_3^3 K_{0001} K_{0010} K_{0200} \sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma\left(A_3^3 K_{0001} K_{0010} K_{0200}+A_3 A_4^2 L_{0001}(K_{0200} L_{0010}+K_{0010} L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma\left(A_3^3 K_{0001} K_{0010} K_{0200}+A_3 A_4^2 L_{0010}(K_{0200} L_{0001}+K_{0001} L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\sigma\left(A_3^3K_{0001}K_{0010}K_{0200}+A_3A_4^2L_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})\right)}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_4\sigma(A_3^2K_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})+A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0200}L_{0001}L_{0010}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_3^2A_4\sigma(K_{0010}K_{0200}L_{0001}+K_{0001}K_{0200}L_{0010}+K_{0001}K_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2\sigma(K_{0200}L_{0001}L_{0010}+K_{0010}L_{0001}L_{0200}+K_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_4^3L_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+3\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0010}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0001}K_{0200}L_{0010}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_4\sigma\left(A_4^2L_{0001}L_{0010}L_{0200}+A_3^2K_{0001}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0001}L_{0010}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$+\frac{A_4\sigma\left(A_4^2L_{0001}L_{0010}L_{0200}+A_3^2K_{0010}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0010}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0001}K_{0010}L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3^3K_{0001}K_{0010}K_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\omega_d\left(A_3^3K_{0001}K_{0010}K_{0200}+A_3A_4^2L_{0001}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d\left(A_3^3K_{0001}K_{0010}K_{0200}+A_3A_4^2L_{0010}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\,\cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d\left(A_3^3K_{0001}K_{0010}K_{0200}+A_3A_4^2L_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})\right)}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(3\big(t+\min(-\tau_1,-\tau_2)\big)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_4\omega_d(A_3^2K_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})+A_4^2L_{0001}L_{0010}L_{0200})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$+\frac{3A_3A_4^2K_{0200}L_{0001}L_{0010}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_3^2A_4\omega_d(K_{0010}K_{0200}L_{0001}+K_{0001}K_{0200}L_{0010}+K_{0001}K_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2\omega_d(K_{0200}L_{0001}L_{0010}+K_{0010}L_{0001}L_{0200}+K_{0001}L_{0010}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+2\varphi_2\right)$$

$$-\frac{A_4^3L_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\big(t+\min(-\tau_1,-\tau_2)\big)\right)+3\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0010}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0001}K_{0200}L_{0010}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)$$

$$-\frac{A_4\omega_d\left(A_4^2L_{0001}L_{0010}L_{0200}+A_3^2K_{0001}(K_{0200}L_{0010}+K_{0010}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2K_{0001}L_{0010}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_4\omega_d\left(A_4^2L_{0001}L_{0010}L_{0200}+A_3^2K_{0010}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2K_{0010}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$+\frac{3A_3^2A_4K_{0001}K_{0010}L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)\Bigg\}$$

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$$\boldsymbol{h_{2}}_{\left(x_{2,2}^{qs2},u_2,u_2\right)}(\boldsymbol{t-\tau_1, t-\tau_2}) = \frac{1}{4}e^{-\sigma\left((t-\tau_1)+(t-\tau_2)\right)}$$

$$\times\left\{-\frac{2\sigma\left(-4A_3^3K_{0001}^2K_{0200}\omega_d^2 + A_3A_4^2K_{0001}L_{0001}L_{0200}(\sigma^2+\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}\right.$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{A_4\sigma\left(A_4^2L_{0001}^2L_{0200}(\sigma^2+\omega_d^2) - A_3^2K_{0001}\left(16K_{0200}L_{0001}\omega_d^2 + K_{0001}L_{0200}(\sigma^2+9\omega_d^2)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{2A_3A_4^2L_{0001}\sigma\left(4K_{0200}L_{0001}\omega_d^2 + K_{0001}L_{0200}(\sigma^2+9\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_4^3L_{0001}^2L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+3\varphi_2\right)$$

$$-\frac{A_4\sigma(A_3^2K_{0001}^2L_{0200} + A_4^2L_{0001}^2L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2K_{0001}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_2\right)$$

$$-\frac{A_4\sigma(A_3^2K_{0001}^2L_{0200} + A_4^2L_{0001}^2L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2K_{0001}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_3^2A_4K_{0001}^2L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times\sin\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{2\omega_d\left(A_3^3K_{0001}^2K_{0200}(\sigma^2-3\omega_d^2)+3A_3A_4^2K_{0001}L_{0001}L_{0200}(\sigma^2+\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{2\omega_d\left(A_3^3K_{0001}^2K_{0200}+A_3A_4^2L_{0001}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_2)-(t-\tau_1)\right)\right)$$

$$-\frac{A_4\omega_d\left(3A_4^2L_{0001}^2L_{0200}(\sigma^2+\omega_d^2)-A_3^2K_{0001}\left(-4K_{0200}L_{0001}(\sigma^2-3\omega_d^2)+K_{0001}L_{0200}(\sigma^2+9\omega_d^2)\right)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{2A_3A_4^2L_{0001}\omega_d\left(-K_{0200}L_{0001}(\sigma^2-3\omega_d^2)+K_{0001}L_{0200}(\sigma^2+9\omega_d^2)\right)}{(\sigma^2+\omega_d^2)(\sigma^2+9\omega_d^2)}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_4^3L_{0001}^2L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+3\varphi_2\right)$$

$$+\frac{A_4\omega_d\left(A_4^2L_{0001}^2L_{0200}+A_3^2K_{0001}(2K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0001}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_2\right)$$

$$+\frac{A_4\omega_d\left(A_4^2L_{0001}^2L_{0200}+A_3^2K_{0001}(2K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0001}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times \cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)\right)+2\varphi_2\right)$$

$$-\frac{3A_3^2A_4K_{0001}^2L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times \cos\left(-\omega_d\left((t-\tau_1)+(t-\tau_2)\right)+\varphi_2\right)$$

$$-\frac{A_3^3K_{0001}^2K_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$+\frac{\sigma\left(A_3^3K_{0001}^2K_{0200}+A_3A_4^2L_{0001}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$+\frac{\sigma\left(A_3^3K_{0001}^2K_{0200}+A_3A_4^2L_{0001}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\sigma(A_3^3K_{0001}^2K_{0200}+2A_3A_4^2K_{0001}L_{0001}L_{0200})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{A_4L_{0001}\sigma(2A_3^2K_{0001}K_{0200}+A_4^2L_{0001}L_{0200})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_3A_4^2K_{0200}L_{0001}^2\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_3^2 A_4 K_{0001}\sigma(2K_{0200}L_{0001}+K_{0001}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3 A_4^2 L_{0001}\sigma(K_{0200}L_{0001}+2K_{0001}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_4^3 L_{0001}^2 L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+3\varphi_2\right)$$

$$-\frac{A_3^2 A_4 K_{0001}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3^2 A_4 K_{0001}K_{0200}L_{0001}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_4\sigma\left(A_4^2 L_{0001}^2 L_{0200}+A_3^2 K_{0001}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_3 A_4^2 K_{0001}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$+\frac{A_4\sigma\left(A_4^2 L_{0001}^2 L_{0200}+A_3^2 K_{0001}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times\, e^{\sigma(t+\min(-\tau_1,-\tau_2))}\ \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{A_3 A_4^2 K_{0001}L_{0001}L_{0200}\sigma}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0001}^2L_{0200}\sigma}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \sin\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3^3K_{0001}^2K_{0200}\omega_d}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)\right)$$

$$-\frac{\omega_d\left(A_3^3K_{0001}^2K_{0200}+A_3A_4^2L_{0001}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)+(t-\tau_2)\right)\right)$$

$$-\frac{\omega_d\left(A_3^3K_{0001}^2K_{0200}+A_3A_4^2L_{0001}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(\left(t+\min(-\tau_1,-\tau_2)\right)+(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3\omega_d(A_3^3K_{0001}^2K_{0200}+2A_3A_4^2K_{0001}L_{0001}L_{0200})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left(3\left(t+\min(-\tau_1,-\tau_2)\right)-(t-\tau_1)-(t-\tau_2)\right)\right)$$

$$+\frac{3A_4L_{0001}\omega_d(2A_3^2K_{0001}K_{0200}+A_4^2L_{0001}L_{0200})}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$+\frac{3A_3A_4^2K_{0200}L_{0001}^2\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-3\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$

$$-\frac{A_3^2A_4K_{0001}\omega_d(2K_{0200}L_{0001}+K_{0001}L_{0200})}{\sigma^2+\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))} \cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$

$$-\frac{A_3A_4^2L_{0001}\omega_d(K_{0200}L_{0001}+2K_{0001}L_{0200})}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$
$$-\frac{A_4^3L_{0001}^2L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)+(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+3\varphi_2\right)$$
$$-\frac{A_3^2A_4K_{0001}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$
$$-\frac{A_3^2A_4K_{0001}K_{0200}L_{0001}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$
$$-\frac{A_4\omega_d\left(A_4^2L_{0001}^2L_{0200}+A_3^2K_{0001}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)+(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$
$$-\frac{A_3A_4^2K_{0001}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(-\omega_d\left((t-\tau_1)-(t-\tau_2)-\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$
$$-\frac{A_4\omega_d\left(A_4^2L_{0001}^2L_{0200}+A_3^2K_{0001}(K_{0200}L_{0001}+K_{0001}L_{0200})\right)}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+\varphi_2\right)$$
$$-\frac{A_3A_4^2K_{0001}L_{0001}L_{0200}\omega_d}{\sigma^2+\omega_d^2}$$
$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left((t-\tau_1)-(t-\tau_2)+\left(t+\min(-\tau_1,-\tau_2)\right)\right)+2\varphi_2\right)$$
$$+\frac{3A_3^2A_4K_{0001}^2L_{0200}\omega_d}{\sigma^2+9\omega_d^2}$$

$$\times e^{\sigma(t+\min(-\tau_1,-\tau_2))}\cos\left(\omega_d\left(-(t-\tau_1)-(t-\tau_2)+3\big(t+\min(-\tau_1,-\tau_2)\big)\right)+\varphi_2\right)\Big\} \tag{A.50}$$

$$x_{2,2}^{bs1i1} = \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{bs1i1},u_1,u_1\right)}}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2 + \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{bs1i1},u_2,u_1\right)}}(t-\tau_1, t-\tau_2)u_2(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2 \quad (A.51)$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{2,2}^{bs1i1},u_1,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) =$$

$$\frac{1}{2}\Big(\big(A_3K_{1010}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)$$

$$+A_4L_{1010}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_2)\big)$$

$$\times\big(A_1K_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+A_2L_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\Big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\big)\big)\Big)$$

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$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{2,2}^{bs1i1},u_2,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_1,\boldsymbol{t}-\boldsymbol{\tau}_2)=$$

$$\frac{1}{2}\Big(\big(A_3K_{1010}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)$$

$$+A_4L_{1010}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_2)\big)$$

$$\times\big(A_1K_{0001}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+A_2L_{0001}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\Big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\Big)\Big)\Big)$$

(A.53)

$$x_{2,2}^{bs2i1} = \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{bs2i1}, u_1, u_1\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_1(\tau_2)\, d\tau_1 d\tau_2 + \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{bs2i1}, u_2, u_1\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_1(\tau_2)\, d\tau_1 d\tau_2 \tag{A.54}$$

$$\boldsymbol{h}_{\mathbf{2}_{\left(x_{2,2}^{bs2i1},u_1,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_1,\boldsymbol{t}-\boldsymbol{\tau}_2)=$$

$$\frac{1}{2}\Big(\big(A_3K_{0110}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)$$

$$+A_4L_{0110}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_2)\big)$$

$$\times\big(A_3K_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\Big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\Big)$$

$$+A_4L_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_2\big)\big)\Big) \qquad (A.55)$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{2,2}^{bs2i1},u_2,u_1\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) =$$

$$\frac{1}{2}\Big(\big(A_3K_{0110}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)$$

$$+A_4L_{0110}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_2)\big)$$

$$\times\big(A_3K_{0001}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\Big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\Big)$$

$$+A_4L_{0001}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_2\big)\big)\Big)$$

(A.56)

$$x_{2,2}^{bs1i2} = \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{bs1i2},u_1,u_2\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2$$

$$+ \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{bs1i2},u_2,u_2\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2$$

(A. 57)

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{2,2}^{bs1i2}, u_1, u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}}, \boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) =$$

$$\frac{1}{2}\Big(\big(A_3K_{1001}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)$$

$$+A_4L_{1001}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_2)\big)$$

$$\times\big(A_1K_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+A_2L_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\Big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\Big)\big)\Big)$$

(A. 58)

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{2,2}^{bs1i2},u_2,u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) =$$

$$\frac{1}{2}\Big(\big(A_3K_{1001}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)$$

$$+A_4L_{1001}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_2)\big)$$

$$\times\big(A_1K_{0001}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_1\big)$$

$$+A_2L_{0001}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\Big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\Big)\Big)\Big)$$

(A. 59)

$$x_{2,2}^{bs2i2} = \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{bs2i2},u_1,u_2\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 + \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{bs2i2},u_2,u_2\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 \quad (A.60)$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{2,2}^{bs2i2},u_1,u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) =$$

$$\frac{1}{2}\Big(\big(A_3K_{0101}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)$$

$$+A_4L_{0101}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_2)\big)$$

$$\times\left(A_3K_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}\right.$$

$$\times\sin\left(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\right)$$

$$+A_4L_{0010}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\left.\times\sin\big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_2\big)\big)\right)$$

(A. 61)

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(x_{2,2}^{bs2i2},u_2,u_2\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}},\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) =$$

$$\frac{1}{2}\Big(\big(A_3K_{0101}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin\big(\omega_d\min(t-\tau_1,t-\tau_2)\big)$$

$$+A_4L_{0101}e^{-\sigma\min(t-\tau_1,t-\tau_2)}\sin(\omega_d\min(t-\tau_1,t-\tau_2)+\varphi_2)\big)$$

$$\times\left(A_3K_{0001}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}\right.$$

$$\times\sin\left(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)\right)$$

$$+A_4L_{0001}e^{-\sigma\max\left((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\right)}$$

$$\times\sin\big(\omega_d\max\big((t-\tau_2)-(t-\tau_1),(t-\tau_1)-(t-\tau_2)\big)+\varphi_2\big)\big)\Big)$$

(A. 62)

$$x_{2,2}^{qi1} = \int_0^t \int_0^t h_{2\left(x_{2,2}^{qi1},u_1,u_1\right)}(t-\tau_1, t-\tau_2)u_1(\tau_1)u_1(\tau_2)\, d\tau_1 d\tau_2 \tag{A.63}$$

$$\boldsymbol{h}_{\boldsymbol{2}\left(\boldsymbol{x_{2,2}^{qi1}},\boldsymbol{u_1},\boldsymbol{u_1}\right)}(\boldsymbol{t}-\boldsymbol{\tau_1},\boldsymbol{t}-\boldsymbol{\tau_2}) = A_3 K_{0020} e^{-\sigma(t-\tau_1)} \sin\left(\omega_d(t-\tau_1)\right) \delta\left((t-\tau_2)-(t-\tau_1)\right)$$

$$+A_4 L_{0020} e^{-\sigma(t-\tau_1)} \sin\left(\varphi_2 + \omega_d(t-\tau_1)\right) \delta\left((t-\tau_2)-(t-\tau_1)\right) \tag{A.64}$$

$$x_{2,2}^{bi1i2} = \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{bi1i2},u_1,u_2\right)}}(t-\tau_1, t-\tau_2) u_1(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 \qquad \text{(A. 65)}$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(\boldsymbol{x}_{\boldsymbol{2,2}}^{\boldsymbol{bi1i2}},\boldsymbol{u_1},\boldsymbol{u_2}\right)}}(\boldsymbol{t}-\boldsymbol{\tau_1},\boldsymbol{t}-\boldsymbol{\tau_2}) = A_3 K_{0011} e^{-\sigma(t-\tau_1)} \sin\left(\omega_d(t-\tau_1)\right) \delta\left((t-\tau_2)-(t-\tau_1)\right)$$
$$+A_4 L_{0011} e^{-\sigma(t-\tau_1)} \sin\left(\varphi_2 + \omega_d(t-\tau_1)\right) \delta\left((t-\tau_2)-(t-\tau_1)\right)$$

(A. 66)

$$x_{2,2}^{qi2} = \int_0^t \int_0^t h_{2_{\left(x_{2,2}^{qi2},u_2,u_2\right)}}(t-\tau_1, t-\tau_2) u_2(\tau_1) u_2(\tau_2)\, d\tau_1 d\tau_2 \tag{A.67}$$

$$\boldsymbol{h}_{\boldsymbol{2}_{\left(\boldsymbol{x}_{\boldsymbol{2,2}}^{\boldsymbol{qi2}},\boldsymbol{u}_{\boldsymbol{2}},\boldsymbol{u}_{\boldsymbol{2}}\right)}}(\boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{1}}, \boldsymbol{t}-\boldsymbol{\tau}_{\boldsymbol{2}}) = A_3 K_{0002} e^{-\sigma(t-\tau_1)} \sin\big(\omega_d (t-\tau_1)\big)\, \delta\big((t-\tau_2)-(t-\tau_1)\big)$$

$$+A_4 L_{0002} e^{-\sigma(t-\tau_1)} \sin\left(\varphi_2 + \omega_d (t-\tau_1)\right) \delta\big((t-\tau_2)-(t-\tau_1)\big)$$

(A. 68)

Appendix B

Surface Plots for 2DOF Volterra Kernels

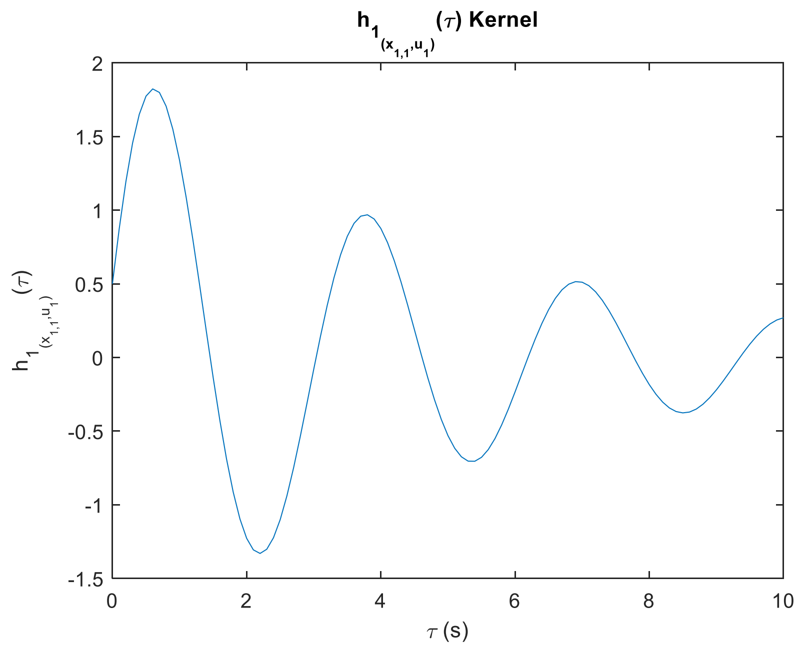

Figure B.1 1st Order Kernel Plot for State 1 w.r.t. Input 1

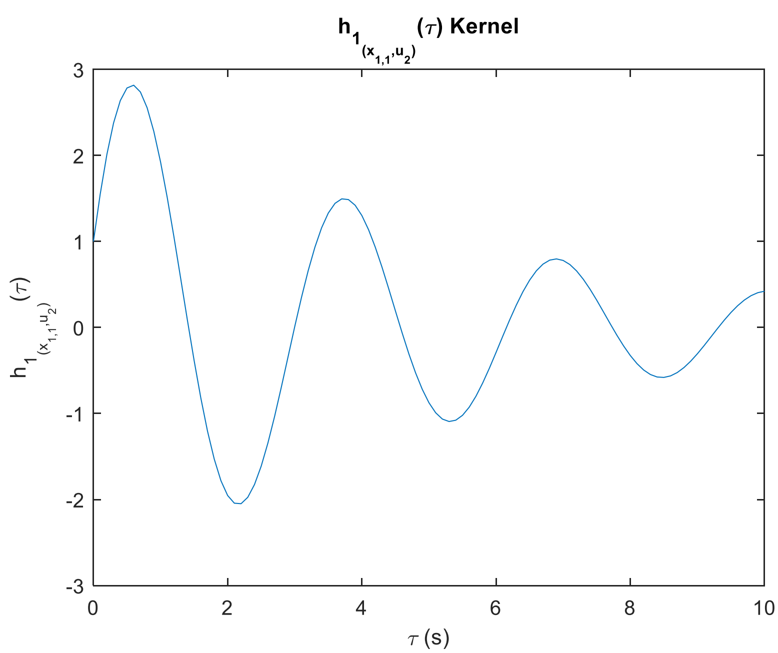

Figure B.2 1st Order Kernel Plot for State 1 w.r.t. Input 2

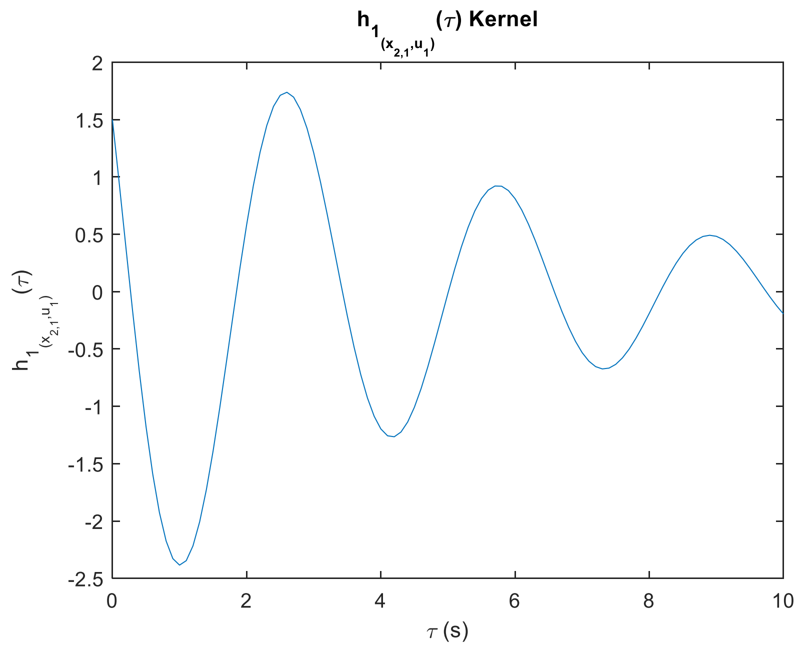

Figure B.3 1st Order Kernel Plot for State 2 w.r.t. Input 1

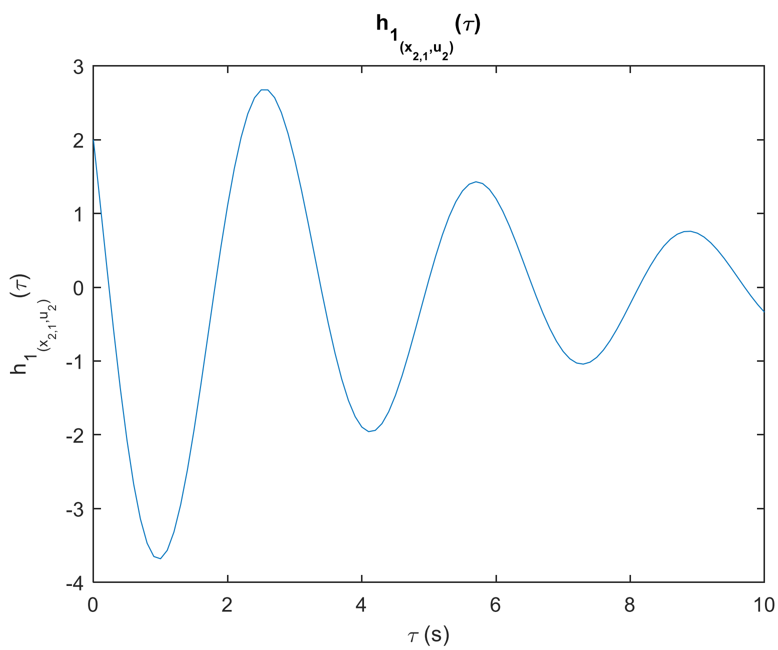

Figure B.4 1st Order Kernel Plot for State 2 w.r.t. Input 2

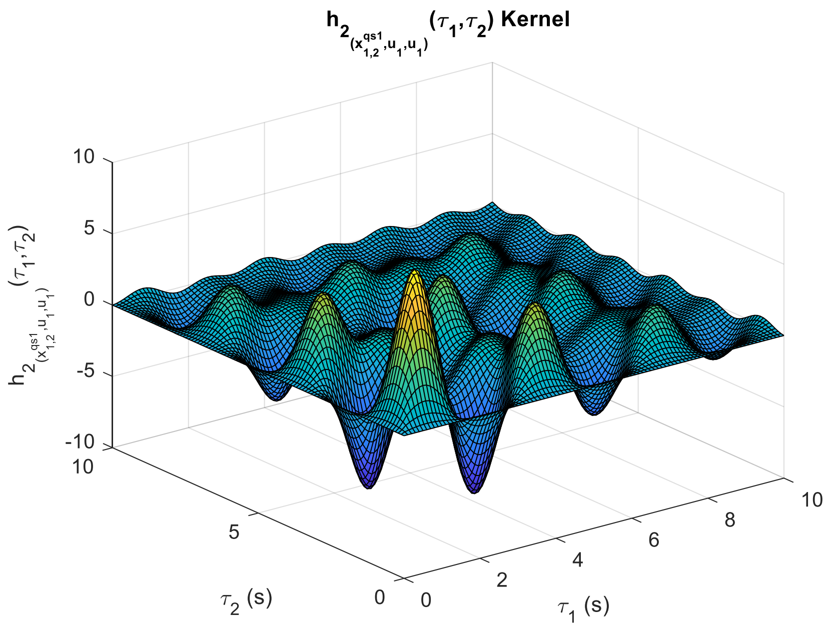

Figure B.5 2nd Order Kernel Plot for State 1 (qs1 Component) w.r.t. Quadratic Input 1

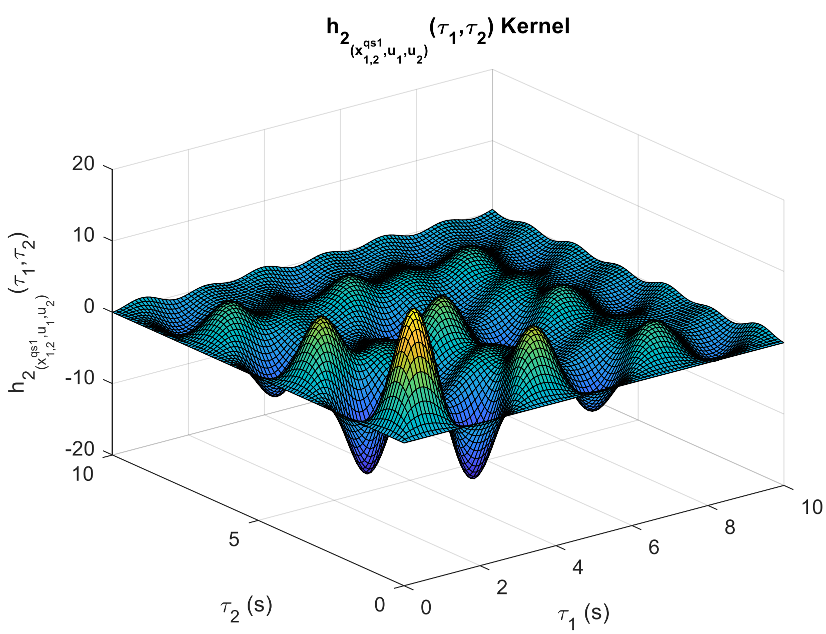

Figure B.6 2nd Order Kernel Plot for State 1 (qs1 Component) w.r.t. Input 1 and Input 2

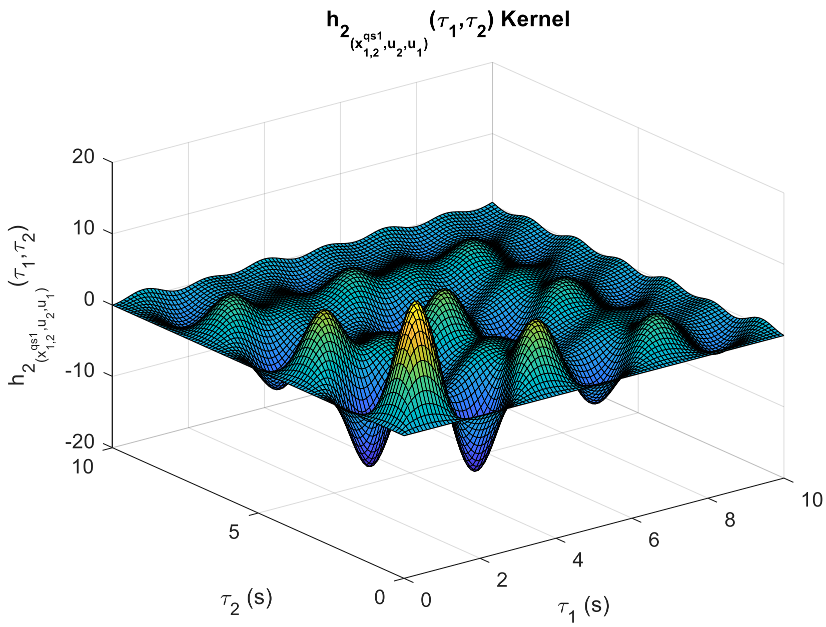

Figure B.7 2nd Order Kernel Plot for State 1 (qs1 Component) w.r.t. Input 2 and Input 1

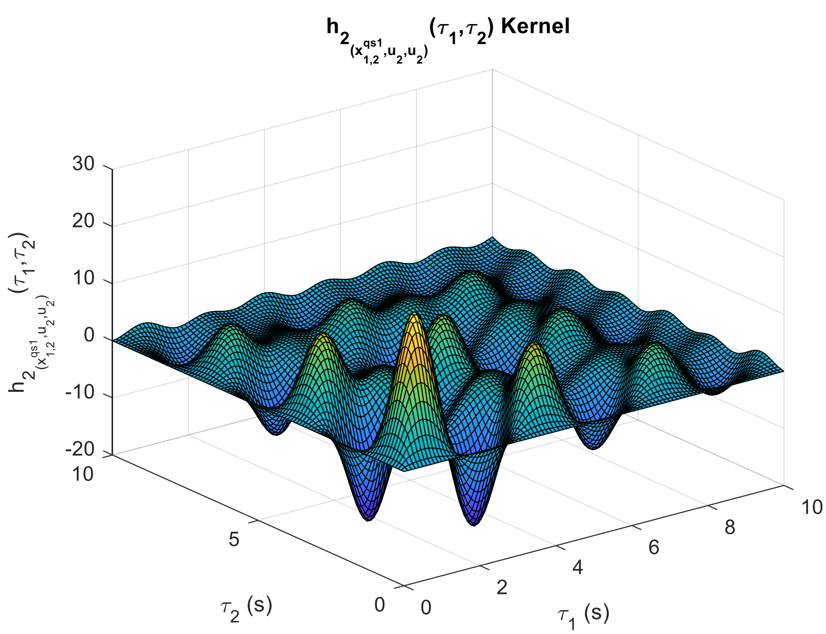

Figure B.8 2nd Order Kernel Plot for State 1 (qs1 Component) w.r.t. Quadratic Input 2

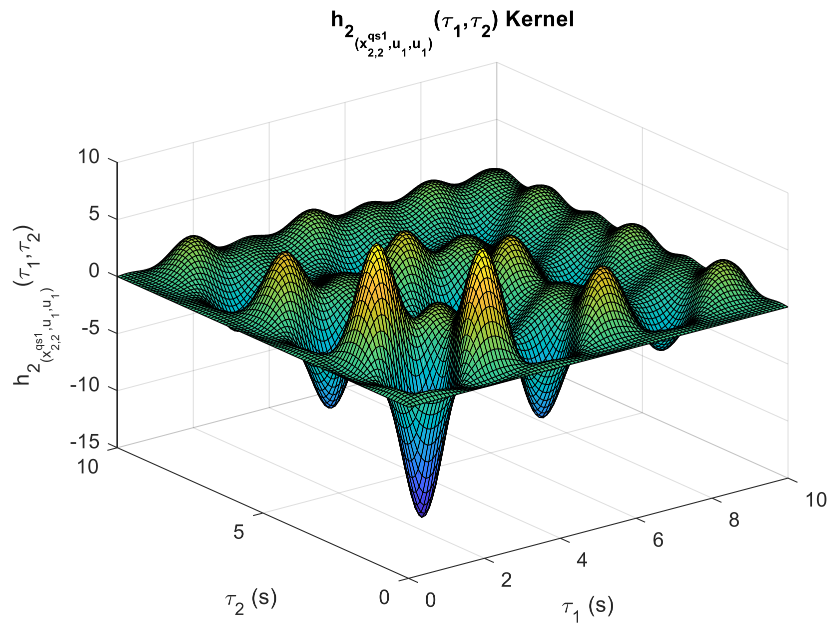

Figure B.9 2nd Order Kernel Plot for State 2 (qs1 Component) w.r.t. Quadratic Input 1

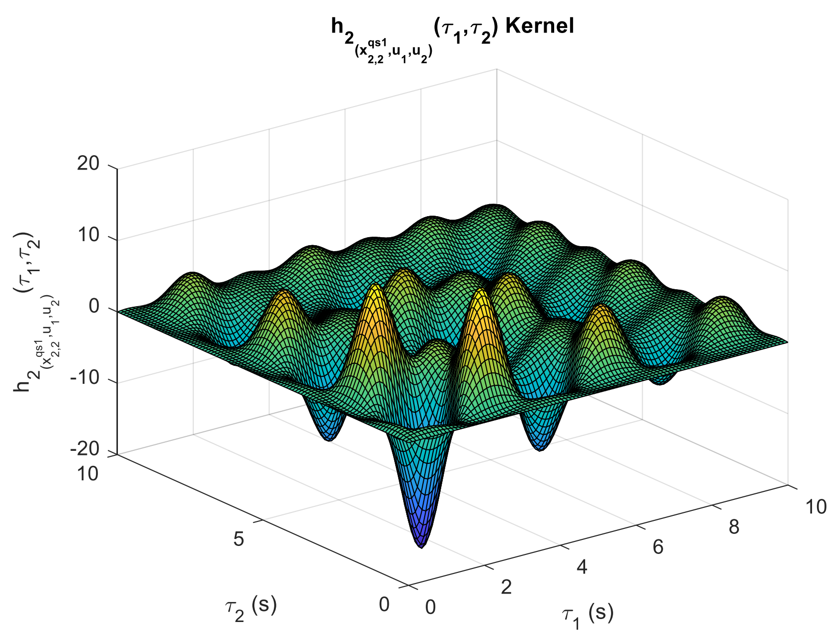

Figure B.10 2nd Order Kernel Plot for State 2 (qs1 Component) w.r.t. Input 1 and Input 2

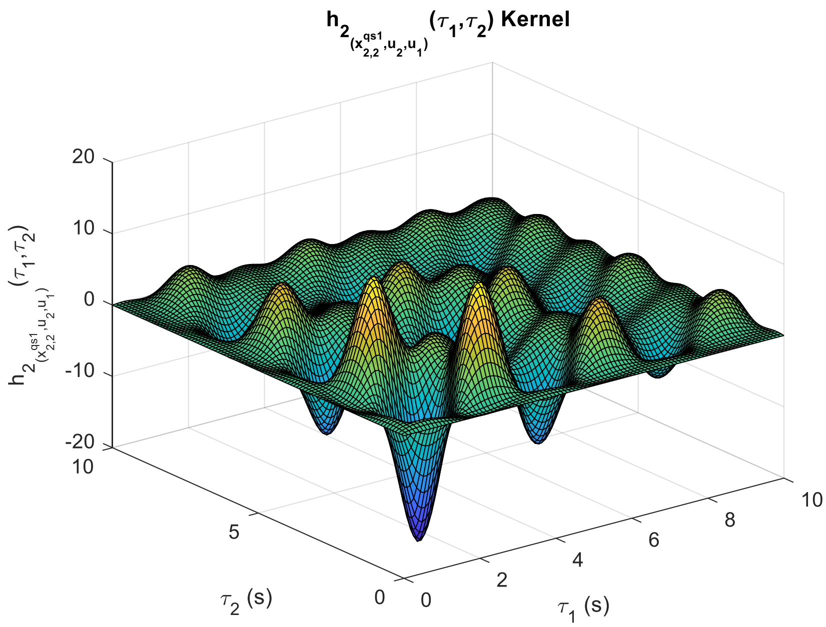

Figure B.11 2nd Order Kernel Plot for State 2 (qs1 Component) w.r.t. Input 2 and Input 1

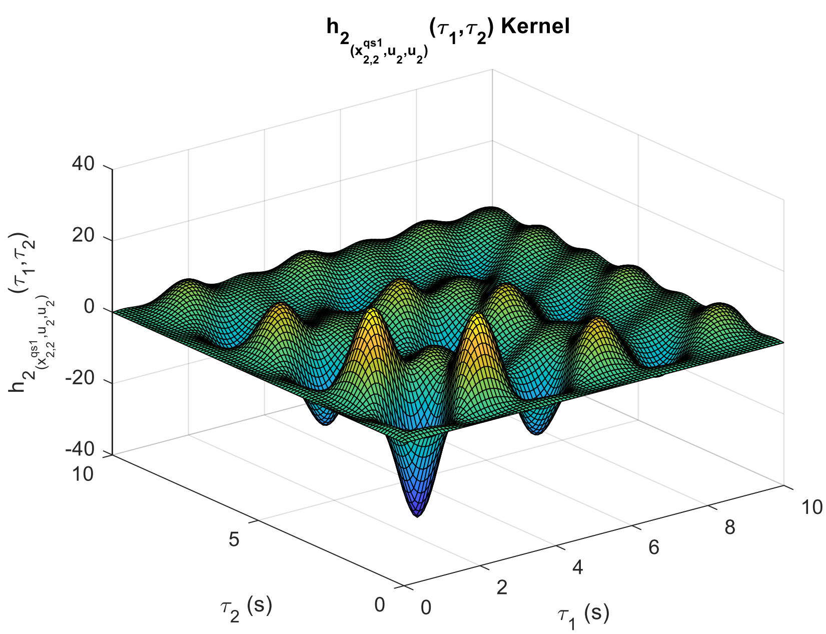

Figure B.12 2nd Order Kernel Plot for State 2 (qs1 Component) w.r.t. Quadratic Input 2

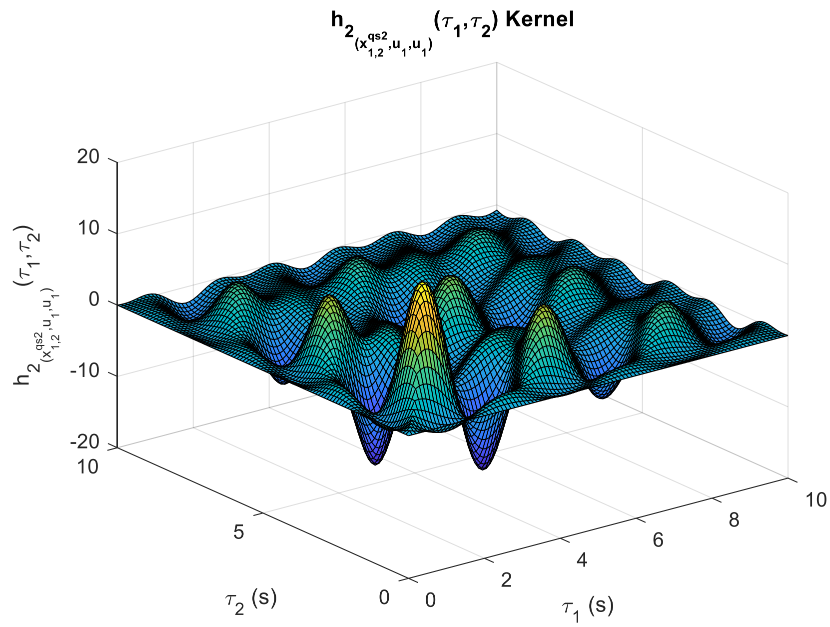

Figure B.13 2nd Order Kernel Plot for State 1 (qs2 Component) w.r.t. Quadratic Input 1

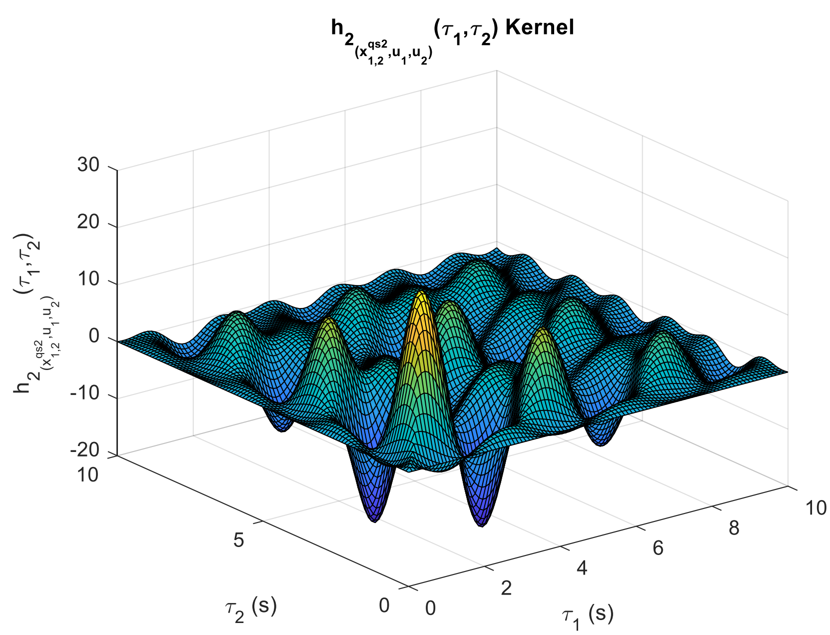

Figure B.14 2nd Order Kernel Plot for State 1 (qs2 Component) w.r.t. Input 1 and Input 2

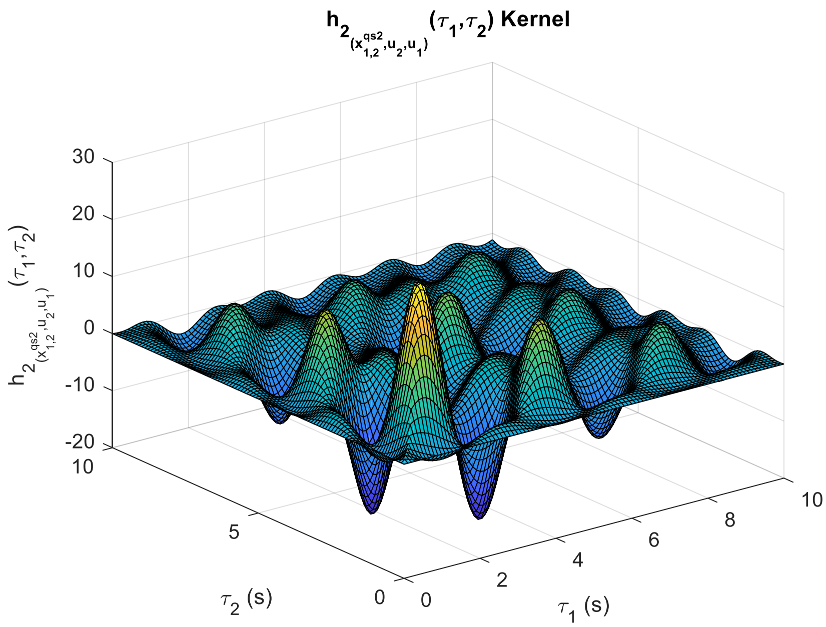

Figure B.15 2nd Order Kernel Plot for State 1 (qs2 Component) w.r.t. Input 2 and Input 1

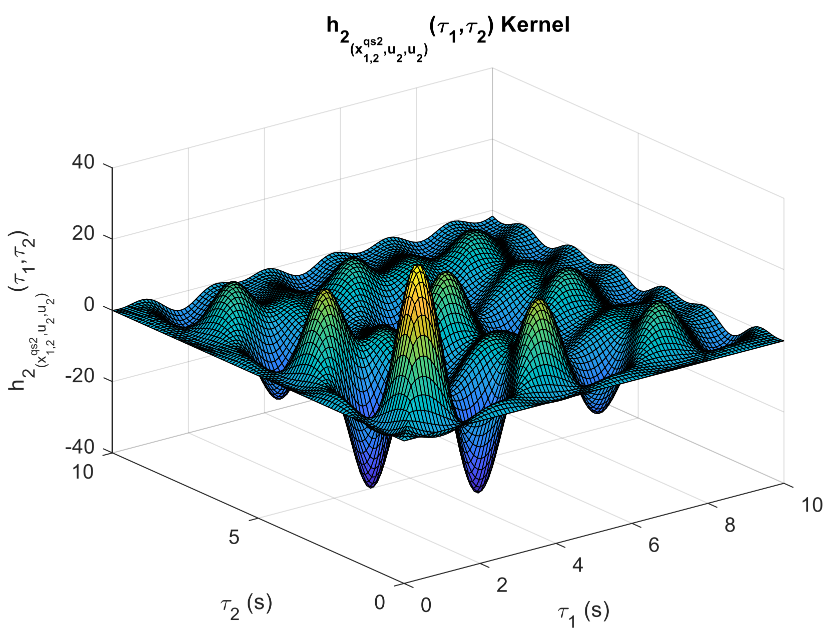

Figure B.16 2nd Order Kernel Plot for State 1 (qs2 Component) w.r.t. Quadratic Input 2

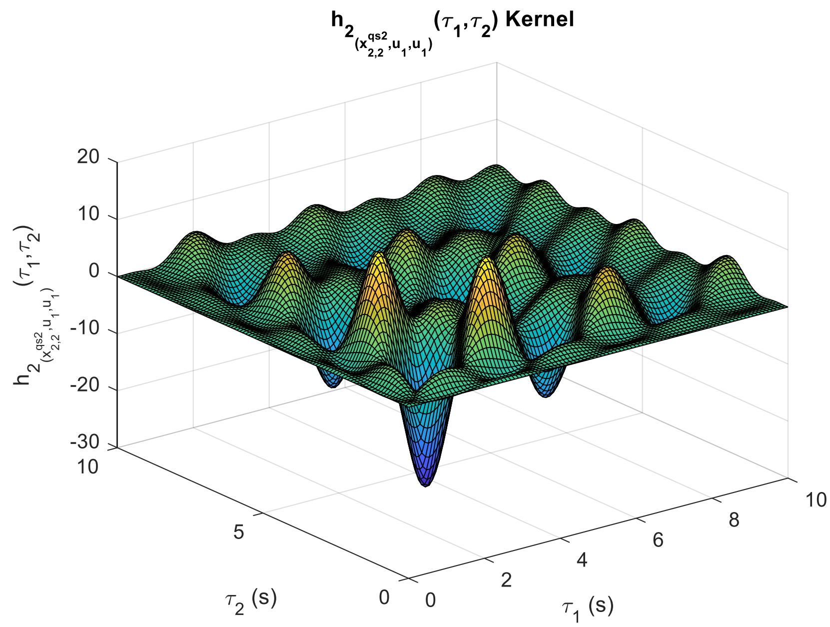

Figure B.17 2nd Order Kernel Plot for State 2 (qs2 Component) w.r.t. Quadratic Input 1

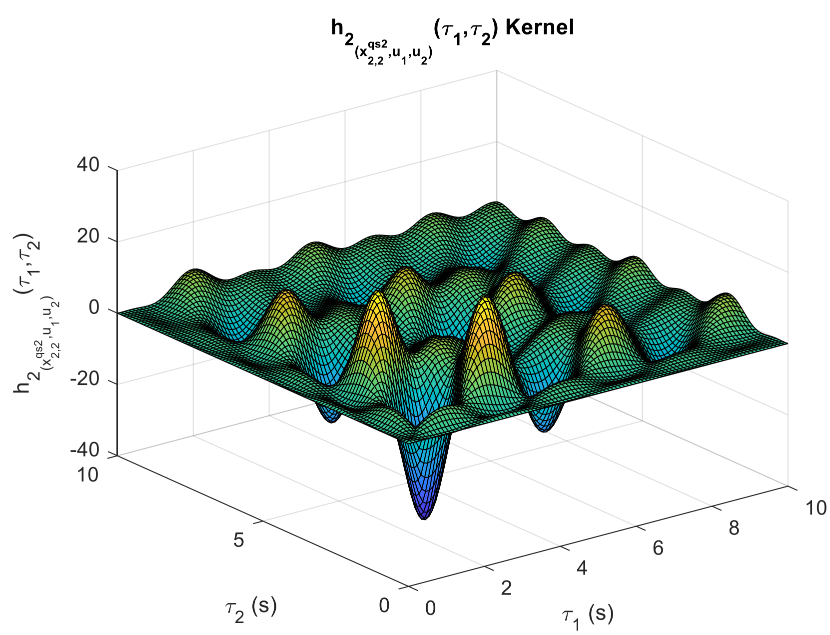

Figure B.18 2nd Order Kernel Plot for State 2 (qs2 Component) w.r.t. Input 1 and Input 2

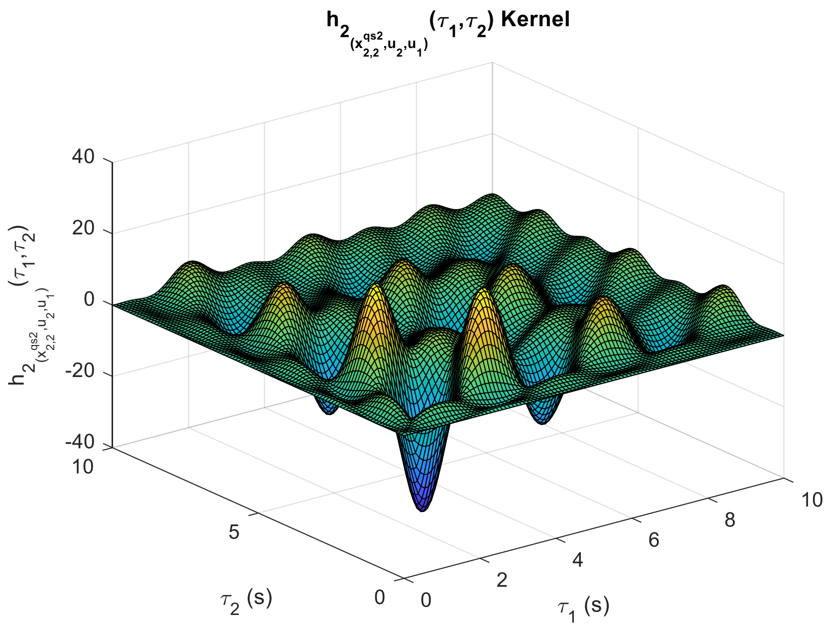

Figure B.19 2nd Order Kernel Plot for State 2 (qs2 Component) w.r.t. Input 2 and Input 1

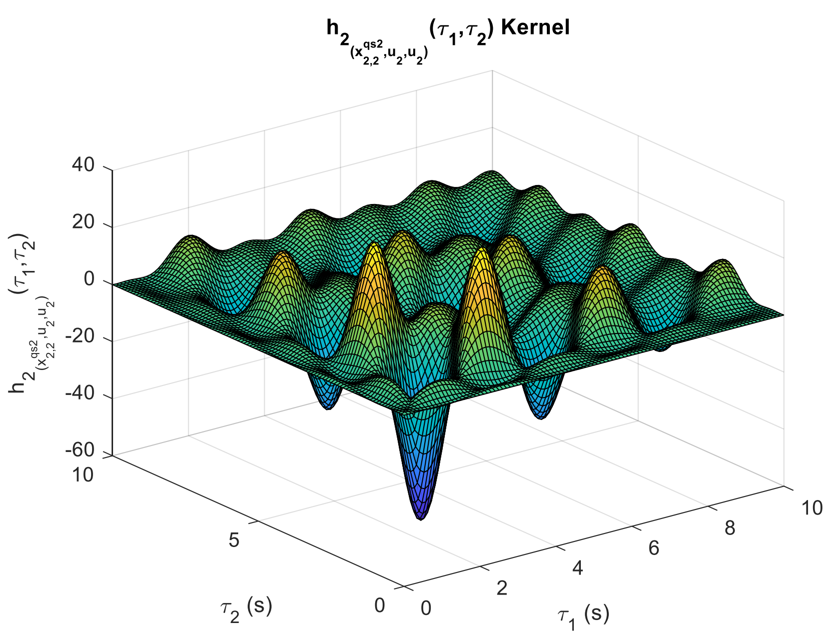

Figure B.20 2nd Order Kernel Plot for State 2 (qs2 Component) w.r.t. Quadratic Input 2

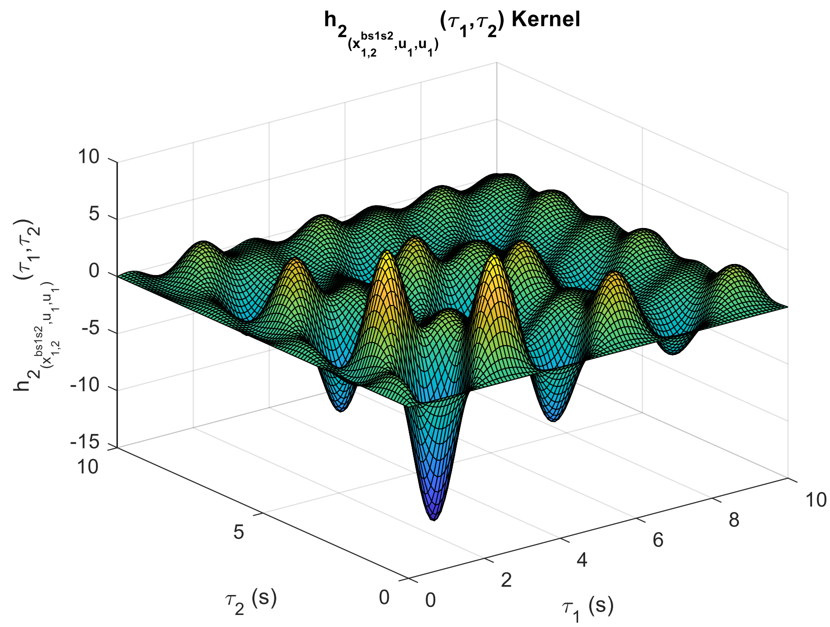

Figure B.21 2nd Order Kernel Plot for State 1 (bs1s2 Component) w.r.t. Quadratic Input 1

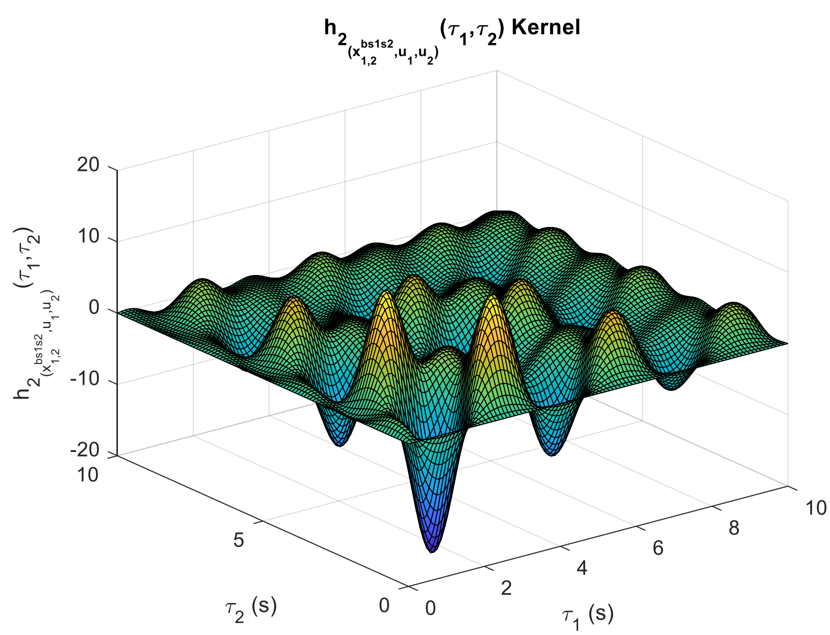

Figure B.22 2nd Order Kernel Plot for State 1 (bs1s2 Component) w.r.t. Input 1 and Input 2

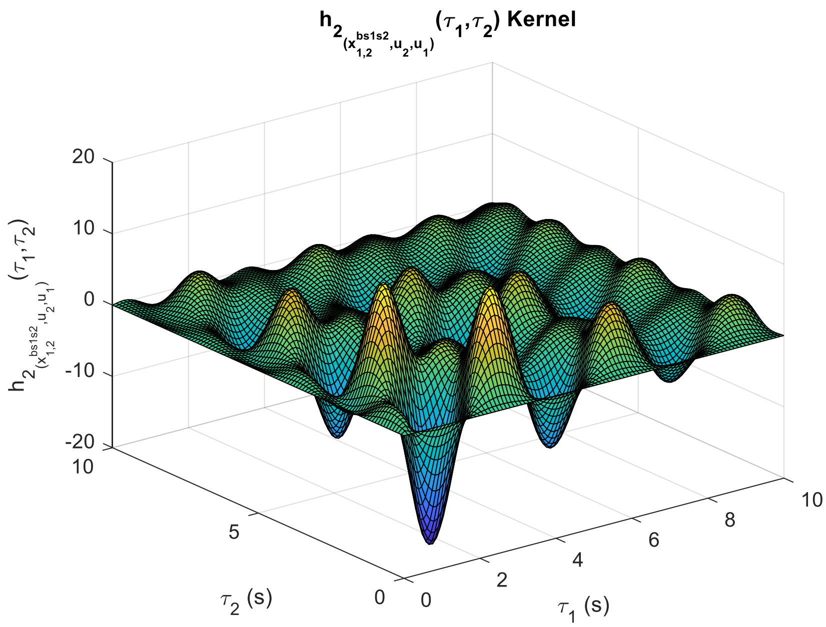

Figure B.23 2nd Order Kernel Plot for State 1 (bs1s2 Component) w.r.t. Input 2 and Input 1

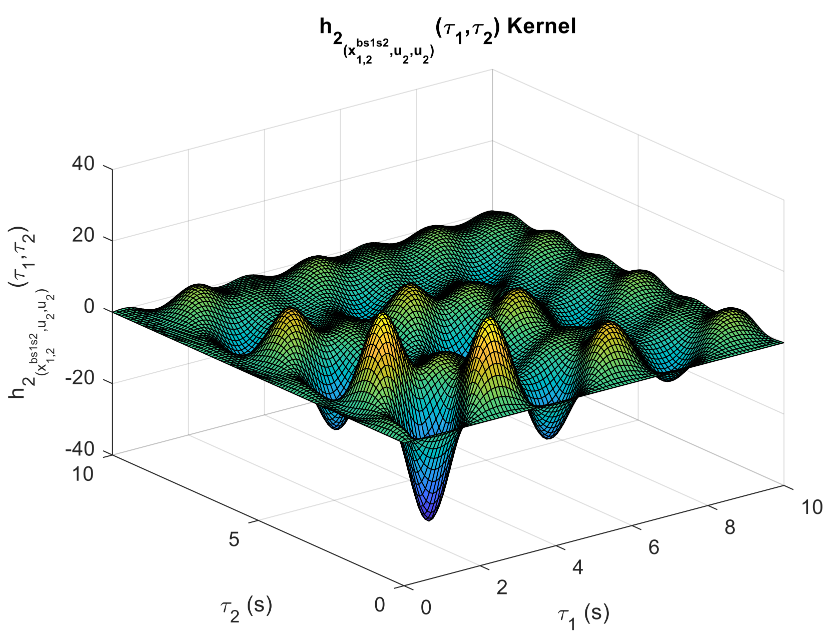

Figure B.24 2nd Order Kernel Plot for State 1 (bs1s2 Component) w.r.t. Quadratic Input 2

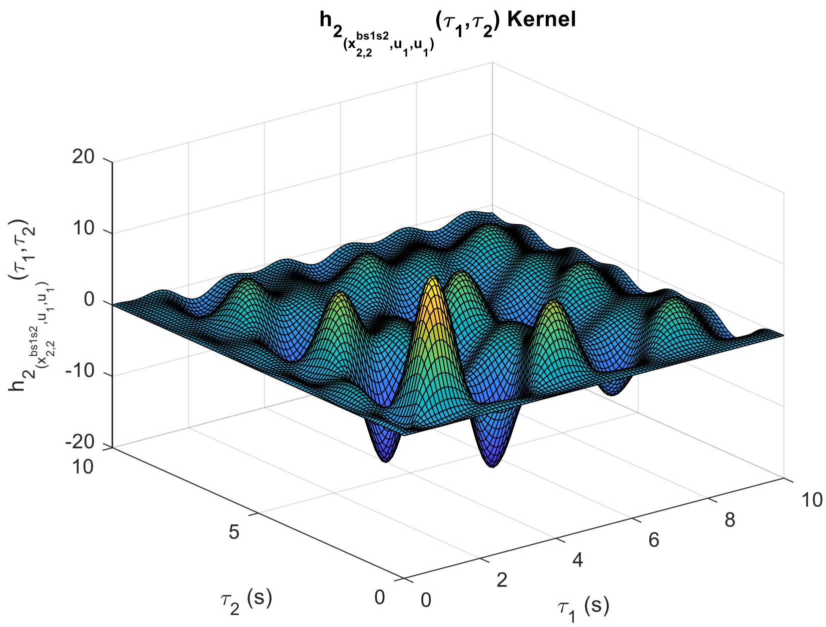

Figure B.25 2nd Order Kernel Plot for State 2 (bs1s2 Component) w.r.t. Quadratic Input 1

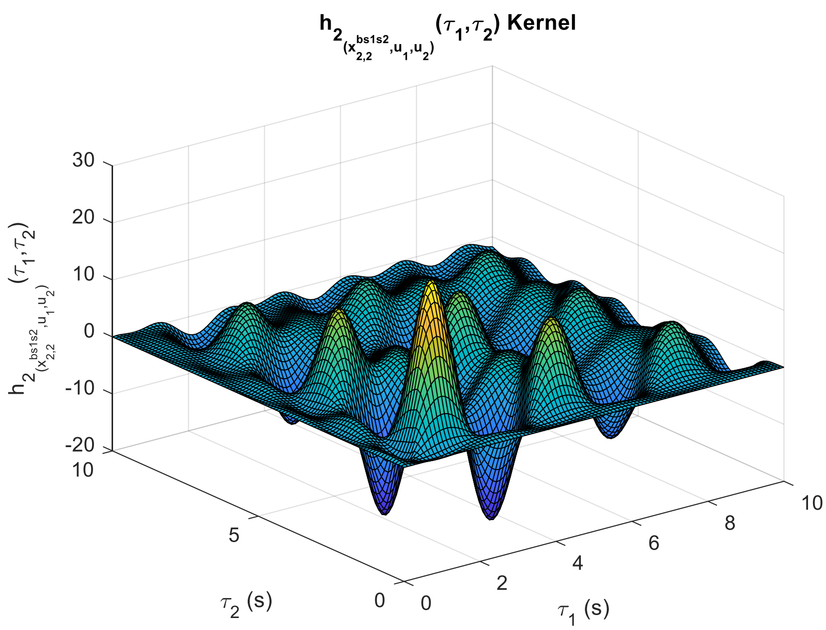

Figure B.26 2nd Order Kernel Plot for State 2 (bs1s2 Component) w.r.t. Input 1 and Input 2

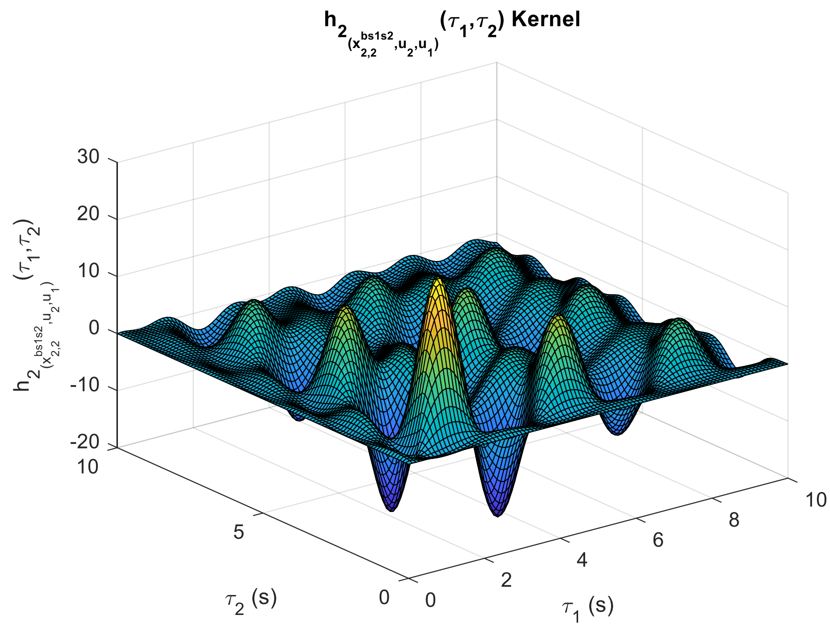

Figure B.27 2nd Order Kernel Plot for State 2 (bs1s2 Component) w.r.t. Input 2 and Input 1

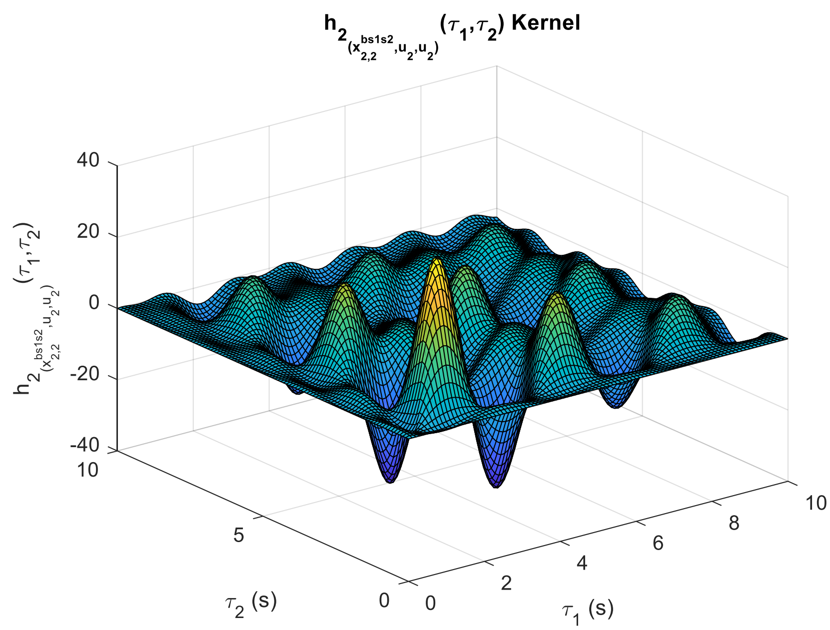

Figure B.28 2nd Order Kernel Plot for State 2 (bs1s2 Component) w.r.t. Quadratic Input 2

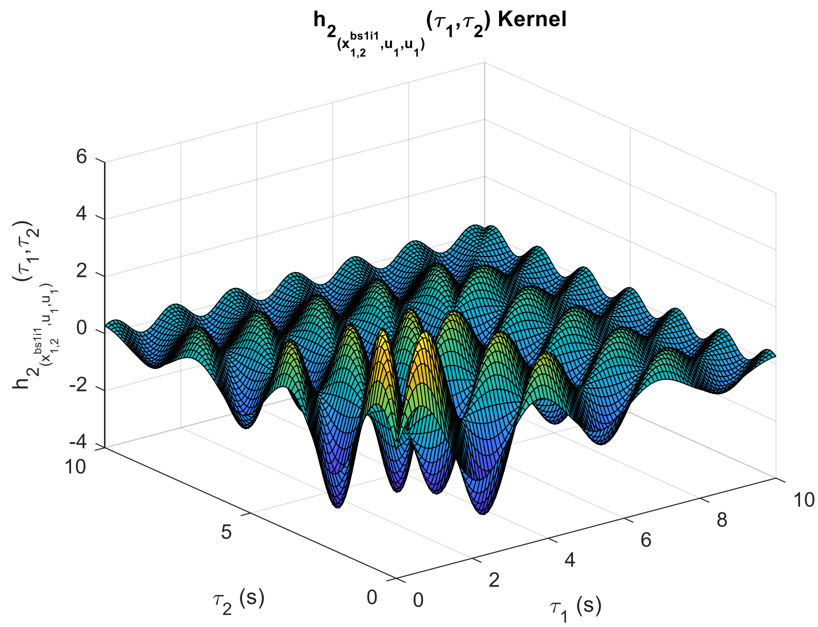

Figure B.29 2nd Order Kernel Plot for State 1 (bs1i1 Component) w.r.t. Quadratic Input 1

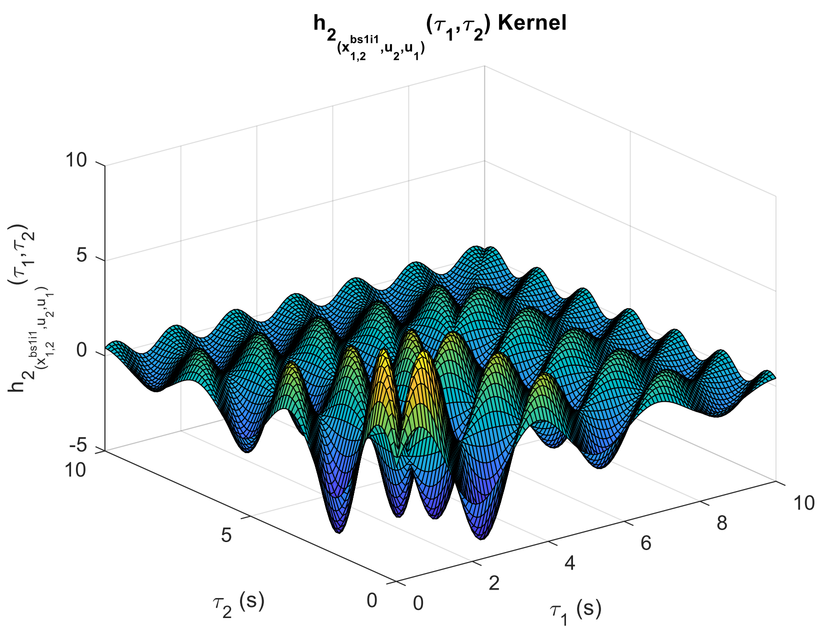

Figure B.30 2nd Order Kernel Plot for State 1 (bs1i1 Component) w.r.t. Input 2 and Input 1

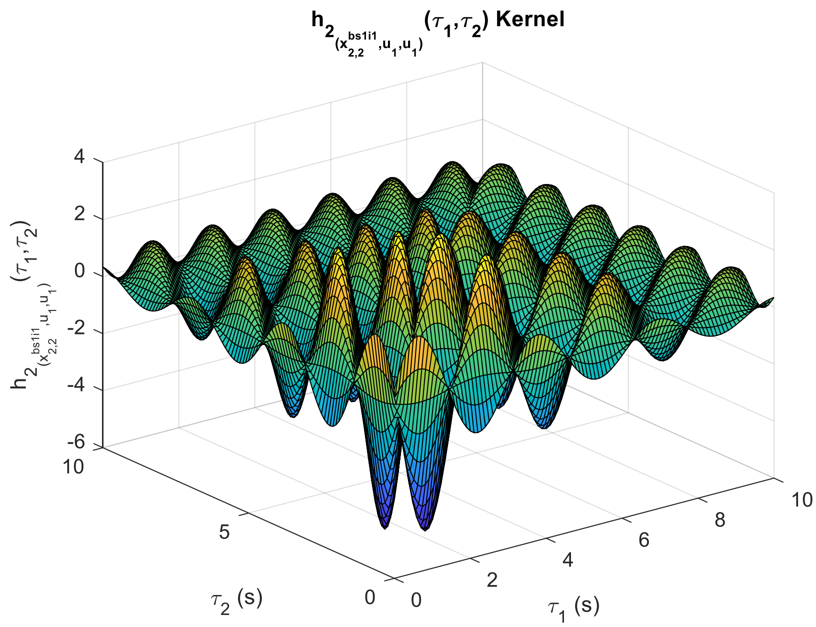

Figure B.31 2nd Order Kernel Plot for State 2 (bs1i1 Component) w.r.t. Quadratic Input 1

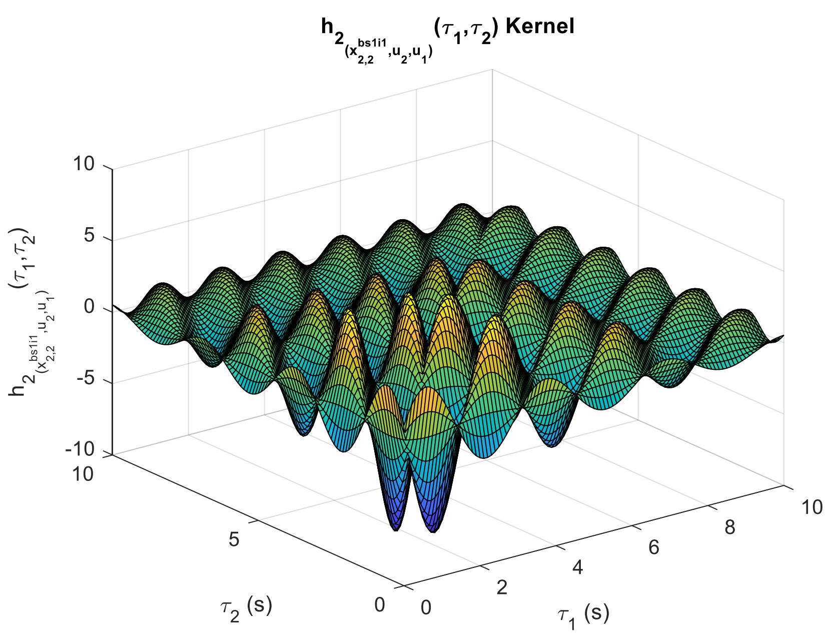

Figure B.32 2nd Order Kernel Plot for State 2 (bs1i1 Component) w.r.t. Input 2 and Input 1

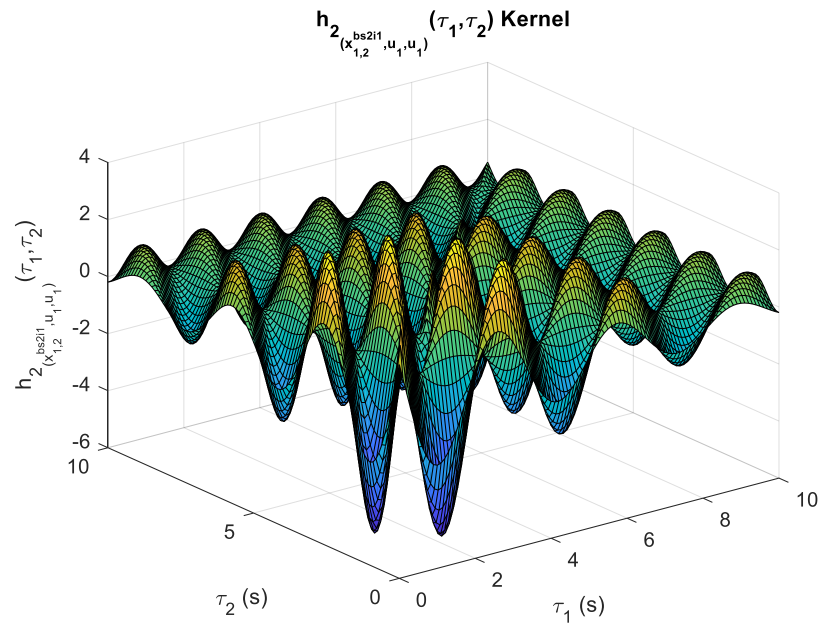

Figure B.33 2nd Order Kernel Plot for State 1 (bs2i1 Component) w.r.t. Quadratic Input 1

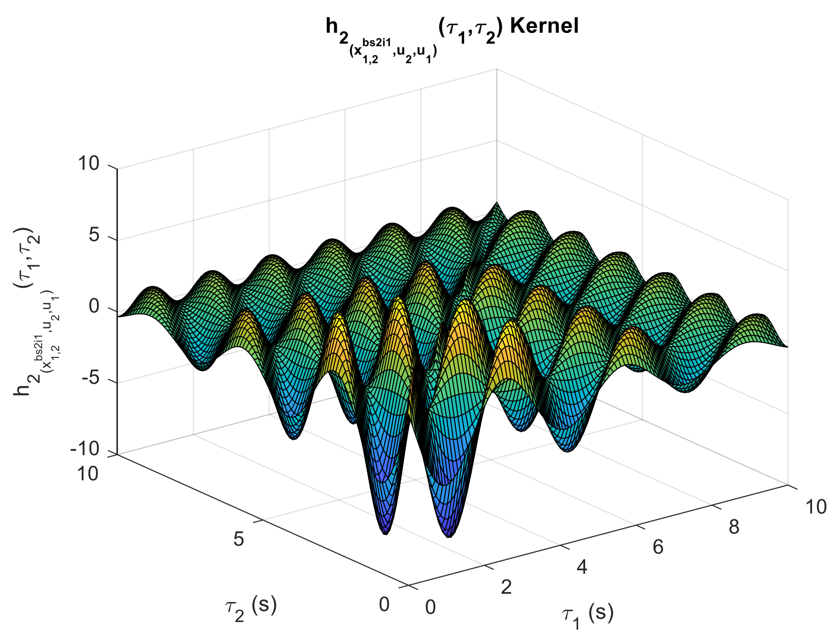

Figure B.34 2nd Order Kernel Plot for State 1 (bs2i1 Component) w.r.t. Input 2 and Input 1

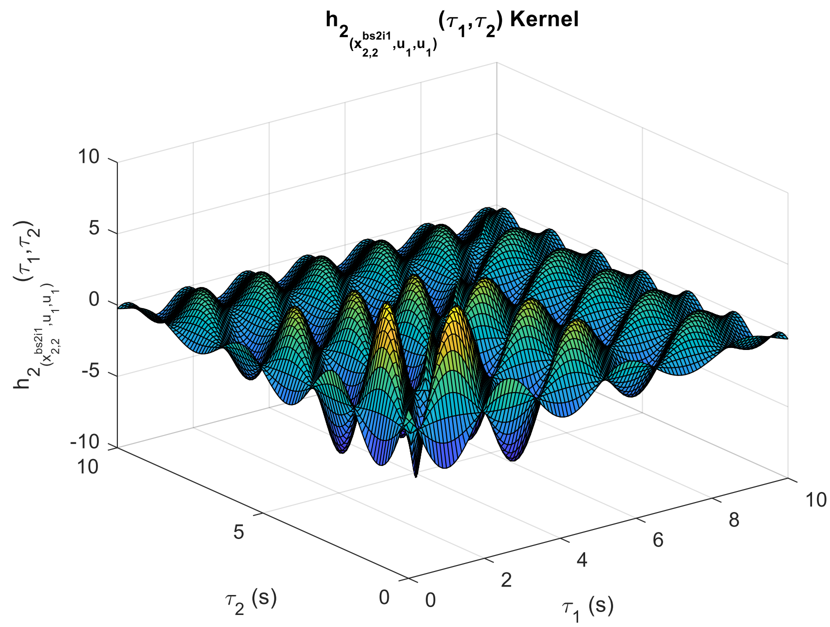

Figure B.35 2nd Order Kernel Plot for State 2 (bs2i1 Component) w.r.t. Quadratic Input 1

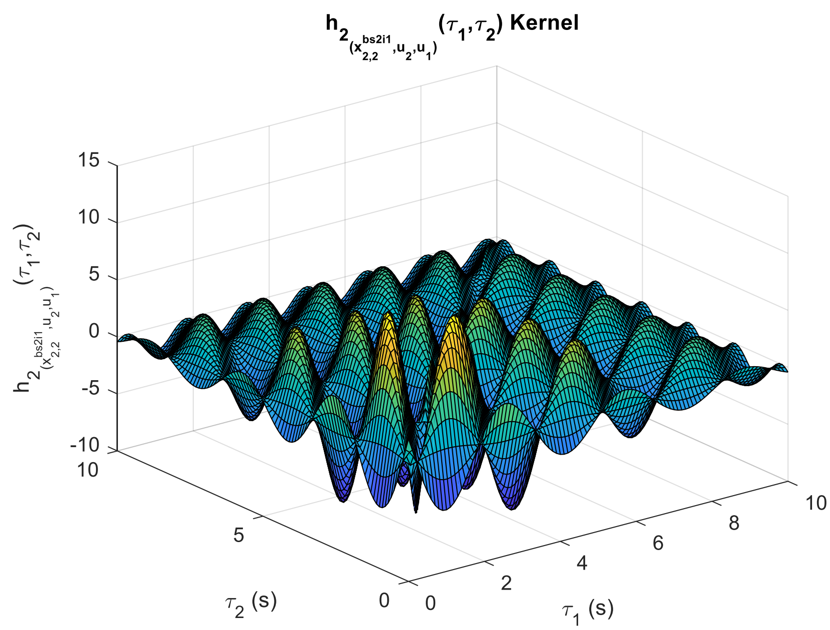

Figure B.36 2nd Order Kernel Plot for State 2 (bs2i1 Component) w.r.t. Input 2 and Input 1

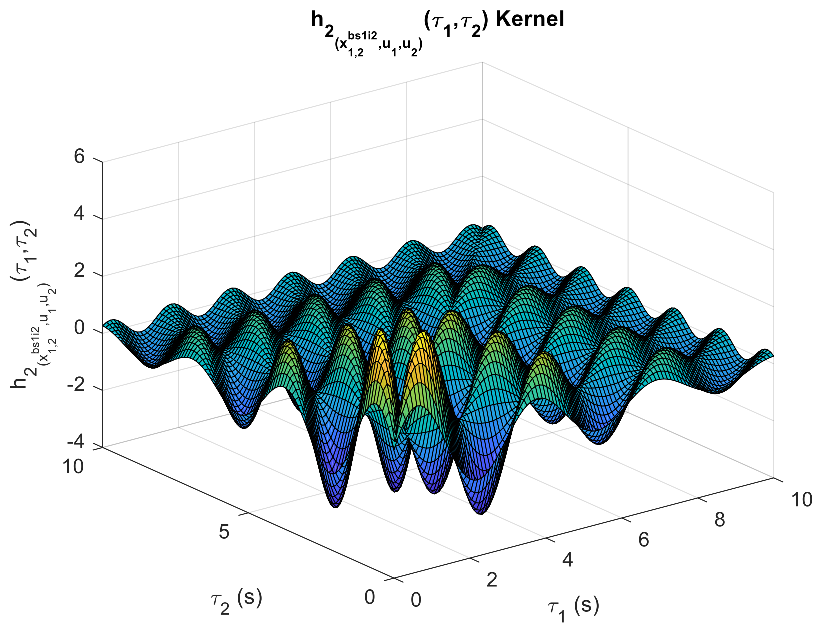

Figure B.37 2nd Order Kernel Plot for State 1 (bs1i2 Component) w.r.t. Input 1 and Input 2

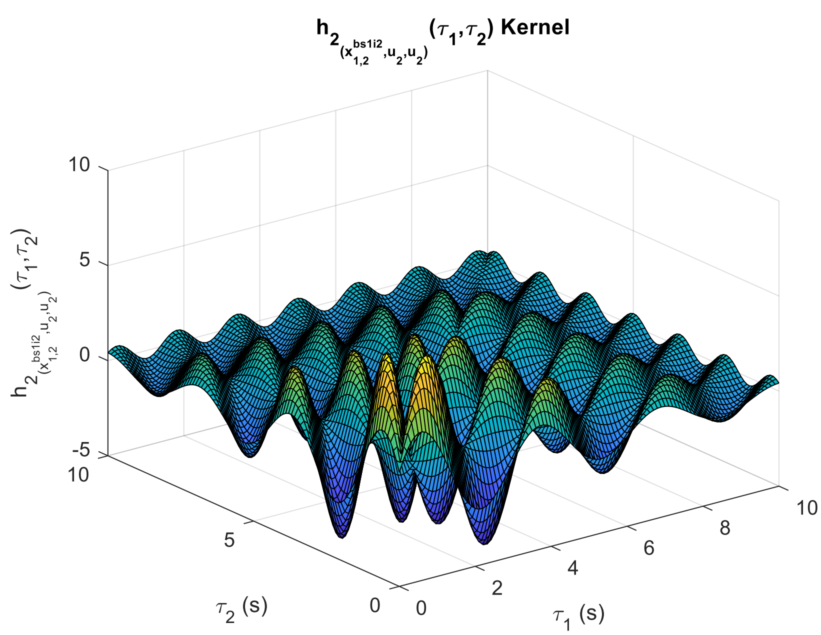

Figure B.38 2nd Order Kernel Plot for State 1 (bs1i2 Component) w.r.t. Quadratic Input 2

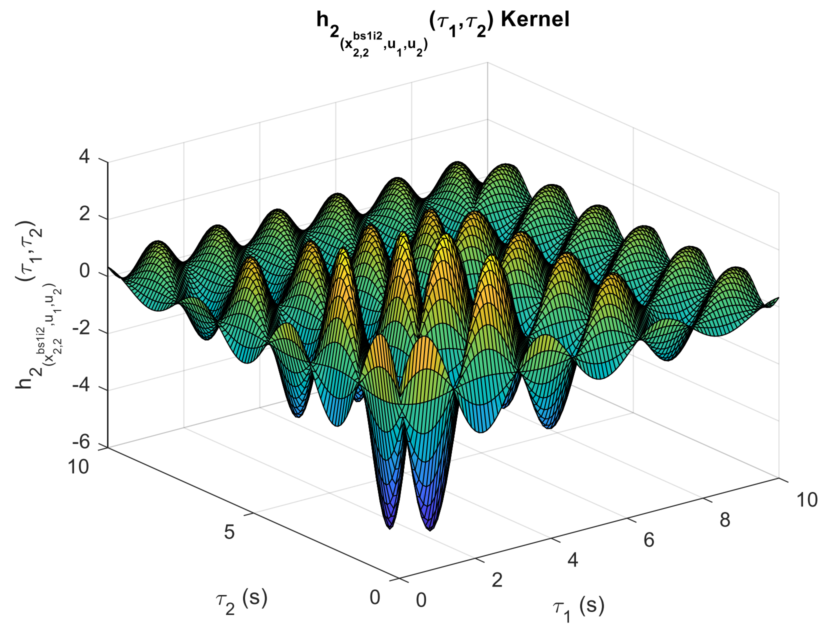

Figure B.39 2nd Order Kernel Plot for State 2 (bs1i2 Component) w.r.t. Input 1 and Input 2

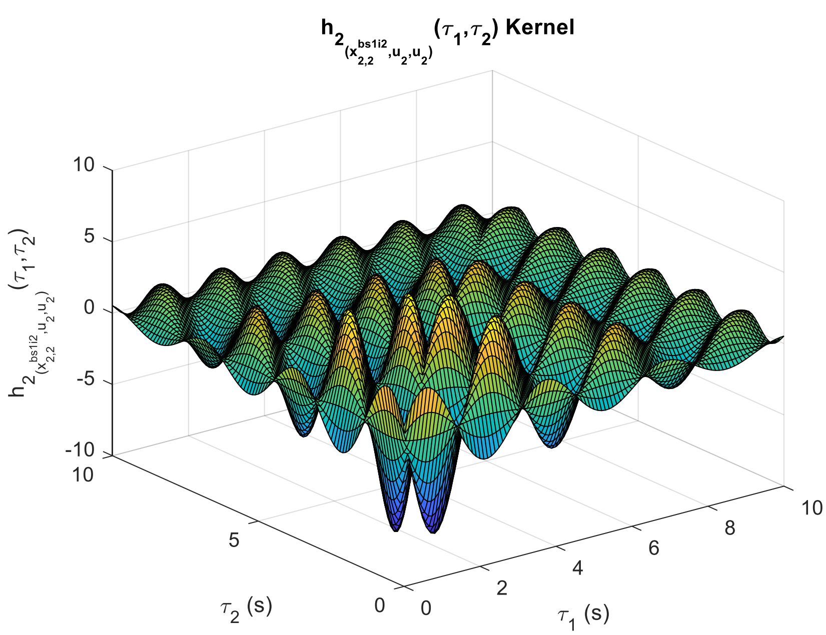

Figure B.40 2nd Order Kernel Plot for State 2 (bs1i2 Component) w.r.t. Quadratic Input 2

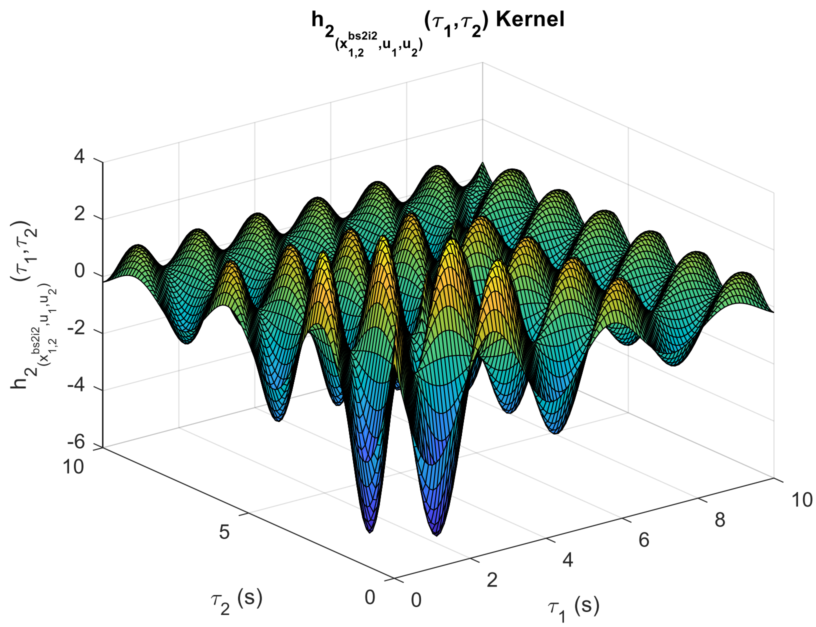

Figure B.41 2nd Order Kernel Plot for State 1 (bs2i2 Component) w.r.t. Input 1 and Input 2

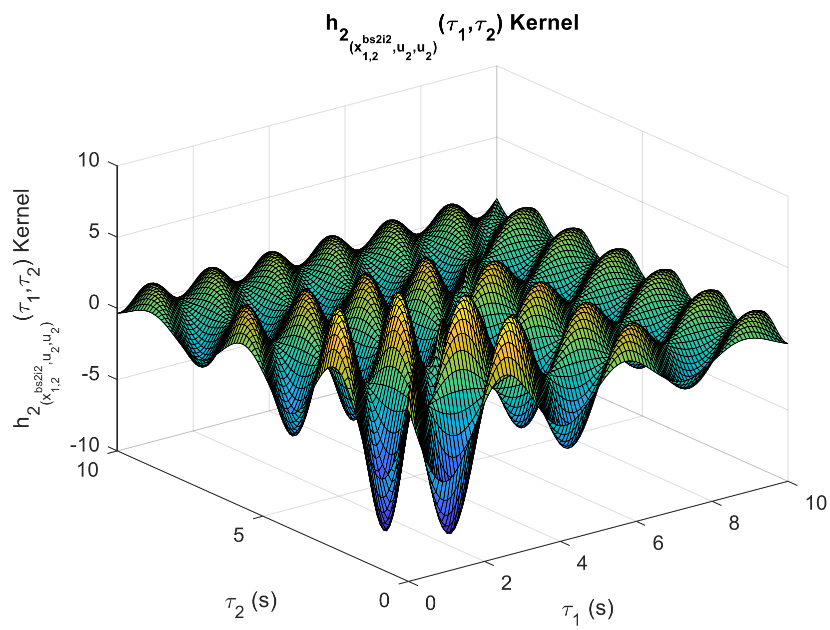

Figure B.42 2nd Order Kernel Plot for State 1 (bs2i2 Component) w.r.t. Quadratic Input 2

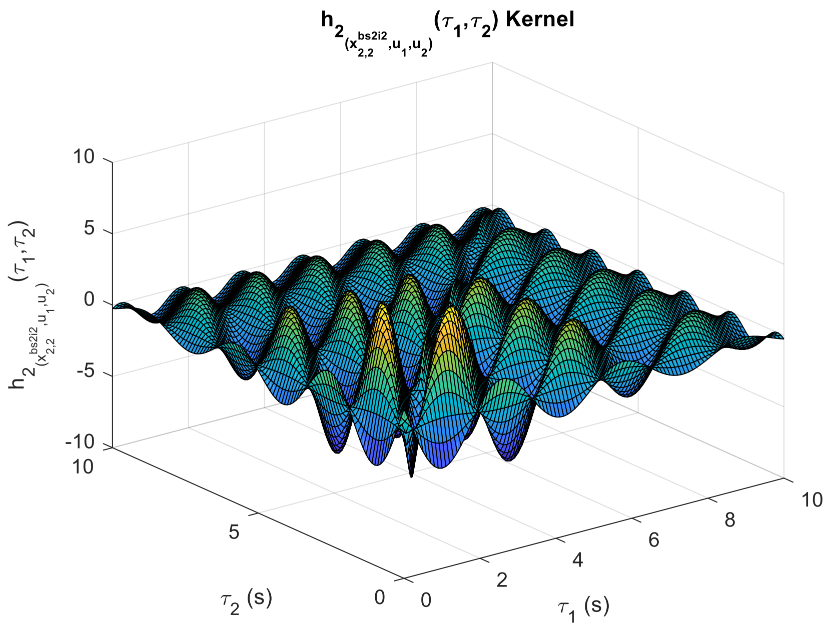

Figure B.43 2nd Order Kernel Plot for State 2 (bs2i2 Component) w.r.t. Input 1 and Input 2

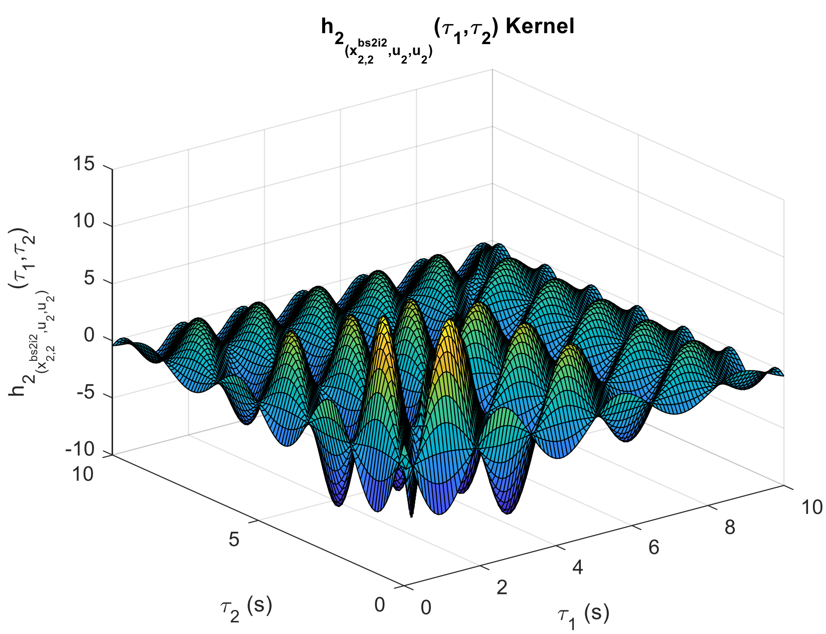

Figure B.44 2nd Order Kernel Plot for State 2 (bs2i2 Component) w.r.t. Quadratic Input 2

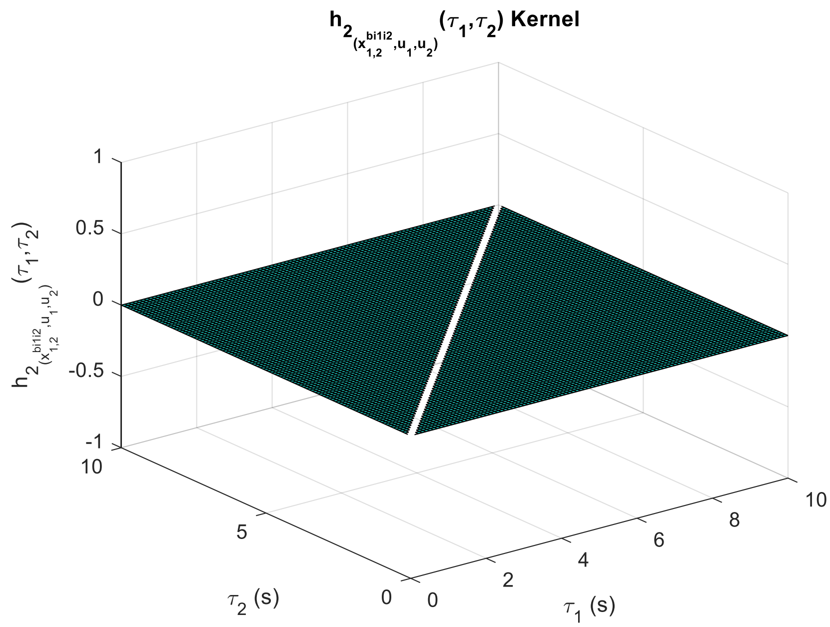

Figure B.45 2nd Order Kernel Plot for State 1 (bi1i2 Component) w.r.t. Input 1 and Input 2

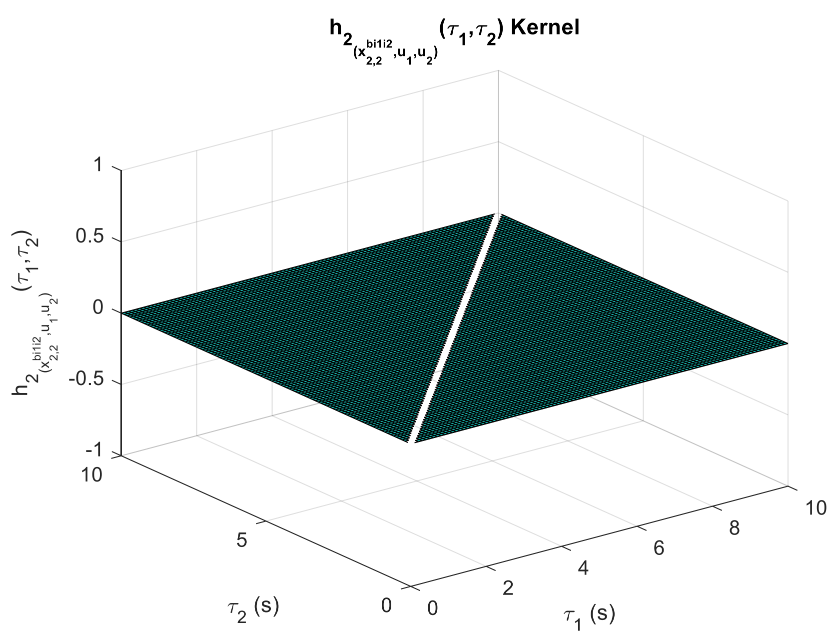

Figure B.46 2nd Order Kernel Plot for State 2 (bi1i2 Component) w.r.t. Input 1 and Input 2

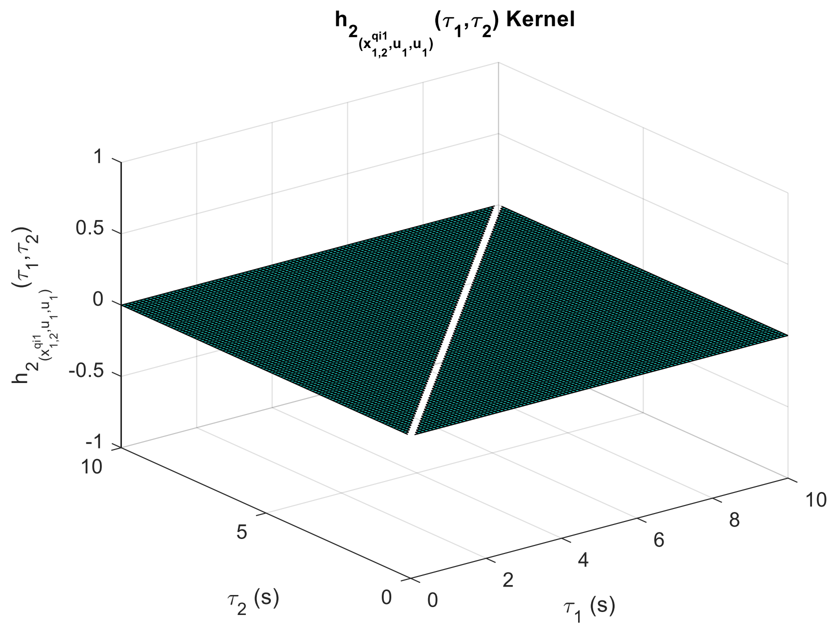

Figure B.47 2nd Order Kernel Plot for State 1 (qi1 Component) w.r.t. Quadratic Input 1

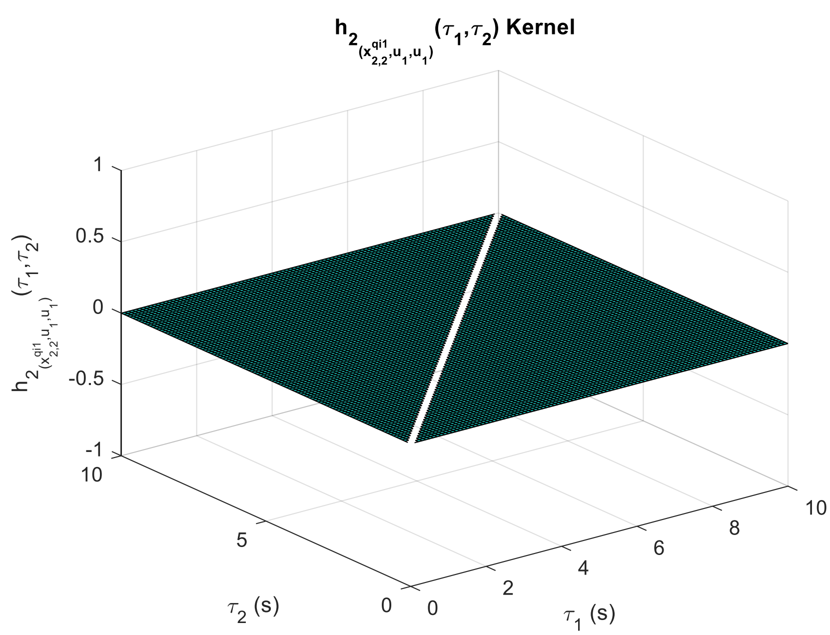

Figure B.48 2nd Order Kernel Plot for State 2 (qi1 Component) w.r.t. Quadratic Input 1

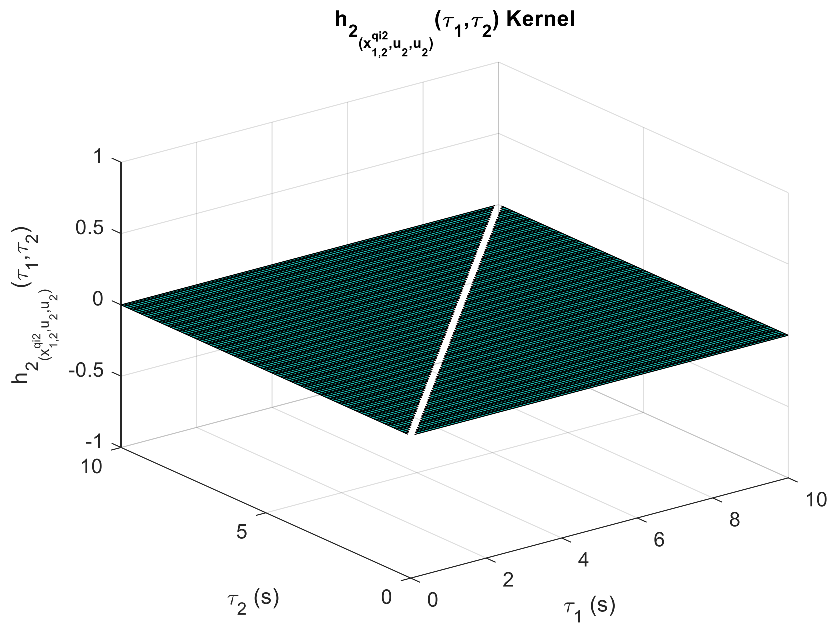

Figure B.49 2nd Order Kernel Plot for State 1 (qi2 Component) w.r.t. Quadratic Input 2

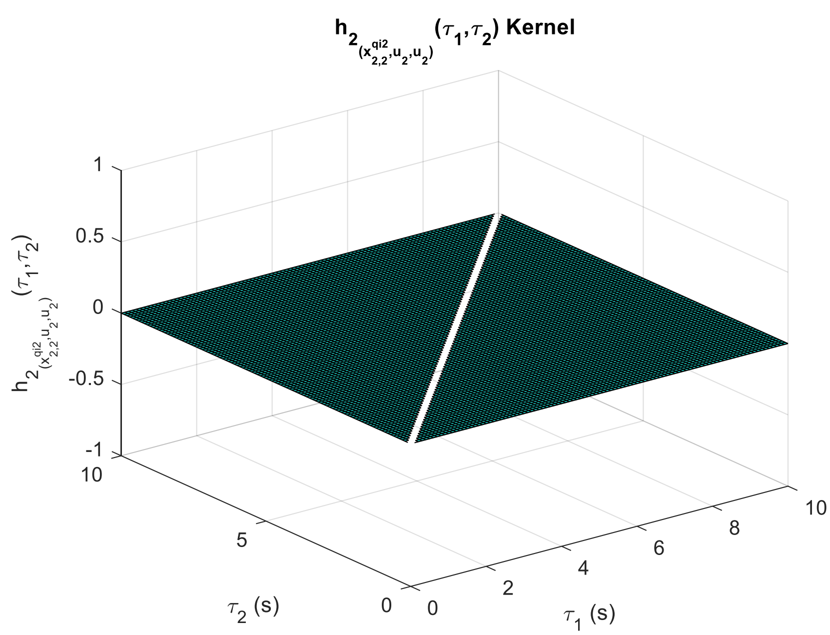

Figure B.50 2nd Order Kernel Plot for State 2 (qi2 Component) w.r.t. Quadratic Input 2

Appendix C

Second Order 2DOF Step Response Equations

$$x_{1,2} = x_{1,2}^{qs1} + x_{1,2}^{bs1s2} + x_{1,2}^{qs2} + x_{1,2}^{bs1i1} + x_{1,2}^{bs2i1} + x_{1,2}^{bs1i2} + x_{1,2}^{bs2i2} + x_{1,2}^{qi1} + x_{1,2}^{bi1i2} + x_{1,2}^{qi2} \quad (C.1)$$

$$x_{1,2}^{qs1}(t) = x_{1,2}^{qs1,u_1,u_1}(t) \times u_1u_1 + x_{1,2}^{qs1,u_1,u_2}(t) \times u_1u_2 + x_{1,2}^{qs1,u_2,u_1}(t) \times u_2u_1 + x_{1,2}^{qs1,u_2,u_2}(t) \times u_2u_2 \quad (C.2)$$

$$x_{1,2}^{qs1,u_1,u_1}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{\omega_d{}^2(\sigma^2+9\omega_d{}^2)(2A_1{}^2A_2K_{0010}((1$$

$$+2e^{2\sigma t})K_{2000}L_{0010}+(1+e^{2\sigma t})K_{0010}L_{2000})(\sigma^2+\omega_d{}^2)+2A_2{}^3L_{0010}{}^2L_{2000}(\sigma^2+(1$$

$$+2e^{2\sigma t})\omega_d{}^2)+(A_1{}^3(2+3e^{2\sigma t})K_{0010}{}^2K_{2000}(\sigma^2+\omega_d{}^2)+2A_1A_2{}^2L_{0010}(K_{2000}L_{0010}$$

$$+2K_{0010}L_{2000})(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2))\cos(\varphi_1))+2A_1{}^2A_2K_{0010}\omega_d{}^2(\sigma^2$$

$$+9\omega_d{}^2)(-e^{2\sigma t}K_{0010}L_{2000}(\sigma^2-\omega_d{}^2)+K_{2000}L_{0010}((1-2e^{2\sigma t})\sigma^2+(1$$

$$+2e^{2\sigma t})\omega_d{}^2))\cos(2\varphi_1)+A_1{}^3e^{2\sigma t}K_{0010}{}^2K_{2000}\omega_d{}^2(-3\sigma^4-26\sigma^2\omega_d{}^2$$

$$+9\omega_d{}^4)\cos(3\varphi_1)+4A_2e^{\sigma t}\omega_d{}^2(\sigma^2+\omega_d{}^2)(A_2{}^2L_{0010}{}^2L_{2000}(\sigma^3t-12\omega_d{}^2$$

$$+9\sigma t\omega_d{}^2)-A_1{}^2K_{0010}(-K_{2000}L_{0010}(\sigma^2-15\omega_d{}^2)+K_{0010}L_{2000}(\sigma^2$$

$$+9\omega_d{}^2)))\cos(t\omega_d)-2A_2L_{0010}\omega_d{}^2(A_1{}^2K_{0010}K_{2000}(5\sigma^4+2\sigma^2\omega_d{}^2-3\omega_d{}^4)$$

$$+A_2{}^2L_{0010}L_{2000}(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(2t\omega_d)+A_1A_2{}^2K_{2000}L_{0010}{}^2\omega_d{}^2(-5\sigma^4$$

$$-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1-2t\omega_d)-2A_1e^{\sigma t}(\sigma^2+\omega_d{}^2)(A_1{}^2K_{0010}{}^2K_{2000}(\sigma^4$$

$$+7\sigma^2\omega_d{}^2+6\omega_d{}^4)+A_2{}^2L_{0010}(K_{2000}L_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2)+K_{0010}L_{2000}(\sigma^4$$

$$+6\sigma^2\omega_d{}^2+21\omega_d{}^4)))\cos(\varphi_1-t\omega_d)-A_1{}^2A_2e^{\sigma t}K_{0010}(\sigma^2+\omega_d{}^2)^2(K_{0010}L_{2000}(\sigma^2$$

$$+3\omega_d{}^2)+K_{2000}L_{0010}(\sigma^2+9\omega_d{}^2))\cos(2\varphi_1-t\omega_d)+2A_1e^{\sigma t}(\sigma^2$$

$$+\omega_d{}^2)(A_1{}^2K_{0010}{}^2K_{2000}(\sigma^4+7\sigma^2\omega_d{}^2-18\omega_d{}^4)+A_2{}^2L_{0010}(K_{0010}L_{2000}(\sigma^2$$

$$+9\omega_d{}^2)(\sigma^2-3\omega_d{}^2+4\sigma t\omega_d{}^2)+K_{2000}L_{0010}\omega_d{}^2(\sigma^2+2\sigma^3t-15\omega_d{}^2$$

$$+18\sigma t\omega_d{}^2)))\cos(\varphi_1+t\omega_d)-2A_1{}^2A_2K_{0010}\omega_d{}^2(K_{0010}L_{2000}(\sigma^4-12\sigma^2\omega_d{}^2$$

$$+3\omega_d{}^4)+K_{2000}L_{0010}(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4))\cos(2(\varphi_1+t\omega_d))$$

$$+A_1{}^2A_2e^{\sigma t}K_{0010}(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)(K_{0010}L_{2000}(\sigma^2-\omega_d{}^2+4\sigma t\omega_d{}^2)$$

$$+K_{2000}L_{0010}(\sigma^2-3\omega_d{}^2+8\sigma t\omega_d{}^2))\cos(2\varphi_1+t\omega_d)$$

$$+4A_1{}^3e^{\sigma t}K_{0010}{}^2K_{2000}\sigma t\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(3\varphi_1+t\omega_d)$$

$$+A_1\omega_d{}^2(A_1{}^2K_{0010}{}^2K_{2000}(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)+A_2{}^2L_{0010}(K_{2000}L_{0010}(3\sigma^4$$

$$+26\sigma^2\omega_d{}^2-9\omega_d{}^4)-4K_{0010}L_{2000}(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4)))\cos(\varphi_1+2t\omega_d)$$

$$+A_1{}^3K_{0010}{}^2K_{2000}\omega_d{}^2(3\sigma^4+26\sigma^2\omega_d{}^2-9\omega_d{}^4)\cos(3\varphi_1+2t\omega_d)+A_1\sigma\omega_d(\sigma^2$$

$$+ 9{\omega_d}^2)({A_1}^2(-2 + 3e^{2\sigma t}){K_{0010}}^2 K_{2000}(\sigma^2 + {\omega_d}^2)$$

$$- 2{A_2}^2 L_{0010}(-4e^{2\sigma t}K_{0010}L_{2000}{\omega_d}^2 + K_{2000}L_{0010}(\sigma^2 + {\omega_d}^2 - 2e^{2\sigma t}{\omega_d}^2)))\sin(\varphi_1)$$

$$+ 2{A_1}^2 A_2 K_{0010}\sigma\omega_d(\sigma^2 + 9{\omega_d}^2)(2e^{2\sigma t}K_{0010}L_{2000}{\omega_d}^2 - K_{2000}L_{0010}(\sigma^2 + (1$$

$$- 4e^{2\sigma t}){\omega_d}^2))\sin(2\varphi_1) - {A_1}^3 e^{2\sigma t}{K_{0010}}^2 K_{2000}\omega_d(\sigma^5 + 6\sigma^3{\omega_d}^2 - 27\sigma{\omega_d}^4)\sin(3\varphi_1)$$

$$- 2A_2 e^{\sigma t}\omega_d(\sigma^2 + {\omega_d}^2)(2{A_2}^2{L_{0010}}^2 L_{2000}(\sigma^3 + 5\sigma{\omega_d}^2 + \sigma^2 t{\omega_d}^2 + 9t{\omega_d}^4)$$

$$+ {A_1}^2 K_{0010}(K_{0010}L_{2000}t(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4) + K_{2000}L_{0010}(4\sigma^3 + \sigma^4 t$$

$$+ 20\sigma{\omega_d}^2 + 10\sigma^2 t{\omega_d}^2 + 9t{\omega_d}^4)))\sin(t\omega_d)$$

$$+ 2A_2 L_{0010}\sigma\omega_d(2{A_2}^2 L_{0010}L_{2000}{\omega_d}^2(3\sigma^2 - 5{\omega_d}^2) + {A_1}^2 K_{0010}K_{2000}(-\sigma^4 + 6\sigma^2{\omega_d}^2$$

$$+ 7{\omega_d}^4))\sin(2t\omega_d) + A_1{A_2}^2 K_{2000}{L_{0010}}^2\sigma\omega_d(\sigma^4 - 6\sigma^2{\omega_d}^2 - 7{\omega_d}^4)\sin(\varphi_1 - 2t\omega_d)$$

$$+ 2A_1 e^{\sigma t}\omega_d(\sigma^2 + {\omega_d}^2)({A_1}^2{K_{0010}}^2 K_{2000}\sigma(\sigma^2 + {\omega_d}^2) + {A_2}^2 L_{0010}(K_{2000}L_{0010}\sigma(\sigma^2$$

$$+ 9{\omega_d}^2) + K_{0010}L_{2000}(\sigma^2 + {\omega_d}^2)(2\sigma + \sigma^2 t + 9t{\omega_d}^2)))\sin(\varphi_1 - t\omega_d)$$

$$+ 2{A_1}^2 A_2 e^{\sigma t}{K_{0010}}^2 L_{2000}\sigma\omega_d(\sigma^2 + {\omega_d}^2)^2\sin(2\varphi_1 - t\omega_d) - 2A_1 e^{\sigma t}\omega_d(\sigma^2$$

$$+ {\omega_d}^2)({A_1}^2{K_{0010}}^2 K_{2000}(\sigma^2 + 9{\omega_d}^2)(\sigma + \sigma^2 t + t{\omega_d}^2) + {A_2}^2 L_{0010}(-K_{0010}L_{2000}(\sigma^2$$

$$+ 9{\omega_d}^2)(-2\sigma + \sigma^2 t - 3t{\omega_d}^2) + K_{2000}L_{0010}(\sigma^3 + \sigma{\omega_d}^2 + 2\sigma^2 t{\omega_d}^2$$

$$+ 18t{\omega_d}^4)))\sin(\varphi_1 + t\omega_d) + 2{A_1}^2 A_2 K_{0010}\sigma\omega_d(2K_{0010}L_{2000}{\omega_d}^2(3\sigma^2 - 5{\omega_d}^2)$$

$$+ K_{2000}L_{0010}(\sigma^4 + 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4))\sin(2(\varphi_1 + t\omega_d)) - 2{A_1}^2 A_2 e^{\sigma t}K_{0010}\omega_d(\sigma^4$$

$$+ 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)(-K_{2000}L_{0010}t(\sigma^2 - 3{\omega_d}^2) + K_{0010}L_{2000}(\sigma - \sigma^2 t$$

$$+ t{\omega_d}^2))\sin(2\varphi_1 + t\omega_d) + 2{A_1}^3 e^{\sigma t}{K_{0010}}^2 K_{2000}t\omega_d(\sigma^6 + 9\sigma^4{\omega_d}^2 - \sigma^2{\omega_d}^4$$

$$- 9{\omega_d}^6)\sin(3\varphi_1 + t\omega_d) + A_1\sigma\omega_d({A_1}^2{K_{0010}}^2 K_{2000}(-\sigma^4 + 6\sigma^2{\omega_d}^2 + 7{\omega_d}^4)$$

$$+ {A_2}^2 L_{0010}(8K_{0010}L_{2000}{\omega_d}^2(3\sigma^2 - 5{\omega_d}^2) + K_{2000}L_{0010}(\sigma^4 + 6\sigma^2{\omega_d}^2$$

$$- 27{\omega_d}^4)))\sin(\varphi_1 + 2t\omega_d) + {A_1}^3{K_{0010}}^2 K_{2000}\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4)\sin(3\varphi_1$$

$$+ 2t\omega_d)\}$$

$$(C.3)$$

$$x_{1,2}^{qs1,u_1,u_2}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)}e^{-2\sigma t}\{\omega_d(A_2\omega_d(\sigma^2$$

$$+9\omega_d{}^2)(A_1{}^2(K_{0010}K_{2000}L_{0001}+K_{0001}K_{2000}L_{0010}+2K_{0001}K_{0010}L_{2000}$$

$$+2e^{2\sigma t}(K_{0010}K_{2000}L_{0001}+K_{0001}K_{2000}L_{0010}+K_{0001}K_{0010}L_{2000}))(\sigma^2+\omega_d{}^2)$$

$$+2A_2{}^2L_{0001}L_{0010}L_{2000}(\sigma^2+\omega_d{}^2+2e^{2\sigma t}\omega_d{}^2))+A_1\omega_d(\sigma^2+9\omega_d{}^2)(A_1{}^2(2$$

$$+3e^{2\sigma t})K_{0001}K_{0010}K_{2000}(\sigma^2+\omega_d{}^2)+2A_2{}^2(K_{2000}L_{0001}L_{0010}$$

$$+K_{0010}L_{0001}L_{2000}+K_{0001}L_{0010}L_{2000})(\sigma^2+\omega_d{}^2+2e^{2\sigma t}\omega_d{}^2))\cos(\varphi_1)$$

$$+A_1{}^2A_2\omega_d(\sigma^2+9\omega_d{}^2)(-2e^{2\sigma t}(K_{0010}K_{2000}L_{0001}+K_{0001}K_{2000}L_{0010}$$

$$+K_{0001}K_{0010}L_{2000})(\sigma-\omega_d)(\sigma+\omega_d)+K_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^2$$

$$+\omega_d{}^2))\cos(2\varphi_1)+A_1{}^3e^{2\sigma t}K_{0001}K_{0010}K_{2000}\omega_d(-3\sigma^2+\omega_d{}^2)(\sigma^2$$

$$+9\omega_d{}^2)\cos(3\varphi_1)+A_1\sigma(\sigma^2+9\omega_d{}^2)(A_1{}^2(-2+3e^{2\sigma t})K_{0001}K_{0010}K_{2000}(\sigma^2$$

$$+\omega_d{}^2)+2A_2{}^2(2e^{2\sigma t}(K_{2000}L_{0001}L_{0010}+K_{0010}L_{0001}L_{2000}$$

$$+K_{0001}L_{0010}L_{2000})\omega_d{}^2-K_{2000}L_{0001}L_{0010}(\sigma^2+\omega_d{}^2)))\sin(\varphi_1)+A_1{}^2A_2\sigma(\sigma^2$$

$$+9\omega_d{}^2)(4e^{2\sigma t}(K_{0010}K_{2000}L_{0001}+K_{0001}K_{2000}L_{0010}+K_{0001}K_{0010}L_{2000})\omega_d{}^2$$

$$-K_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^2+\omega_d{}^2))\sin(2\varphi_1)$$

$$-A_1{}^3e^{2\sigma t}K_{0001}K_{0010}K_{2000}\sigma(\sigma^4+6\sigma^2\omega_d{}^2-27\omega_d{}^4)\sin(3\varphi_1))$$

$$+\omega_d\cos(2t\omega_d)(A_2\omega_d(-A_1{}^2K_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})(5\sigma^2-3\omega_d{}^2)(\sigma^2$$

$$+\omega_d{}^2)-2A_2{}^2L_{0001}L_{0010}L_{2000}(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4))$$

$$+A_1(-\omega_d(A_1{}^2K_{0001}K_{0010}K_{2000}(5\sigma^2-3\omega_d{}^2)(\sigma^2+\omega_d{}^2)$$

$$+2A_2{}^2(K_{2000}L_{0001}L_{0010}+K_{0010}L_{0001}L_{2000}+K_{0001}L_{0010}L_{2000})(\sigma^4$$

$$-12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(\varphi_1)+A_1A_2\omega_d((3K_{0010}K_{2000}L_{0001}$$

$$+3K_{0001}K_{2000}L_{0010}-2K_{0001}K_{0010}L_{2000})\sigma^4+2(13K_{2000}(K_{0010}L_{0001}$$

$$+K_{0001}L_{0010})+12K_{0001}K_{0010}L_{2000})\sigma^2\omega_d{}^2-3(3K_{0010}K_{2000}L_{0001}$$

$$+ 3K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000})\omega_d{}^4)\cos(2\varphi_1)$$

$$+ A_1{}^2K_{0001}K_{0010}K_{2000}\omega_d(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4)\cos(3\varphi_1)$$

$$+ \sigma(-A_1{}^2K_{0001}K_{0010}K_{2000}(\sigma^2 - 7\omega_d{}^2)(\sigma^2 + \omega_d{}^2) + 2A_2{}^2(2(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{2000}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + K_{2000}L_{0001}L_{0010}(\sigma^4$$

$$- 17\omega_d{}^4)))\sin(\varphi_1) + A_1A_2\sigma(K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma^4$$

$$+ 6(K_{0010}K_{2000}L_{0001} + K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000})\sigma^2\omega_d{}^2$$

$$- (27K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010}) + 20K_{0001}K_{0010}L_{2000})\omega_d{}^4)\sin(2\varphi_1)$$

$$+ A_1{}^2K_{0001}K_{0010}K_{2000}\sigma(\sigma^4 + 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4)\sin(3\varphi_1))) + e^{\sigma t}\omega_d(\sigma^2$$

$$+ \omega_d{}^2)\cos(t\omega_d)(2A_1{}^2A_2\omega_d((K_{0010}K_{2000}L_{0001} + K_{0001}K_{2000}L_{0010}$$

$$- 2K_{0001}K_{0010}L_{2000})\sigma^2 - 3(5K_{0010}K_{2000}L_{0001} + 5K_{0001}K_{2000}L_{0010}$$

$$+ 6K_{0001}K_{0010}L_{2000})\omega_d{}^2) + 4A_2{}^3L_{0001}L_{0010}L_{2000}\omega_d(\sigma^3t + 3(-4 + 3\sigma t)\omega_d{}^2)$$

$$+ A_1(4\omega_d(-12A_1{}^2K_{0001}K_{0010}K_{2000}\omega_d{}^2 + A_2{}^2(K_{2000}L_{0001}L_{0010}$$

$$+ K_{0010}L_{0001}L_{2000} + K_{0001}L_{0010}L_{2000})(\sigma^3t + 3(-4 + 3\sigma t)\omega_d{}^2))\cos(\varphi_1)$$

$$+ 2A_1A_2\omega_d(K_{0010}\sigma^2(2K_{0001}L_{2000}(1 + \sigma t) + K_{2000}L_{0001}(-1 + 2\sigma t))$$

$$+ 3K_{0010}(-3K_{2000}L_{0001} - 2K_{0001}L_{2000} + 6(K_{2000}L_{0001} + K_{0001}L_{2000})\sigma t)\omega_d{}^2$$

$$+ K_{0001}K_{2000}L_{0010}(-1 + 2\sigma t)(\sigma^2 + 9\omega_d{}^2))\cos(2\varphi_1)$$

$$+ 4A_1{}^2K_{0001}K_{0010}K_{2000}\sigma t\omega_d(\sigma^2 + 9\omega_d{}^2)\cos(3\varphi_1)$$

$$- 2(A_1{}^2K_{0001}K_{0010}K_{2000}(\sigma^4t + 2\sigma(4 + 5\sigma t)\omega_d{}^2 + 9t\omega_d{}^4)$$

$$+ A_2{}^2(-(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}\sigma^4t + 2\sigma(K_{2000}L_{0001}L_{0010}(-4 + \sigma t)$$

$$- 4(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}(-1 + \sigma t))\omega_d{}^2 + 9(2K_{2000}L_{0001}L_{0010}$$

$$+ K_{0010}L_{0001}L_{2000} + K_{0001}L_{0010}L_{2000})t\omega_d{}^4))\sin(\varphi_1)$$

$$+ A_1A_2((K_{0010}K_{2000}L_{0001} + K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000})\sigma^4t$$

$$+ 2\sigma(-8K_{0001}K_{0010}L_{2000} + (3K_{0010}K_{2000}L_{0001} + 3K_{0001}K_{2000}L_{0010}$$

$$+ 8K_{0001}K_{0010}L_{2000})\sigma t)\omega_d{}^2 - 9(3K_{0010}K_{2000}L_{0001} + 3K_{0001}K_{2000}L_{0010}$$

$$+ 2K_{0001}K_{0010}L_{2000})t\omega_d{}^4)\sin(2\varphi_1) + 2A_1{}^2K_{0001}K_{0010}K_{2000}t(\sigma^4 + 8\sigma^2\omega_d{}^2$$

$$- 9\omega_d{}^4)\sin(3\varphi_1))) + e^{\sigma t}(\sigma^2 + \omega_d{}^2)(-A_2\omega_d(4A_2{}^2L_{0001}L_{0010}L_{2000}(\sigma^3 + \sigma(5$$

$$+ \sigma t)\omega_d{}^2 + 9t\omega_d{}^4) + A_1{}^2(\sigma^3(2K_{0001}K_{0010}L_{2000}\sigma t + K_{0010}K_{2000}L_{0001}(4 + \sigma t)$$

$$+ K_{0001}K_{2000}L_{0010}(4 + \sigma t)) + 10\sigma(2K_{0001}K_{0010}L_{2000}\sigma t + K_{0010}K_{2000}L_{0001}(2$$

$$+ \sigma t) + K_{0001}K_{2000}L_{0010}(2 + \sigma t))\omega_d{}^2 + 9(K_{0010}K_{2000}L_{0001}$$

$$+ K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000})t\omega_d{}^4))$$

$$- 2A_1\omega_d(A_1{}^2K_{0001}K_{0010}K_{2000}(\sigma^3(2 + \sigma t) + 10\sigma(1 + \sigma t)\omega_d{}^2 + 9t\omega_d{}^4)$$

$$+ 2A_2{}^2(K_{2000}L_{0001}L_{0010} + K_{0010}L_{0001}L_{2000} + K_{0001}L_{0010}L_{2000})(\sigma^3 + \sigma(5$$

$$+ \sigma t)\omega_d{}^2 + 9t\omega_d{}^4))\cos(\varphi_1) + A_1{}^2A_2\omega_d(\sigma^3(-4K_{0001}K_{0010}L_{2000}$$

$$+ (K_{0010}K_{2000}L_{0001} + K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000})\sigma t)$$

$$+ 2\sigma(-10K_{0001}K_{0010}L_{2000} + (3K_{0010}K_{2000}L_{0001} + 3K_{0001}K_{2000}L_{0010}$$

$$+ 8K_{0001}K_{0010}L_{2000})\sigma t)\omega_d{}^2 - 9(3K_{0010}K_{2000}L_{0001} + 3K_{0001}K_{2000}L_{0010}$$

$$+ 2K_{0001}K_{0010}L_{2000})t\omega_d{}^4)\cos(2\varphi_1) + 2A_1{}^3K_{0001}K_{0010}K_{2000}t\omega_d(\sigma^4$$

$$+ 8\sigma^2\omega_d{}^2 - 9\omega_d{}^4)\cos(3\varphi_1) - 2A_1(2A_1{}^2K_{0001}K_{0010}K_{2000}(\sigma^4 + 7\sigma^2\omega_d{}^2$$

$$- 6\omega_d{}^4) + A_2{}^2((K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}\sigma^4 + 2\sigma^2(K_{2000}L_{0001}L_{0010}(1$$

$$+ \sigma t) + (K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}(3 + \sigma t))\omega_d{}^2$$

$$+ 3(2K_{2000}L_{0001}L_{0010}(-1 + 3\sigma t) + (K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}(-1$$

$$+ 6\sigma t))\omega_d{}^4))\sin(\varphi_1) - A_1{}^2A_2(K_{0001}K_{2000}L_{0010}(\sigma^2 + 9\omega_d{}^2)(\sigma^2 + (-1$$

$$+ 4\sigma t)\omega_d{}^2) + K_{0010}(K_{2000}L_{0001}(\sigma^2 + 9\omega_d{}^2)(\sigma^2 + (-1 + 4\sigma t)\omega_d{}^2)$$

$$+ 2K_{0001}L_{2000}(\sigma^4 + 2\sigma^2(3 + \sigma t)\omega_d{}^2 + 3(-1 + 6\sigma t)\omega_d{}^4)))\sin(2\varphi_1)$$

$$- 4A_1{}^3K_{0001}K_{0010}K_{2000}\sigma t\omega_d{}^2(\sigma^2 + 9\omega_d{}^2)\sin(3\varphi_1))\sin(t\omega_d)$$

$$+ \omega_d(A_2\sigma(4A_2{}^2L_{0001}L_{0010}L_{2000}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) - A_1{}^2K_{2000}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})(\sigma^2 - 7{\omega_d}^2)(\sigma^2 + {\omega_d}^2)) + A_1(\sigma(4{A_2}^2(K_{2000}L_{0001}L_{0010}$$

$$+ K_{0010}L_{0001}L_{2000} + K_{0001}L_{0010}L_{2000}){\omega_d}^2(3\sigma^2 - 5{\omega_d}^2)$$

$$- {A_1}^2K_{0001}K_{0010}K_{2000}(\sigma^2 - 7{\omega_d}^2)(\sigma^2 + {\omega_d}^2))\cos(\varphi_1)$$

$$+ A_1A_2\sigma(K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma^4 + 6(K_{0010}K_{2000}L_{0001}$$

$$+ K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000})\sigma^2{\omega_d}^2 - (27K_{2000}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010}) + 20K_{0001}K_{0010}L_{2000}){\omega_d}^4)\cos(2\varphi_1)$$

$$+ {A_1}^2K_{0001}K_{0010}K_{2000}\sigma(\sigma^2 - 3{\omega_d}^2)(\sigma^2 + 9{\omega_d}^2)\cos(3\varphi_1)$$

$$+ \omega_d({A_1}^2K_{0001}K_{0010}K_{2000}(5\sigma^2 - 3{\omega_d}^2)(\sigma^2 + {\omega_d}^2)$$

$$+ 2{A_2}^2(-2K_{2000}L_{0001}L_{0010}(2\sigma^4 + 7\sigma^2{\omega_d}^2 - 3{\omega_d}^4) + (K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{2000}(\sigma^4 - 12\sigma^2{\omega_d}^2 + 3{\omega_d}^4)))\sin(\varphi_1)$$

$$+ A_1A_2\omega_d((-3K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010}) + 2K_{0001}K_{0010}L_{2000})\sigma^4$$

$$- 2(13K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010}) + 12K_{0001}K_{0010}L_{2000})\sigma^2{\omega_d}^2$$

$$+ 3(3K_{0010}K_{2000}L_{0001} + 3K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000}){\omega_d}^4)\sin(2\varphi_1)$$

$$+ {A_1}^2K_{0001}K_{0010}K_{2000}\omega_d(-3\sigma^2 + {\omega_d}^2)(\sigma^2 + 9{\omega_d}^2)\sin(3\varphi_1)))\sin(2t\omega_d)\}$$

$$(C.4)$$

$$x_{1,2}^{qs1,u_2,u_1}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{\omega_d(A_2\omega_d(\sigma^2$$

$$+ 9\omega_d{}^2)(A_1{}^2(K_{0010}K_{2000}L_{0001} + K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000}$$

$$+ 2e^{2\sigma t}(K_{0010}K_{2000}L_{0001} + K_{0001}K_{2000}L_{0010} + K_{0001}K_{0010}L_{2000}))(\sigma^2 + \omega_d{}^2)$$

$$+ 2A_2{}^2L_{0001}L_{0010}L_{2000}(\sigma^2 + \omega_d{}^2 + 2e^{2\sigma t}\omega_d{}^2)) + A_1\omega_d(\sigma^2 + 9\omega_d{}^2)(A_1{}^2(2$$

$$+ 3e^{2\sigma t})K_{0001}K_{0010}K_{2000}(\sigma^2 + \omega_d{}^2) + 2A_2{}^2(K_{2000}L_{0001}L_{0010}$$

$$+ K_{0010}L_{0001}L_{2000} + K_{0001}L_{0010}L_{2000})(\sigma^2 + \omega_d{}^2 + 2e^{2\sigma t}\omega_d{}^2))\cos(\varphi_1)$$

$$+ A_1{}^2A_2\omega_d(\sigma^2 + 9\omega_d{}^2)(-2e^{2\sigma t}(K_{0010}K_{2000}L_{0001} + K_{0001}K_{2000}L_{0010}$$

$$+ K_{0001}K_{0010}L_{2000})(\sigma - \omega_d)(\sigma + \omega_d) + K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010})(\sigma^2$$

$$+ \omega_d{}^2))\cos(2\varphi_1) + A_1{}^3e^{2\sigma t}K_{0001}K_{0010}K_{2000}\omega_d(-3\sigma^2 + \omega_d{}^2)(\sigma^2$$

$$+ 9\omega_d{}^2)\cos(3\varphi_1) + A_1\sigma(\sigma^2 + 9\omega_d{}^2)(A_1{}^2(-2 + 3e^{2\sigma t})K_{0001}K_{0010}K_{2000}(\sigma^2$$

$$+ \omega_d{}^2) + 2A_2{}^2(2e^{2\sigma t}(K_{2000}L_{0001}L_{0010} + K_{0010}L_{0001}L_{2000}$$

$$+ K_{0001}L_{0010}L_{2000})\omega_d{}^2 - K_{2000}L_{0001}L_{0010}(\sigma^2 + \omega_d{}^2)))\sin(\varphi_1) + A_1{}^2A_2\sigma(\sigma^2$$

$$+ 9\omega_d{}^2)(4e^{2\sigma t}(K_{0010}K_{2000}L_{0001} + K_{0001}K_{2000}L_{0010} + K_{0001}K_{0010}L_{2000})\omega_d{}^2$$

$$- K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010})(\sigma^2 + \omega_d{}^2))\sin(2\varphi_1)$$

$$- A_1{}^3e^{2\sigma t}K_{0001}K_{0010}K_{2000}\sigma(\sigma^4 + 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4)\sin(3\varphi_1))$$

$$+ \omega_d\cos(2t\omega_d)(A_2\omega_d(-A_1{}^2K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010})(5\sigma^2 - 3\omega_d{}^2)(\sigma^2$$

$$+ \omega_d{}^2) - 2A_2{}^2L_{0001}L_{0010}L_{2000}(\sigma^4 - 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))$$

$$+ A_1(-\omega_d(A_1{}^2K_{0001}K_{0010}K_{2000}(5\sigma^2 - 3\omega_d{}^2)(\sigma^2 + \omega_d{}^2)$$

$$+ 2A_2{}^2(K_{2000}L_{0001}L_{0010} + K_{0010}L_{0001}L_{2000} + K_{0001}L_{0010}L_{2000})(\sigma^4$$

$$- 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))\cos(\varphi_1) + A_1A_2\omega_d((3K_{0010}K_{2000}L_{0001}$$

$$+ 3K_{0001}K_{2000}L_{0010} - 2K_{0001}K_{0010}L_{2000})\sigma^4 + 2(13K_{2000}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010}) + 12K_{0001}K_{0010}L_{2000})\sigma^2\omega_d{}^2 - 3(3K_{0010}K_{2000}L_{0001}$$

$$+ 3K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000})\omega_d{}^4)\cos(2\varphi_1)$$

$$+ A_1{}^2K_{0001}K_{0010}K_{2000}\omega_d(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4)\cos(3\varphi_1)$$

$$+ \sigma(-A_1{}^2K_{0001}K_{0010}K_{2000}(\sigma^2 - 7\omega_d{}^2)(\sigma^2 + \omega_d{}^2) + 2A_2{}^2(2(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{2000}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + K_{2000}L_{0001}L_{0010}(\sigma^4$$

$$- 17\omega_d{}^4)))\sin(\varphi_1) + A_1A_2\sigma(K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma^4$$

$$+ 6(K_{0010}K_{2000}L_{0001} + K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000})\sigma^2\omega_d{}^2$$

$$- (27K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010}) + 20K_{0001}K_{0010}L_{2000})\omega_d{}^4)\sin(2\varphi_1)$$

$$+ A_1{}^2K_{0001}K_{0010}K_{2000}\sigma(\sigma^4 + 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4)\sin(3\varphi_1))) + e^{\sigma t}\omega_d(\sigma^2$$

$$+ \omega_d{}^2)\cos(t\omega_d)(2A_1{}^2A_2\omega_d((K_{0010}K_{2000}L_{0001} + K_{0001}K_{2000}L_{0010}$$

$$- 2K_{0001}K_{0010}L_{2000})\sigma^2 - 3(5K_{0010}K_{2000}L_{0001} + 5K_{0001}K_{2000}L_{0010}$$

$$+ 6K_{0001}K_{0010}L_{2000})\omega_d{}^2) + 4A_2{}^3L_{0001}L_{0010}L_{2000}\omega_d(\sigma^3t + 3(-4 + 3\sigma t)\omega_d{}^2)$$

$$+ A_1(4\omega_d(-12A_1{}^2K_{0001}K_{0010}K_{2000}\omega_d{}^2 + A_2{}^2(K_{2000}L_{0001}L_{0010}$$

$$+ K_{0010}L_{0001}L_{2000} + K_{0001}L_{0010}L_{2000})(\sigma^3t + 3(-4 + 3\sigma t)\omega_d{}^2))\cos(\varphi_1)$$

$$+ 2A_1A_2\omega_d(K_{0010}\sigma^2(2K_{0001}L_{2000}(1 + \sigma t) + K_{2000}L_{0001}(-1 + 2\sigma t))$$

$$+ 3K_{0010}(-3K_{2000}L_{0001} - 2K_{0001}L_{2000} + 6(K_{2000}L_{0001} + K_{0001}L_{2000})\sigma t)\omega_d{}^2$$

$$+ K_{0001}K_{2000}L_{0010}(-1 + 2\sigma t)(\sigma^2 + 9\omega_d{}^2))\cos(2\varphi_1)$$

$$+ 4A_1{}^2K_{0001}K_{0010}K_{2000}\sigma t\omega_d(\sigma^2 + 9\omega_d{}^2)\cos(3\varphi_1)$$

$$- 2(A_1{}^2K_{0001}K_{0010}K_{2000}(\sigma^4t + 2\sigma(4 + 5\sigma t)\omega_d{}^2 + 9t\omega_d{}^4)$$

$$+ A_2{}^2(-(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}\sigma^4t + 2\sigma(K_{2000}L_{0001}L_{0010}(-4 + \sigma t)$$

$$- 4(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}(-1 + \sigma t))\omega_d{}^2 + 9(2K_{2000}L_{0001}L_{0010}$$

$$+ K_{0010}L_{0001}L_{2000} + K_{0001}L_{0010}L_{2000})t\omega_d{}^4))\sin(\varphi_1)$$

$$+ A_1A_2((K_{0010}K_{2000}L_{0001} + K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000})\sigma^4t$$

$$+ 2\sigma(-8K_{0001}K_{0010}L_{2000} + (3K_{0010}K_{2000}L_{0001} + 3K_{0001}K_{2000}L_{0010}$$

$$+ 8K_{0001}K_{0010}L_{2000})\sigma t)\omega_d{}^2 - 9(3K_{0010}K_{2000}L_{0001} + 3K_{0001}K_{2000}L_{0010}$$

$$+ 2K_{0001}K_{0010}L_{2000})t\omega_d{}^4)\sin(2\varphi_1) + 2A_1{}^2K_{0001}K_{0010}K_{2000}t(\sigma^4 + 8\sigma^2\omega_d{}^2$$

$$- 9\omega_d{}^4)\sin(3\varphi_1))) + e^{\sigma t}(\sigma^2 + \omega_d{}^2)(-A_2\omega_d(4A_2{}^2L_{0001}L_{0010}L_{2000}(\sigma^3 + \sigma(5$$

$$+ \sigma t)\omega_d{}^2 + 9t\omega_d{}^4) + A_1{}^2(\sigma^3(2K_{0001}K_{0010}L_{2000}\sigma t + K_{0010}K_{2000}L_{0001}(4 + \sigma t)$$

$$+ K_{0001}K_{2000}L_{0010}(4 + \sigma t)) + 10\sigma(2K_{0001}K_{0010}L_{2000}\sigma t + K_{0010}K_{2000}L_{0001}(2$$

$$+ \sigma t) + K_{0001}K_{2000}L_{0010}(2 + \sigma t))\omega_d{}^2 + 9(K_{0010}K_{2000}L_{0001}$$

$$+ K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000})t\omega_d{}^4))$$

$$- 2A_1\omega_d(A_1{}^2K_{0001}K_{0010}K_{2000}(\sigma^3(2 + \sigma t) + 10\sigma(1 + \sigma t)\omega_d{}^2 + 9t\omega_d{}^4)$$

$$+ 2A_2{}^2(K_{2000}L_{0001}L_{0010} + K_{0010}L_{0001}L_{2000} + K_{0001}L_{0010}L_{2000})(\sigma^3 + \sigma(5$$

$$+ \sigma t)\omega_d{}^2 + 9t\omega_d{}^4))\cos(\varphi_1) + A_1{}^2A_2\omega_d(\sigma^3(-4K_{0001}K_{0010}L_{2000}$$

$$+ (K_{0010}K_{2000}L_{0001} + K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000})\sigma t)$$

$$+ 2\sigma(-10K_{0001}K_{0010}L_{2000} + (3K_{0010}K_{2000}L_{0001} + 3K_{0001}K_{2000}L_{0010}$$

$$+ 8K_{0001}K_{0010}L_{2000})\sigma t)\omega_d{}^2 - 9(3K_{0010}K_{2000}L_{0001} + 3K_{0001}K_{2000}L_{0010}$$

$$+ 2K_{0001}K_{0010}L_{2000})t\omega_d{}^4)\cos(2\varphi_1) + 2A_1{}^3K_{0001}K_{0010}K_{2000}t\omega_d(\sigma^4$$

$$+ 8\sigma^2\omega_d{}^2 - 9\omega_d{}^4)\cos(3\varphi_1) - 2A_1(2A_1{}^2K_{0001}K_{0010}K_{2000}(\sigma^4 + 7\sigma^2\omega_d{}^2$$

$$- 6\omega_d{}^4) + A_2{}^2((K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}\sigma^4 + 2\sigma^2(K_{2000}L_{0001}L_{0010}(1$$

$$+ \sigma t) + (K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}(3 + \sigma t))\omega_d{}^2$$

$$+ 3(2K_{2000}L_{0001}L_{0010}(-1 + 3\sigma t) + (K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}(-1$$

$$+ 6\sigma t))\omega_d{}^4))\sin(\varphi_1) - A_1{}^2A_2(K_{0001}K_{2000}L_{0010}(\sigma^2 + 9\omega_d{}^2)(\sigma^2 + (-1$$

$$+ 4\sigma t)\omega_d{}^2) + K_{0010}(K_{2000}L_{0001}(\sigma^2 + 9\omega_d{}^2)(\sigma^2 + (-1 + 4\sigma t)\omega_d{}^2)$$

$$+ 2K_{0001}L_{2000}(\sigma^4 + 2\sigma^2(3 + \sigma t)\omega_d{}^2 + 3(-1 + 6\sigma t)\omega_d{}^4)))\sin(2\varphi_1)$$

$$- 4A_1{}^3K_{0001}K_{0010}K_{2000}\sigma t\omega_d{}^2(\sigma^2 + 9\omega_d{}^2)\sin(3\varphi_1))\sin(t\omega_d)$$

$$+ \omega_d(A_2\sigma(4A_2{}^2L_{0001}L_{0010}L_{2000}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) - A_1{}^2K_{2000}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})(\sigma^2 - 7{\omega_d}^2)(\sigma^2 + {\omega_d}^2)) + A_1(\sigma(4{A_2}^2(K_{2000}L_{0001}L_{0010}$$

$$+ K_{0010}L_{0001}L_{2000} + K_{0001}L_{0010}L_{2000}){\omega_d}^2(3\sigma^2 - 5{\omega_d}^2)$$

$$- {A_1}^2K_{0001}K_{0010}K_{2000}(\sigma^2 - 7{\omega_d}^2)(\sigma^2 + {\omega_d}^2))\cos(\varphi_1)$$

$$+ A_1A_2\sigma(K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma^4 + 6(K_{0010}K_{2000}L_{0001}$$

$$+ K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000})\sigma^2{\omega_d}^2 - (27K_{2000}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010}) + 20K_{0001}K_{0010}L_{2000}){\omega_d}^4)\cos(2\varphi_1)$$

$$+ {A_1}^2K_{0001}K_{0010}K_{2000}\sigma(\sigma^2 - 3{\omega_d}^2)(\sigma^2 + 9{\omega_d}^2)\cos(3\varphi_1)$$

$$+ \omega_d({A_1}^2K_{0001}K_{0010}K_{2000}(5\sigma^2 - 3{\omega_d}^2)(\sigma^2 + {\omega_d}^2)$$

$$+ 2{A_2}^2(-2K_{2000}L_{0001}L_{0010}(2\sigma^4 + 7\sigma^2{\omega_d}^2 - 3{\omega_d}^4) + (K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{2000}(\sigma^4 - 12\sigma^2{\omega_d}^2 + 3{\omega_d}^4)))\sin(\varphi_1)$$

$$+ A_1A_2\omega_d((-3K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010}) + 2K_{0001}K_{0010}L_{2000})\sigma^4$$

$$- 2(13K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010}) + 12K_{0001}K_{0010}L_{2000})\sigma^2{\omega_d}^2$$

$$+ 3(3K_{0010}K_{2000}L_{0001} + 3K_{0001}K_{2000}L_{0010} + 2K_{0001}K_{0010}L_{2000}){\omega_d}^4)\sin(2\varphi_1)$$

$$+ {A_1}^2K_{0001}K_{0010}K_{2000}\omega_d(-3\sigma^2 + {\omega_d}^2)(\sigma^2 + 9{\omega_d}^2)\sin(3\varphi_1)))\sin(2t\omega_d)\}$$

(C.5)

$$x_{1,2}^{qs1,u_2,u_2}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{\omega_d{}^2(\sigma^2+9\omega_d{}^2)(2A_1{}^2A_2K_{0001}((1$$

$$+2e^{2\sigma t})K_{2000}L_{0001}+(1+e^{2\sigma t})K_{0001}L_{2000})(\sigma^2+\omega_d{}^2)+2A_2{}^3L_{0001}{}^2L_{2000}(\sigma^2+(1$$

$$+2e^{2\sigma t})\omega_d{}^2)+(A_1{}^3(2+3e^{2\sigma t})K_{0001}{}^2K_{2000}(\sigma^2+\omega_d{}^2)+2A_1A_2{}^2L_{0001}(K_{2000}L_{0001}$$

$$+2K_{0001}L_{2000})(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2))\cos(\varphi_1))+2A_1{}^2A_2K_{0001}\omega_d{}^2(\sigma^2$$

$$+9\omega_d{}^2)(-e^{2\sigma t}K_{0001}L_{2000}(\sigma^2-\omega_d{}^2)+K_{2000}L_{0001}((1-2e^{2\sigma t})\sigma^2+(1$$

$$+2e^{2\sigma t})\omega_d{}^2))\cos(2\varphi_1)+A_1{}^3e^{2\sigma t}K_{0001}{}^2K_{2000}\omega_d{}^2(-3\sigma^4-26\sigma^2\omega_d{}^2$$

$$+9\omega_d{}^4)\cos(3\varphi_1)+4A_2e^{\sigma t}\omega_d{}^2(\sigma^2+\omega_d{}^2)(A_2{}^2L_{0001}{}^2L_{2000}(\sigma^3t-12\omega_d{}^2$$

$$+9\sigma t\omega_d{}^2)-A_1{}^2K_{0001}(-K_{2000}L_{0001}(\sigma^2-15\omega_d{}^2)+K_{0001}L_{2000}(\sigma^2$$

$$+9\omega_d{}^2)))\cos(t\omega_d)-2A_2L_{0001}\omega_d{}^2(A_1{}^2K_{0001}K_{2000}(5\sigma^4+2\sigma^2\omega_d{}^2-3\omega_d{}^4)$$

$$+A_2{}^2L_{0001}L_{2000}(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(2t\omega_d)+A_1A_2{}^2K_{2000}L_{0001}{}^2\omega_d{}^2(-5\sigma^4$$

$$-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1-2t\omega_d)-2A_1e^{\sigma t}(\sigma^2+\omega_d{}^2)(A_1{}^2K_{0001}{}^2K_{2000}(\sigma^4$$

$$+7\sigma^2\omega_d{}^2+6\omega_d{}^4)+A_2{}^2L_{0001}(K_{2000}L_{0001}\omega_d{}^2(\sigma^2+9\omega_d{}^2)+K_{0001}L_{2000}(\sigma^4$$

$$+6\sigma^2\omega_d{}^2+21\omega_d{}^4)))\cos(\varphi_1-t\omega_d)-A_1{}^2A_2e^{\sigma t}K_{0001}(\sigma^2+\omega_d{}^2)^2(K_{0001}L_{2000}(\sigma^2$$

$$+3\omega_d{}^2)+K_{2000}L_{0001}(\sigma^2+9\omega_d{}^2))\cos(2\varphi_1-t\omega_d)+2A_1e^{\sigma t}(\sigma^2$$

$$+\omega_d{}^2)(A_1{}^2K_{0001}{}^2K_{2000}(\sigma^4+7\sigma^2\omega_d{}^2-18\omega_d{}^4)+A_2{}^2L_{0001}(K_{0001}L_{2000}(\sigma^2$$

$$+9\omega_d{}^2)(\sigma^2-3\omega_d{}^2+4\sigma t\omega_d{}^2)+K_{2000}L_{0001}\omega_d{}^2(\sigma^2+2\sigma^3t-15\omega_d{}^2$$

$$+18\sigma t\omega_d{}^2)))\cos(\varphi_1+t\omega_d)-2A_1{}^2A_2K_{0001}\omega_d{}^2(K_{0001}L_{2000}(\sigma^4-12\sigma^2\omega_d{}^2$$

$$+3\omega_d{}^4)+K_{2000}L_{0001}(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4))\cos(2(\varphi_1+t\omega_d))$$

$$+A_1{}^2A_2e^{\sigma t}K_{0001}(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)(K_{0001}L_{2000}(\sigma^2-\omega_d{}^2+4\sigma t\omega_d{}^2)$$

$$+K_{2000}L_{0001}(\sigma^2-3\omega_d{}^2+8\sigma t\omega_d{}^2))\cos(2\varphi_1+t\omega_d)$$

$$+4A_1{}^3e^{\sigma t}K_{0001}{}^2K_{2000}\sigma t\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(3\varphi_1+t\omega_d)$$

$$+A_1\omega_d{}^2(A_1{}^2K_{0001}{}^2K_{2000}(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)+A_2{}^2L_{0001}(K_{2000}L_{0001}(3\sigma^4$$

$$+26\sigma^2\omega_d{}^2-9\omega_d{}^4)-4K_{0001}L_{2000}(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4)))\cos(\varphi_1+2t\omega_d)$$

$$+A_1{}^3K_{0001}{}^2K_{2000}\omega_d{}^2(3\sigma^4+26\sigma^2\omega_d{}^2-9\omega_d{}^4)\cos(3\varphi_1+2t\omega_d)+A_1\sigma\omega_d(\sigma^2$$

$$+ 9\omega_d{}^2)(A_1{}^2(-2 + 3e^{2\sigma t})K_{0001}{}^2K_{2000}(\sigma^2 + \omega_d{}^2)$$

$$- 2A_2{}^2L_{0001}(-4e^{2\sigma t}K_{0001}L_{2000}\omega_d{}^2 + K_{2000}L_{0001}(\sigma^2 + \omega_d{}^2 - 2e^{2\sigma t}\omega_d{}^2)))\sin(\varphi_1)$$

$$+ 2A_1{}^2A_2K_{0001}\sigma\omega_d(\sigma^2 + 9\omega_d{}^2)(2e^{2\sigma t}K_{0001}L_{2000}\omega_d{}^2 - K_{2000}L_{0001}(\sigma^2 + (1$$

$$- 4e^{2\sigma t})\omega_d{}^2))\sin(2\varphi_1) - A_1{}^3e^{2\sigma t}K_{0001}{}^2K_{2000}\omega_d(\sigma^5 + 6\sigma^3\omega_d{}^2 - 27\sigma\omega_d{}^4)\sin(3\varphi_1)$$

$$- 2A_2e^{\sigma t}\omega_d(\sigma^2 + \omega_d{}^2)(2A_2{}^2L_{0001}{}^2L_{2000}(\sigma^3 + 5\sigma\omega_d{}^2 + \sigma^2t\omega_d{}^2 + 9t\omega_d{}^4)$$

$$+ A_1{}^2K_{0001}(K_{0001}L_{2000}t(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4) + K_{2000}L_{0001}(4\sigma^3 + \sigma^4t$$

$$+ 20\sigma\omega_d{}^2 + 10\sigma^2t\omega_d{}^2 + 9t\omega_d{}^4)))\sin(t\omega_d)$$

$$+ 2A_2L_{0001}\sigma\omega_d(2A_2{}^2L_{0001}L_{2000}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + A_1{}^2K_{0001}K_{2000}(-\sigma^4 + 6\sigma^2\omega_d{}^2$$

$$+ 7\omega_d{}^4))\sin(2t\omega_d) + A_1A_2{}^2K_{2000}L_{0001}{}^2\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_1 - 2t\omega_d)$$

$$+ 2A_1e^{\sigma t}\omega_d(\sigma^2 + \omega_d{}^2)(A_1{}^2K_{0001}{}^2K_{2000}\sigma(\sigma^2 + \omega_d{}^2) + A_2{}^2L_{0001}(K_{2000}L_{0001}\sigma(\sigma^2$$

$$+ 9\omega_d{}^2) + K_{0001}L_{2000}(\sigma^2 + \omega_d{}^2)(2\sigma + \sigma^2t + 9t\omega_d{}^2)))\sin(\varphi_1 - t\omega_d)$$

$$+ 2A_1{}^2A_2e^{\sigma t}K_{0001}{}^2L_{2000}\sigma\omega_d(\sigma^2 + \omega_d{}^2)^2\sin(2\varphi_1 - t\omega_d) - 2A_1e^{\sigma t}\omega_d(\sigma^2$$

$$+ \omega_d{}^2)(A_1{}^2K_{0001}{}^2K_{2000}(\sigma^2 + 9\omega_d{}^2)(\sigma + \sigma^2t + t\omega_d{}^2) + A_2{}^2L_{0001}(-K_{0001}L_{2000}(\sigma^2$$

$$+ 9\omega_d{}^2)(-2\sigma + \sigma^2t - 3t\omega_d{}^2) + K_{2000}L_{0001}(\sigma^3 + \sigma\omega_d{}^2 + 2\sigma^2t\omega_d{}^2$$

$$+ 18t\omega_d{}^4)))\sin(\varphi_1 + t\omega_d) + 2A_1{}^2A_2K_{0001}\sigma\omega_d(2K_{0001}L_{2000}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2)$$

$$+ K_{2000}L_{0001}(\sigma^4 + 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4))\sin(2(\varphi_1 + t\omega_d)) - 2A_1{}^2A_2e^{\sigma t}K_{0001}\omega_d(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(-K_{2000}L_{0001}t(\sigma^2 - 3\omega_d{}^2) + K_{0001}L_{2000}(\sigma - \sigma^2t$$

$$+ t\omega_d{}^2))\sin(2\varphi_1 + t\omega_d) + 2A_1{}^3e^{\sigma t}K_{0001}{}^2K_{2000}t\omega_d(\sigma^6 + 9\sigma^4\omega_d{}^2 - \sigma^2\omega_d{}^4$$

$$- 9\omega_d{}^6)\sin(3\varphi_1 + t\omega_d) + A_1\sigma\omega_d(A_1{}^2K_{0001}{}^2K_{2000}(-\sigma^4 + 6\sigma^2\omega_d{}^2 + 7\omega_d{}^4)$$

$$+ A_2{}^2L_{0001}(8K_{0001}L_{2000}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + K_{2000}L_{0001}(\sigma^4 + 6\sigma^2\omega_d{}^2$$

$$- 27\omega_d{}^4)))\sin(\varphi_1 + 2t\omega_d) + A_1{}^3K_{0001}{}^2K_{2000}\sigma\omega_d(\sigma^4 + 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4)\sin(3\varphi_1$$

$$+ 2t\omega_d)\}$$

$$(C.6)$$

$$x_{1,2}^{bs1s2}(t) = x_{1,2}^{bs1s2,u_1,u_1}(t) \times u_1u_1 + x_{1,2}^{bs1s2,u_1,u_2}(t) \times u_1u_2 + x_{1,2}^{bs1s2,u_2,u_1}(t) \times u_2u_1 + x_{1,2}^{bs1s2,u_2,u_2}(t) \times u_2u_2 \qquad (C.7)$$

$$x_{1,2}^{bs1s2,u_1,u_1}(t) = \frac{1}{8\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{2A_3K_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_1{}^2(1$$

$$+ 2e^{2\sigma t})K_{0010}K_{1100}(\sigma^2+\omega_d{}^2) + 2A_2{}^2L_{0010}L_{1100}(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)$$

$$+ 2A_1A_2(K_{1100}L_{0010}+K_{0010}L_{1100})(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_1))$$

$$+ 2A_1{}^2A_3K_{0010}{}^2K_{1100}\omega_d{}^2(\sigma^2+9\omega_d{}^2)((1-2e^{2\sigma t})\sigma^2+(1$$

$$+ 2e^{2\sigma t})\omega_d{}^2)\cos(2\varphi_1) + 2A_1A_2A_4L_{0010}((1+2e^{2\sigma t})K_{1100}L_{0010}+2(1$$

$$+ e^{2\sigma t})K_{0010}L_{1100})\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1-\varphi_2) + 2A_1{}^2A_4(1$$

$$+ e^{2\sigma t})K_{0010}K_{1100}L_{0010}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(2\varphi_1-\varphi_2)$$

$$+ 2A_4L_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_1{}^2(1+2e^{2\sigma t})K_{0010}K_{1100}(\sigma^2+\omega_d{}^2)$$

$$+ 2A_2{}^2L_{0010}L_{1100}(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2))\cos(\varphi_2) + 2A_1A_2A_4L_{0010}\omega_d{}^2(\sigma^2$$

$$+ 9\omega_d{}^2)(-2e^{2\sigma t}K_{0010}L_{1100}(\sigma^2-\omega_d{}^2) + K_{1100}L_{0010}((1-2e^{2\sigma t})\sigma^2+(1$$

$$+ 2e^{2\sigma t})\omega_d{}^2))\cos(\varphi_1+\varphi_2) + 2A_1{}^2A_4e^{2\sigma t}K_{0010}K_{1100}L_{0010}\omega_d{}^2(-3\sigma^4$$

$$- 26\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\cos(2\varphi_1+\varphi_2) + 4A_3e^{\sigma t}K_{0010}\omega_d{}^2(\sigma^2$$

$$+ \omega_d{}^2)(A_1{}^2K_{0010}K_{1100}(\sigma^2-15\omega_d{}^2) + 2A_2{}^2L_{0010}L_{1100}(\sigma^3t-12\omega_d{}^2$$

$$+ 9\sigma t\omega_d{}^2))\cos(t\omega_d) - 2A_3K_{0010}\omega_d{}^2(A_1{}^2K_{0010}K_{1100}(5\sigma^4+2\sigma^2\omega_d{}^2-3\omega_d{}^4)$$

$$+ 2A_2{}^2L_{0010}L_{1100}(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(2t\omega_d)$$

$$+ 2A_1A_2A_3K_{0010}K_{1100}L_{0010}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1-2t\omega_d)$$

$$+ 2A_1A_2A_4K_{1100}L_{0010}{}^2\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1-\varphi_2-2t\omega_d)$$

$$- 2A_1A_2A_3e^{\sigma t}K_{0010}(\sigma^2+\omega_d{}^2)(2K_{1100}L_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2) + K_{0010}L_{1100}(\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2+21\omega_d{}^4))\cos(\varphi_1-t\omega_d) - A_1{}^2A_3e^{\sigma t}K_{0010}{}^2K_{1100}(\sigma^2+\omega_d{}^2)^2(\sigma^2$$

$$+ 9\omega_d{}^2)\cos(2\varphi_1-t\omega_d) + A_1A_2A_4e^{\sigma t}L_{0010}(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)(-4K_{0010}L_{1100}\omega_d{}^2 + K_{1100}L_{0010}(\sigma^2-3\omega_d{}^2))\cos(\varphi_1-\varphi_2-t\omega_d)$$

$$- A_1{}^2A_4e^{\sigma t}K_{0010}K_{1100}L_{0010}(\sigma^2+\omega_d{}^2)^2(\sigma^2+9\omega_d{}^2)\cos(2\varphi_1-\varphi_2-t\omega_d)$$

$$- A_4 e^{\sigma t} L_{0010}(\sigma^2 + {\omega_d}^2)(3{A_1}^2 K_{0010} K_{1100}(\sigma^4 + 6\sigma^2{\omega_d}^2 + 5{\omega_d}^4)$$

$$+ 2{A_2}^2 L_{0010} L_{1100}(\sigma^4 + 6\sigma^2{\omega_d}^2 + 21{\omega_d}^4))\cos(\varphi_2 - t\omega_d)$$

$$- A_1 A_2 A_4 e^{\sigma t} L_{0010}(\sigma^2 + {\omega_d}^2)^2(2K_{0010} L_{1100}(\sigma^2 + 3{\omega_d}^2) + K_{1100} L_{0010}(\sigma^2$$

$$+ 9{\omega_d}^2))\cos(\varphi_1 + \varphi_2 - t\omega_d) + 2A_1 A_2 A_3 e^{\sigma t} K_{0010}(\sigma^2 + {\omega_d}^2)(K_{0010} L_{1100}(\sigma^2$$

$$+ 9{\omega_d}^2)(\sigma^2 - 3{\omega_d}^2 + 4\sigma t{\omega_d}^2) + 2K_{1100} L_{0010}{\omega_d}^2(\sigma^2 + 2\sigma^3 t - 15{\omega_d}^2$$

$$+ 18\sigma t{\omega_d}^2))\cos(\varphi_1 + t\omega_d) + {A_1}^2 A_3 {K_{0010}}^2 K_{1100}(\sigma^2 + 9{\omega_d}^2)((6\sigma^2{\omega_d}^2$$

$$- 2{\omega_d}^4)\cos(2(\varphi_1 + t\omega_d)) + e^{\sigma t}(\sigma^2 + {\omega_d}^2)(\sigma^2 - 3{\omega_d}^2 + 8\sigma t{\omega_d}^2)\cos(2\varphi_1$$

$$+ t\omega_d)) - A_1 A_2 A_4 e^{\sigma t} L_{0010}(\sigma^2 + {\omega_d}^2)(4K_{0010} L_{1100}{\omega_d}^2(\sigma^2 + 9{\omega_d}^2)$$

$$+ K_{1100} L_{0010}(\sigma^4 + 2\sigma^2{\omega_d}^2 + 33{\omega_d}^4))\cos(\varphi_1 - \varphi_2 + t\omega_d)$$

$$+ {A_1}^2 A_4 e^{\sigma t} K_{0010} K_{1100} L_{0010}(\sigma^6 + 7\sigma^4{\omega_d}^2 - 21\sigma^2{\omega_d}^4 - 27{\omega_d}^6)\cos(2\varphi_1$$

$$- \varphi_2 + t\omega_d) + A_4 e^{\sigma t} L_{0010}(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)({A_1}^2 K_{0010} K_{1100}(3\sigma^2$$

$$- 5{\omega_d}^2) + 2{A_2}^2 L_{0010} L_{1100}(\sigma^2 - 3{\omega_d}^2 + 4\sigma t{\omega_d}^2))\cos(\varphi_2 + t\omega_d)$$

$$+ A_1 A_2 A_4 e^{\sigma t} L_{0010}(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)(2K_{0010} L_{1100}(\sigma^2 - {\omega_d}^2 + 4\sigma t{\omega_d}^2)$$

$$+ K_{1100} L_{0010}(\sigma^2 - 3{\omega_d}^2 + 8\sigma t{\omega_d}^2))\cos(\varphi_1 + \varphi_2 + t\omega_d)$$

$$+ 8{A_1}^2 A_4 e^{\sigma t} K_{0010} K_{1100} L_{0010}\sigma t{\omega_d}^2(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\cos(2\varphi_1 + \varphi_2$$

$$+ t\omega_d) + 2A_1 A_2 A_3 K_{0010}{\omega_d}^2(K_{1100} L_{0010}(3\sigma^4 + 26\sigma^2{\omega_d}^2 - 9{\omega_d}^4)$$

$$- 2K_{0010} L_{1100}(\sigma^4 - 12\sigma^2{\omega_d}^2 + 3{\omega_d}^4))\cos(\varphi_1 + 2t\omega_d)$$

$$- 2A_4 L_{0010}{\omega_d}^2({A_1}^2 K_{0010} K_{1100}(5\sigma^4 + 2\sigma^2{\omega_d}^2 - 3{\omega_d}^4) + 2{A_2}^2 L_{0010} L_{1100}(\sigma^4$$

$$- 12\sigma^2{\omega_d}^2 + 3{\omega_d}^4))\cos(\varphi_2 + 2t\omega_d) + 2A_1 A_2 A_4 L_{0010}{\omega_d}^2(K_{1100} L_{0010}(3\sigma^4$$

$$+ 26\sigma^2{\omega_d}^2 - 9{\omega_d}^4) - 2K_{0010} L_{1100}(\sigma^4 - 12\sigma^2{\omega_d}^2 + 3{\omega_d}^4))\cos(\varphi_1 + \varphi_2$$

$$+ 2t\omega_d) + 2{A_1}^2 A_4 K_{0010} K_{1100} L_{0010}{\omega_d}^2(3\sigma^4 + 26\sigma^2{\omega_d}^2 - 9{\omega_d}^4)\cos(2\varphi_1$$

$$+ \varphi_2 + 2t\omega_d) + 4A_1 A_2 A_3 K_{0010}\sigma\omega_d(\sigma^2 + 9{\omega_d}^2)(2e^{2\sigma t} K_{0010} L_{1100}{\omega_d}^2$$

$$- K_{1100} L_{0010}(\sigma^2 + (1 - 2e^{2\sigma t}){\omega_d}^2))\sin(\varphi_1) - 2{A_1}^2 A_3 {K_{0010}}^2 K_{1100}\omega_d(\sigma^2$$

$$+ (1-4e^{2\sigma t}){\omega_d}^2)(\sigma^3+9\sigma{\omega_d}^2)\sin(2\varphi_1) - 2A_1A_2A_4K_{1100}{L_{0010}}^2\sigma\omega_d(\sigma^4$$

$$+ 10\sigma^2{\omega_d}^2+9{\omega_d}^4)\sin(\varphi_1-\varphi_2) + 2{A_1}^2A_4(-1$$

$$+ e^{2\sigma t})K_{0010}K_{1100}L_{0010}\sigma\omega_d(\sigma^4+10\sigma^2{\omega_d}^2+9{\omega_d}^4)\sin(2\varphi_1-\varphi_2)$$

$$+ 2A_4L_{0010}\sigma\omega_d(\sigma^2+9{\omega_d}^2)(4{A_2}^2e^{2\sigma t}L_{0010}L_{1100}{\omega_d}^2+{A_1}^2(-1$$

$$+ 2e^{2\sigma t})K_{0010}K_{1100}(\sigma^2+{\omega_d}^2))\sin(\varphi_2) - 2A_1A_2A_4L_{0010}\omega_d(\sigma^3$$

$$+ 9\sigma{\omega_d}^2)(-4e^{2\sigma t}K_{0010}L_{1100}{\omega_d}^2+K_{1100}L_{0010}(\sigma^2+(1-4e^{2\sigma t}){\omega_d}^2))\sin(\varphi_1$$

$$+ \varphi_2) - 2{A_1}^2A_4e^{2\sigma t}K_{0010}K_{1100}L_{0010}\omega_d(\sigma^5+6\sigma^3{\omega_d}^2-27\sigma{\omega_d}^4)\sin(2\varphi_1$$

$$+ \varphi_2) - 2A_3e^{\sigma t}K_{0010}\omega_d(\sigma^2+{\omega_d}^2)(4{A_2}^2L_{0010}L_{1100}(\sigma^3+5\sigma{\omega_d}^2+\sigma^2t{\omega_d}^2$$

$$+ 9t{\omega_d}^4) + {A_1}^2K_{0010}K_{1100}(4\sigma^3+\sigma^4t+20\sigma{\omega_d}^2+10\sigma^2t{\omega_d}^2$$

$$+ 9t{\omega_d}^4))\sin(t\omega_d) + 2A_3K_{0010}\sigma\omega_d(4{A_2}^2L_{0010}L_{1100}{\omega_d}^2(3\sigma^2-5{\omega_d}^2)$$

$$+ {A_1}^2K_{0010}K_{1100}(-\sigma^4+6\sigma^2{\omega_d}^2+7{\omega_d}^4))\sin(2t\omega_d)$$

$$+ 2A_1A_2A_3K_{0010}K_{1100}L_{0010}\sigma\omega_d(\sigma^4-6\sigma^2{\omega_d}^2-7{\omega_d}^4)\sin(\varphi_1-2t\omega_d)$$

$$+ 2A_1A_2A_4K_{1100}{L_{0010}}^2\sigma\omega_d(\sigma^4-6\sigma^2{\omega_d}^2-7{\omega_d}^4)\sin(\varphi_1-\varphi_2-2t\omega_d)$$

$$+ 2A_1A_2A_3e^{\sigma t}K_{0010}\omega_d(\sigma^2+{\omega_d}^2)(2K_{1100}L_{0010}\sigma(\sigma^2+9{\omega_d}^2)$$

$$+ K_{0010}L_{1100}(\sigma^2+{\omega_d}^2)(2\sigma+\sigma^2t+9t{\omega_d}^2))\sin(\varphi_1-t\omega_d)$$

$$+ 2A_1A_2A_4e^{\sigma t}L_{0010}\omega_d(\sigma^4+10\sigma^2{\omega_d}^2+9{\omega_d}^4)(2K_{1100}L_{0010}\sigma$$

$$+ K_{0010}L_{1100}t(\sigma^2+{\omega_d}^2))\sin(\varphi_1-\varphi_2-t\omega_d)$$

$$+ 2A_4e^{\sigma t}L_{0010}\omega_d(\sigma^2+{\omega_d}^2)^2(2{A_1}^2K_{0010}K_{1100}\sigma+{A_2}^2L_{0010}L_{1100}(2\sigma+\sigma^2t$$

$$+ 9t{\omega_d}^2))\sin(\varphi_2-t\omega_d)$$

$$+ 4A_1A_2A_4e^{\sigma t}K_{0010}L_{0010}L_{1100}\sigma\omega_d(\sigma^2+{\omega_d}^2)^2\sin(\varphi_1+\varphi_2-t\omega_d)$$

$$- 2A_1A_2A_3e^{\sigma t}K_{0010}\omega_d(\sigma^2+{\omega_d}^2)(-K_{0010}L_{1100}(\sigma^2+9{\omega_d}^2)(-2\sigma+\sigma^2t$$

$$- 3t{\omega_d}^2) + 2K_{1100}L_{0010}(\sigma^3+\sigma{\omega_d}^2+2\sigma^2t{\omega_d}^2+18t{\omega_d}^4))\sin(\varphi_1+t\omega_d)$$

$$+ 2{A_1}^2A_3{K_{0010}}^2K_{1100}\sigma\omega_d(\sigma^4+6\sigma^2{\omega_d}^2-27{\omega_d}^4)\sin(2(\varphi_1+t\omega_d))$$

$$+ 2A_1{}^2A_3e^{\sigma t}K_{0010}{}^2K_{1100}t\omega_d(\sigma^6 + 7\sigma^4\omega_d{}^2 - 21\sigma^2\omega_d{}^4 - 27\omega_d{}^6)\sin(2\varphi_1$$

$$+ t\omega_d) - 2A_1A_2A_4e^{\sigma t}L_{0010}\omega_d(\sigma^2 + \omega_d{}^2)^2(K_{0010}L_{1100}t(\sigma^2 + 9\omega_d{}^2)$$

$$+ K_{1100}L_{0010}(2\sigma + \sigma^2 t + 9t\omega_d{}^2))\sin(\varphi_1 - \varphi_2 + t\omega_d)$$

$$- 2A_1{}^2A_4e^{\sigma t}K_{0010}K_{1100}L_{0010}t\omega_d(\sigma^2 + \omega_d{}^2)^2(\sigma^2 + 9\omega_d{}^2)\sin(2\varphi_1 - \varphi_2$$

$$+ t\omega_d) - 2A_4e^{\sigma t}L_{0010}\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(A_1{}^2K_{0010}K_{1100}(2\sigma + \sigma^2 t$$

$$+ t\omega_d{}^2) + A_2{}^2L_{0010}L_{1100}(2\sigma - \sigma^2 t + 3t\omega_d{}^2))\sin(\varphi_2 + t\omega_d)$$

$$- 2A_1A_2A_4e^{\sigma t}L_{0010}\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(-K_{1100}L_{0010}t(\sigma^2 - 3\omega_d{}^2)$$

$$+ 2K_{0010}L_{1100}(\sigma - \sigma^2 t + t\omega_d{}^2))\sin(\varphi_1 + \varphi_2 + t\omega_d)$$

$$+ 4A_1{}^2A_4e^{\sigma t}K_{0010}K_{1100}L_{0010}t\omega_d(\sigma^6 + 9\sigma^4\omega_d{}^2 - \sigma^2\omega_d{}^4 - 9\omega_d{}^6)\sin(2\varphi_1$$

$$+ \varphi_2 + t\omega_d) + 2A_1A_2A_3K_{0010}\sigma\omega_d(4K_{0010}L_{1100}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2)$$

$$+ K_{1100}L_{0010}(\sigma^4 + 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4))\sin(\varphi_1 + 2t\omega_d)$$

$$+ 2A_4L_{0010}\sigma\omega_d(4A_2{}^2L_{0010}L_{1100}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + A_1{}^2K_{0010}K_{1100}(-\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2 + 7\omega_d{}^4))\sin(\varphi_2 + 2t\omega_d)$$

$$+ 2A_1A_2A_4L_{0010}\sigma\omega_d(4K_{0010}L_{1100}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + K_{1100}L_{0010}(\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4))\sin(\varphi_1 + \varphi_2 + 2t\omega_d) + 2A_1{}^2A_4K_{0010}K_{1100}L_{0010}\sigma\omega_d(\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4)\sin(2\varphi_1 + \varphi_2 + 2t\omega_d)\}$$

(C. 8)

$$x_{1,2}^{bs1s2,u_1,u_2}(t) = \frac{1}{8\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{2A_3K_{0001}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_1{}^2(1$$

$$+2e^{2\sigma t})K_{0010}K_{1100}(\sigma^2+\omega_d{}^2)+2A_2{}^2L_{0010}L_{1100}(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)$$

$$+2A_1A_2(K_{1100}L_{0010}+K_{0010}L_{1100})(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_1))$$

$$+2A_1{}^2A_3K_{0001}K_{0010}K_{1100}\omega_d{}^2(\sigma^2+9\omega_d{}^2)((1-2e^{2\sigma t})\sigma^2+(1$$

$$+2e^{2\sigma t})\omega_d{}^2)\cos(2\varphi_1)+2A_1A_2A_4L_{0001}((1+2e^{2\sigma t})K_{1100}L_{0010}+2(1$$

$$+e^{2\sigma t})K_{0010}L_{1100})\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1-\varphi_2)+2A_1{}^2A_4(1$$

$$+e^{2\sigma t})K_{0010}K_{1100}L_{0001}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(2\varphi_1-\varphi_2)$$

$$+2A_4L_{0001}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_1{}^2(1+2e^{2\sigma t})K_{0010}K_{1100}(\sigma^2+\omega_d{}^2)$$

$$+2A_2{}^2L_{0010}L_{1100}(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2))\cos(\varphi_2)+2A_1A_2A_4L_{0001}\omega_d{}^2(\sigma^2$$

$$+9\omega_d{}^2)(-2e^{2\sigma t}K_{0010}L_{1100}(\sigma^2-\omega_d{}^2)+K_{1100}L_{0010}((1-2e^{2\sigma t})\sigma^2+(1$$

$$+2e^{2\sigma t})\omega_d{}^2))\cos(\varphi_1+\varphi_2)+2A_1{}^2A_4e^{2\sigma t}K_{0010}K_{1100}L_{0001}\omega_d{}^2(-3\sigma^4$$

$$-26\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(2\varphi_1+\varphi_2)+4A_3e^{\sigma t}K_{0001}\omega_d{}^2(\sigma^2$$

$$+\omega_d{}^2)(A_1{}^2K_{0010}K_{1100}(\sigma^2-15\omega_d{}^2)+2A_2{}^2L_{0010}L_{1100}(\sigma^3t-12\omega_d{}^2$$

$$+9\sigma t\omega_d{}^2))\cos(t\omega_d)-2A_3K_{0001}\omega_d{}^2(A_1{}^2K_{0010}K_{1100}(5\sigma^4+2\sigma^2\omega_d{}^2-3\omega_d{}^4)$$

$$+2A_2{}^2L_{0010}L_{1100}(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(2t\omega_d)$$

$$+2A_1A_2A_3K_{0001}K_{1100}L_{0010}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1-2t\omega_d)$$

$$+2A_1A_2A_4K_{1100}L_{0001}L_{0010}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1-\varphi_2$$

$$-2t\omega_d)-2A_1A_2A_3e^{\sigma t}K_{0001}(\sigma^2+\omega_d{}^2)(2K_{1100}L_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2)$$

$$+K_{0010}L_{1100}(\sigma^4+6\sigma^2\omega_d{}^2+21\omega_d{}^4))\cos(\varphi_1-t\omega_d)$$

$$-A_1{}^2A_3e^{\sigma t}K_{0001}K_{0010}K_{1100}(\sigma^2+\omega_d{}^2)^2(\sigma^2+9\omega_d{}^2)\cos(2\varphi_1-t\omega_d)$$

$$+A_1A_2A_4e^{\sigma t}L_{0001}(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)(-4K_{0010}L_{1100}\omega_d{}^2$$

$$+K_{1100}L_{0010}(\sigma^2-3\omega_d{}^2))\cos(\varphi_1-\varphi_2-t\omega_d)$$

$$- A_1{}^2 A_4 e^{\sigma t} K_{0010} K_{1100} L_{0001} (\sigma^2 + \omega_d{}^2)^2 (\sigma^2 + 9\omega_d{}^2) \cos(2\varphi_1 - \varphi_2 - t\omega_d)$$

$$- A_4 e^{\sigma t} L_{0001} (\sigma^2 + \omega_d{}^2)(3A_1{}^2 K_{0010} K_{1100} (\sigma^4 + 6\sigma^2 \omega_d{}^2 + 5\omega_d{}^4)$$

$$+ 2A_2{}^2 L_{0010} L_{1100} (\sigma^4 + 6\sigma^2 \omega_d{}^2 + 21\omega_d{}^4)) \cos(\varphi_2 - t\omega_d)$$

$$- A_1 A_2 A_4 e^{\sigma t} L_{0001} (\sigma^2 + \omega_d{}^2)^2 (2K_{0010} L_{1100} (\sigma^2 + 3\omega_d{}^2) + K_{1100} L_{0010} (\sigma^2$$

$$+ 9\omega_d{}^2)) \cos(\varphi_1 + \varphi_2 - t\omega_d) + 2A_1 A_2 A_3 e^{\sigma t} K_{0001} (\sigma^2 + \omega_d{}^2)(K_{0010} L_{1100} (\sigma^2$$

$$+ 9\omega_d{}^2)(\sigma^2 - 3\omega_d{}^2 + 4\sigma t \omega_d{}^2) + 2K_{1100} L_{0010} \omega_d{}^2 (\sigma^2 + 2\sigma^3 t - 15\omega_d{}^2$$

$$+ 18\sigma t \omega_d{}^2)) \cos(\varphi_1 + t\omega_d) + 2A_1{}^2 A_3 K_{0001} K_{0010} K_{1100} \omega_d{}^2 (3\sigma^4 + 26\sigma^2 \omega_d{}^2$$

$$- 9\omega_d{}^4) \cos(2(\varphi_1 + t\omega_d)) + A_1{}^2 A_3 e^{\sigma t} K_{0001} K_{0010} K_{1100} (\sigma^2 - 3\omega_d{}^2$$

$$+ 8\sigma t \omega_d{}^2)(\sigma^4 + 10\sigma^2 \omega_d{}^2 + 9\omega_d{}^4) \cos(2\varphi_1 + t\omega_d) - A_1 A_2 A_4 e^{\sigma t} L_{0001} (\sigma^2$$

$$+ \omega_d{}^2)(4K_{0010} L_{1100} \omega_d{}^2 (\sigma^2 + 9\omega_d{}^2) + K_{1100} L_{0010} (\sigma^4 + 2\sigma^2 \omega_d{}^2$$

$$+ 33\omega_d{}^4)) \cos(\varphi_1 - \varphi_2 + t\omega_d) + A_1{}^2 A_4 e^{\sigma t} K_{0010} K_{1100} L_{0001} (\sigma^6 + 7\sigma^4 \omega_d{}^2$$

$$- 21\sigma^2 \omega_d{}^4 - 27\omega_d{}^6) \cos(2\varphi_1 - \varphi_2 + t\omega_d) + A_4 e^{\sigma t} L_{0001} (\sigma^4 + 10\sigma^2 \omega_d{}^2$$

$$+ 9\omega_d{}^4)(A_1{}^2 K_{0010} K_{1100} (3\sigma^2 - 5\omega_d{}^2) + 2A_2{}^2 L_{0010} L_{1100} (\sigma^2 - 3\omega_d{}^2$$

$$+ 4\sigma t \omega_d{}^2)) \cos(\varphi_2 + t\omega_d) + A_1 A_2 A_4 e^{\sigma t} L_{0001} (\sigma^4 + 10\sigma^2 \omega_d{}^2$$

$$+ 9\omega_d{}^4)(2K_{0010} L_{1100} (\sigma^2 - \omega_d{}^2 + 4\sigma t \omega_d{}^2) + K_{1100} L_{0010} (\sigma^2 - 3\omega_d{}^2$$

$$+ 8\sigma t \omega_d{}^2)) \cos(\varphi_1 + \varphi_2 + t\omega_d) + 8A_1{}^2 A_4 e^{\sigma t} K_{0010} K_{1100} L_{0001} \sigma t \omega_d{}^2 (\sigma^4$$

$$+ 10\sigma^2 \omega_d{}^2 + 9\omega_d{}^4) \cos(2\varphi_1 + \varphi_2 + t\omega_d)$$

$$+ 2A_1 A_2 A_3 K_{0001} \omega_d{}^2 (K_{1100} L_{0010} (3\sigma^4 + 26\sigma^2 \omega_d{}^2 - 9\omega_d{}^4) - 2K_{0010} L_{1100} (\sigma^4$$

$$- 12\sigma^2 \omega_d{}^2 + 3\omega_d{}^4)) \cos(\varphi_1 + 2t\omega_d) - 2A_4 L_{0001} \omega_d{}^2 (A_1{}^2 K_{0010} K_{1100} (5\sigma^4$$

$$+ 2\sigma^2 \omega_d{}^2 - 3\omega_d{}^4) + 2A_2{}^2 L_{0010} L_{1100} (\sigma^4 - 12\sigma^2 \omega_d{}^2 + 3\omega_d{}^4)) \cos(\varphi_2$$

$$+ 2t\omega_d) + 2A_1 A_2 A_4 L_{0001} \omega_d{}^2 (K_{1100} L_{0010} (3\sigma^4 + 26\sigma^2 \omega_d{}^2 - 9\omega_d{}^4)$$

$$- 2K_{0010} L_{1100} (\sigma^4 - 12\sigma^2 \omega_d{}^2 + 3\omega_d{}^4)) \cos(\varphi_1 + \varphi_2 + 2t\omega_d)$$

$$+ 2A_1{}^2 A_4 K_{0010} K_{1100} L_{0001} \omega_d{}^2 (3\sigma^4 + 26\sigma^2 \omega_d{}^2 - 9\omega_d{}^4) \cos(2\varphi_1 + \varphi_2$$

$$+ 2t\omega_d) - 4A_1A_2A_3K_{0001}\omega_d(\sigma^3 + 9\sigma\omega_d{}^2)(-2e^{2\sigma t}K_{0010}L_{1100}\omega_d{}^2$$

$$+ K_{1100}L_{0010}(\sigma^2 + (1 - 2e^{2\sigma t})\omega_d{}^2))\sin(\varphi_1) - 2A_1{}^2A_3K_{0001}K_{0010}K_{1100}\omega_d(\sigma^2$$

$$+ (1 - 4e^{2\sigma t})\omega_d{}^2)(\sigma^3 + 9\sigma\omega_d{}^2)\sin(2\varphi_1) - 2A_1A_2A_4K_{1100}L_{0001}L_{0010}\sigma\omega_d(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 - \varphi_2) + 2A_1{}^2A_4(-1$$

$$+ e^{2\sigma t})K_{0010}K_{1100}L_{0001}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(2\varphi_1 - \varphi_2)$$

$$+ 2A_4L_{0001}\sigma\omega_d(\sigma^2 + 9\omega_d{}^2)(4A_2{}^2e^{2\sigma t}L_{0010}L_{1100}\omega_d{}^2 + A_1{}^2(-1$$

$$+ 2e^{2\sigma t})K_{0010}K_{1100}(\sigma^2 + \omega_d{}^2))\sin(\varphi_2) - 2A_1A_2A_4L_{0001}\omega_d(\sigma^3$$

$$+ 9\sigma\omega_d{}^2)(-4e^{2\sigma t}K_{0010}L_{1100}\omega_d{}^2 + K_{1100}L_{0010}(\sigma^2 + (1 - 4e^{2\sigma t})\omega_d{}^2))\sin(\varphi_1$$

$$+ \varphi_2) - 2A_1{}^2A_4e^{2\sigma t}K_{0010}K_{1100}L_{0001}\omega_d(\sigma^5 + 6\sigma^3\omega_d{}^2 - 27\sigma\omega_d{}^4)\sin(2\varphi_1$$

$$+ \varphi_2) - 2A_3e^{\sigma t}K_{0001}\omega_d(\sigma^2 + \omega_d{}^2)(4A_2{}^2L_{0010}L_{1100}(\sigma^3 + 5\sigma\omega_d{}^2 + \sigma^2t\omega_d{}^2$$

$$+ 9t\omega_d{}^4) + A_1{}^2K_{0010}K_{1100}(4\sigma^3 + \sigma^4t + 20\sigma\omega_d{}^2 + 10\sigma^2t\omega_d{}^2$$

$$+ 9t\omega_d{}^4))\sin(t\omega_d) + 2A_3K_{0001}\sigma\omega_d(4A_2{}^2L_{0010}L_{1100}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2)$$

$$+ A_1{}^2K_{0010}K_{1100}(-\sigma^4 + 6\sigma^2\omega_d{}^2 + 7\omega_d{}^4))\sin(2t\omega_d)$$

$$+ 2A_1A_2A_3K_{0001}K_{1100}L_{0010}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_1 - 2t\omega_d)$$

$$+ 2A_1A_2A_4K_{1100}L_{0001}L_{0010}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_1 - \varphi_2 - 2t\omega_d)$$

$$+ 2A_1A_2A_3e^{\sigma t}K_{0001}\omega_d(\sigma^2 + \omega_d{}^2)(2K_{1100}L_{0010}\sigma(\sigma^2 + 9\omega_d{}^2)$$

$$+ K_{0010}L_{1100}(\sigma^2 + \omega_d{}^2)(2\sigma + \sigma^2t + 9t\omega_d{}^2))\sin(\varphi_1 - t\omega_d)$$

$$+ 2A_1A_2A_4e^{\sigma t}L_{0001}\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(2K_{1100}L_{0010}\sigma$$

$$+ K_{0010}L_{1100}t(\sigma^2 + \omega_d{}^2))\sin(\varphi_1 - \varphi_2 - t\omega_d)$$

$$+ 2A_4e^{\sigma t}L_{0001}\omega_d(\sigma^2 + \omega_d{}^2)^2(2A_1{}^2K_{0010}K_{1100}\sigma + A_2{}^2L_{0010}L_{1100}(2\sigma + \sigma^2t$$

$$+ 9t\omega_d{}^2))\sin(\varphi_2 - t\omega_d)$$

$$+ 4A_1A_2A_4e^{\sigma t}K_{0010}L_{0001}L_{1100}\sigma\omega_d(\sigma^2 + \omega_d{}^2)^2\sin(\varphi_1 + \varphi_2 - t\omega_d)$$

$$- 2A_1A_2A_3e^{\sigma t}K_{0001}\omega_d(\sigma^2 + \omega_d{}^2)(-K_{0010}L_{1100}(\sigma^2 + 9\omega_d{}^2)(-2\sigma + \sigma^2t$$

$$- 3t\omega_d^{\,2}) + 2K_{1100}L_{0010}(\sigma^3 + \sigma\omega_d^{\,2} + 2\sigma^2 t\omega_d^{\,2} + 18t\omega_d^{\,4}))\sin(\varphi_1 + t\omega_d)$$

$$+ 2A_1^{\,2}A_3K_{0001}K_{0010}K_{1100}\sigma\omega_d(\sigma^4 + 6\sigma^2\omega_d^{\,2} - 27\omega_d^{\,4})\sin(2(\varphi_1 + t\omega_d))$$

$$+ 2A_1^{\,2}A_3e^{\sigma t}K_{0001}K_{0010}K_{1100}t\omega_d(\sigma^6 + 7\sigma^4\omega_d^{\,2} - 21\sigma^2\omega_d^{\,4} - 27\omega_d^{\,6})\sin(2\varphi_1$$

$$+ t\omega_d) - 2A_1A_2A_4e^{\sigma t}L_{0001}\omega_d(\sigma^2 + \omega_d^{\,2})^2(K_{0010}L_{1100}t(\sigma^2 + 9\omega_d^{\,2})$$

$$+ K_{1100}L_{0010}(2\sigma + \sigma^2 t + 9t\omega_d^{\,2}))\sin(\varphi_1 - \varphi_2 + t\omega_d)$$

$$- 2A_1^{\,2}A_4e^{\sigma t}K_{0010}K_{1100}L_{0001}t\omega_d(\sigma^2 + \omega_d^{\,2})^2(\sigma^2 + 9\omega_d^{\,2})\sin(2\varphi_1 - \varphi_2$$

$$+ t\omega_d) - 2A_4e^{\sigma t}L_{0001}\omega_d(\sigma^4 + 10\sigma^2\omega_d^{\,2} + 9\omega_d^{\,4})(A_1^{\,2}K_{0010}K_{1100}(2\sigma + \sigma^2 t$$

$$+ t\omega_d^{\,2}) + A_2^{\,2}L_{0010}L_{1100}(2\sigma - \sigma^2 t + 3t\omega_d^{\,2}))\sin(\varphi_2 + t\omega_d)$$

$$- 2A_1A_2A_4e^{\sigma t}L_{0001}\omega_d(\sigma^4 + 10\sigma^2\omega_d^{\,2} + 9\omega_d^{\,4})(-K_{1100}L_{0010}t(\sigma^2 - 3\omega_d^{\,2})$$

$$+ 2K_{0010}L_{1100}(\sigma - \sigma^2 t + t\omega_d^{\,2}))\sin(\varphi_1 + \varphi_2 + t\omega_d)$$

$$+ 4A_1^{\,2}A_4e^{\sigma t}K_{0010}K_{1100}L_{0001}t\omega_d(\sigma^6 + 9\sigma^4\omega_d^{\,2} - \sigma^2\omega_d^{\,4} - 9\omega_d^{\,6})\sin(2\varphi_1$$

$$+ \varphi_2 + t\omega_d) + 2A_1A_2A_3K_{0001}\sigma\omega_d(4K_{0010}L_{1100}\omega_d^{\,2}(3\sigma^2 - 5\omega_d^{\,2})$$

$$+ K_{1100}L_{0010}(\sigma^4 + 6\sigma^2\omega_d^{\,2} - 27\omega_d^{\,4}))\sin(\varphi_1 + 2t\omega_d)$$

$$+ 2A_4L_{0001}\sigma\omega_d(4A_2^{\,2}L_{0010}L_{1100}\omega_d^{\,2}(3\sigma^2 - 5\omega_d^{\,2}) + A_1^{\,2}K_{0010}K_{1100}(-\sigma^4$$

$$+ 6\sigma^2\omega_d^{\,2} + 7\omega_d^{\,4}))\sin(\varphi_2 + 2t\omega_d)$$

$$+ 2A_1A_2A_4L_{0001}\sigma\omega_d(4K_{0010}L_{1100}\omega_d^{\,2}(3\sigma^2 - 5\omega_d^{\,2}) + K_{1100}L_{0010}(\sigma^4$$

$$+ 6\sigma^2\omega_d^{\,2} - 27\omega_d^{\,4}))\sin(\varphi_1 + \varphi_2 + 2t\omega_d) + 2A_1^{\,2}A_4K_{0010}K_{1100}L_{0001}\sigma\omega_d(\sigma^4$$

$$+ 6\sigma^2\omega_d^{\,2} - 27\omega_d^{\,4})\sin(2\varphi_1 + \varphi_2 + 2t\omega_d)\}$$

(C. 9)

$$x_{1,2}^{bs1s2,u_2,u_1}(t) = \frac{1}{8\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{2A_3K_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_1{}^2(1$$

$$+2e^{2\sigma t})K_{0001}K_{1100}(\sigma^2+\omega_d{}^2)+2A_2{}^2L_{0001}L_{1100}(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)$$

$$+2A_1A_2(K_{1100}L_{0001}+K_{0001}L_{1100})(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_1))$$

$$+2A_1{}^2A_3K_{0001}K_{0010}K_{1100}\omega_d{}^2(\sigma^2+9\omega_d{}^2)((1-2e^{2\sigma t})\sigma^2+(1$$

$$+2e^{2\sigma t})\omega_d{}^2)\cos(2\varphi_1)+2A_1A_2A_4L_{0010}((1+2e^{2\sigma t})K_{1100}L_{0001}+2(1$$

$$+e^{2\sigma t})K_{0001}L_{1100})\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1-\varphi_2)+2A_1{}^2A_4(1$$

$$+e^{2\sigma t})K_{0001}K_{1100}L_{0010}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(2\varphi_1-\varphi_2)$$

$$+2A_4L_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_1{}^2(1+2e^{2\sigma t})K_{0001}K_{1100}(\sigma^2+\omega_d{}^2)$$

$$+2A_2{}^2L_{0001}L_{1100}(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2))\cos(\varphi_2)+2A_1A_2A_4L_{0010}\omega_d{}^2(\sigma^2$$

$$+9\omega_d{}^2)(-2e^{2\sigma t}K_{0001}L_{1100}(\sigma^2-\omega_d{}^2)+K_{1100}L_{0001}((1-2e^{2\sigma t})\sigma^2+(1$$

$$+2e^{2\sigma t})\omega_d{}^2))\cos(\varphi_1+\varphi_2)+2A_1{}^2A_4e^{2\sigma t}K_{0001}K_{1100}L_{0010}\omega_d{}^2(-3\sigma^4$$

$$-26\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(2\varphi_1+\varphi_2)+4A_3e^{\sigma t}K_{0010}\omega_d{}^2(\sigma^2$$

$$+\omega_d{}^2)(A_1{}^2K_{0001}K_{1100}(\sigma^2-15\omega_d{}^2)+2A_2{}^2L_{0001}L_{1100}(\sigma^3t-12\omega_d{}^2$$

$$+9\sigma t\omega_d{}^2))\cos(t\omega_d)-2A_3K_{0010}\omega_d{}^2(A_1{}^2K_{0001}K_{1100}(5\sigma^4+2\sigma^2\omega_d{}^2-3\omega_d{}^4)$$

$$+2A_2{}^2L_{0001}L_{1100}(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(2t\omega_d)$$

$$+2A_1A_2A_3K_{0010}K_{1100}L_{0001}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1-2t\omega_d)$$

$$+2A_1A_2A_4K_{1100}L_{0001}L_{0010}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1-\varphi_2$$

$$-2t\omega_d)-2A_1A_2A_3e^{\sigma t}K_{0010}(\sigma^2+\omega_d{}^2)(2K_{1100}L_{0001}\omega_d{}^2(\sigma^2+9\omega_d{}^2)$$

$$+K_{0001}L_{1100}(\sigma^4+6\sigma^2\omega_d{}^2+21\omega_d{}^4))\cos(\varphi_1-t\omega_d)$$

$$-A_1{}^2A_3e^{\sigma t}K_{0001}K_{0010}K_{1100}(\sigma^2+\omega_d{}^2)^2(\sigma^2+9\omega_d{}^2)\cos(2\varphi_1-t\omega_d)$$

$$+A_1A_2A_4e^{\sigma t}L_{0010}(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)(-4K_{0001}L_{1100}\omega_d{}^2$$

$$+K_{1100}L_{0001}(\sigma^2-3\omega_d{}^2))\cos(\varphi_1-\varphi_2-t\omega_d)$$

$$- A_1{}^2 A_4 e^{\sigma t} K_{0001} K_{1100} L_{0010} (\sigma^2 + \omega_d{}^2)^2 (\sigma^2 + 9\omega_d{}^2) \cos(2\varphi_1 - \varphi_2 - t\omega_d)$$

$$- A_4 e^{\sigma t} L_{0010} (\sigma^2 + \omega_d{}^2)(3A_1{}^2 K_{0001} K_{1100} (\sigma^4 + 6\sigma^2 \omega_d{}^2 + 5\omega_d{}^4)$$

$$+ 2A_2{}^2 L_{0001} L_{1100} (\sigma^4 + 6\sigma^2 \omega_d{}^2 + 21\omega_d{}^4)) \cos(\varphi_2 - t\omega_d)$$

$$- A_1 A_2 A_4 e^{\sigma t} L_{0010} (\sigma^2 + \omega_d{}^2)^2 (2K_{0001} L_{1100} (\sigma^2 + 3\omega_d{}^2) + K_{1100} L_{0001} (\sigma^2$$

$$+ 9\omega_d{}^2)) \cos(\varphi_1 + \varphi_2 - t\omega_d) + 2A_1 A_2 A_3 e^{\sigma t} K_{0010} (\sigma^2 + \omega_d{}^2)(K_{0001} L_{1100} (\sigma^2$$

$$+ 9\omega_d{}^2)(\sigma^2 - 3\omega_d{}^2 + 4\sigma t \omega_d{}^2) + 2K_{1100} L_{0001} \omega_d{}^2 (\sigma^2 + 2\sigma^3 t - 15\omega_d{}^2$$

$$+ 18\sigma t \omega_d{}^2)) \cos(\varphi_1 + t\omega_d) + 2A_1{}^2 A_3 K_{0001} K_{0010} K_{1100} \omega_d{}^2 (3\sigma^4 + 26\sigma^2 \omega_d{}^2$$

$$- 9\omega_d{}^4) \cos(2(\varphi_1 + t\omega_d)) + A_1{}^2 A_3 e^{\sigma t} K_{0001} K_{0010} K_{1100} (\sigma^2 - 3\omega_d{}^2$$

$$+ 8\sigma t \omega_d{}^2)(\sigma^4 + 10\sigma^2 \omega_d{}^2 + 9\omega_d{}^4) \cos(2\varphi_1 + t\omega_d) - A_1 A_2 A_4 e^{\sigma t} L_{0010} (\sigma^2$$

$$+ \omega_d{}^2)(4K_{0001} L_{1100} \omega_d{}^2 (\sigma^2 + 9\omega_d{}^2) + K_{1100} L_{0001} (\sigma^4 + 2\sigma^2 \omega_d{}^2$$

$$+ 33\omega_d{}^4)) \cos(\varphi_1 - \varphi_2 + t\omega_d) + A_1{}^2 A_4 e^{\sigma t} K_{0001} K_{1100} L_{0010} (\sigma^6 + 7\sigma^4 \omega_d{}^2$$

$$- 21\sigma^2 \omega_d{}^4 - 27\omega_d{}^6) \cos(2\varphi_1 - \varphi_2 + t\omega_d) + A_4 e^{\sigma t} L_{0010} (\sigma^4 + 10\sigma^2 \omega_d{}^2$$

$$+ 9\omega_d{}^4)(A_1{}^2 K_{0001} K_{1100} (3\sigma^2 - 5\omega_d{}^2) + 2A_2{}^2 L_{0001} L_{1100} (\sigma^2 - 3\omega_d{}^2$$

$$+ 4\sigma t \omega_d{}^2)) \cos(\varphi_2 + t\omega_d) + A_1 A_2 A_4 e^{\sigma t} L_{0010} (\sigma^4 + 10\sigma^2 \omega_d{}^2$$

$$+ 9\omega_d{}^4)(2K_{0001} L_{1100} (\sigma^2 - \omega_d{}^2 + 4\sigma t \omega_d{}^2) + K_{1100} L_{0001} (\sigma^2 - 3\omega_d{}^2$$

$$+ 8\sigma t \omega_d{}^2)) \cos(\varphi_1 + \varphi_2 + t\omega_d) + 8A_1{}^2 A_4 e^{\sigma t} K_{0001} K_{1100} L_{0010} \sigma t \omega_d{}^2 (\sigma^4$$

$$+ 10\sigma^2 \omega_d{}^2 + 9\omega_d{}^4) \cos(2\varphi_1 + \varphi_2 + t\omega_d)$$

$$+ 2A_1 A_2 A_3 K_{0010} \omega_d{}^2 (K_{1100} L_{0001} (3\sigma^4 + 26\sigma^2 \omega_d{}^2 - 9\omega_d{}^4) - 2K_{0001} L_{1100} (\sigma^4$$

$$- 12\sigma^2 \omega_d{}^2 + 3\omega_d{}^4)) \cos(\varphi_1 + 2t\omega_d) - 2A_4 L_{0010} \omega_d{}^2 (A_1{}^2 K_{0001} K_{1100} (5\sigma^4$$

$$+ 2\sigma^2 \omega_d{}^2 - 3\omega_d{}^4) + 2A_2{}^2 L_{0001} L_{1100} (\sigma^4 - 12\sigma^2 \omega_d{}^2 + 3\omega_d{}^4)) \cos(\varphi_2$$

$$+ 2t\omega_d) + 2A_1 A_2 A_4 L_{0010} \omega_d{}^2 (K_{1100} L_{0001} (3\sigma^4 + 26\sigma^2 \omega_d{}^2 - 9\omega_d{}^4)$$

$$- 2K_{0001} L_{1100} (\sigma^4 - 12\sigma^2 \omega_d{}^2 + 3\omega_d{}^4)) \cos(\varphi_1 + \varphi_2 + 2t\omega_d)$$

$$+ 2A_1{}^2 A_4 K_{0001} K_{1100} L_{0010} \omega_d{}^2 (3\sigma^4 + 26\sigma^2 \omega_d{}^2 - 9\omega_d{}^4) \cos(2\varphi_1 + \varphi_2$$

$$+ 2t\omega_d) - 4A_1A_2A_3K_{0010}\omega_d(\sigma^3 + 9\sigma\omega_d{}^2)(-2e^{2\sigma t}K_{0001}L_{1100}\omega_d{}^2$$

$$+ K_{1100}L_{0001}(\sigma^2 + (1 - 2e^{2\sigma t})\omega_d{}^2))\sin(\varphi_1) - 2A_1{}^2A_3K_{0001}K_{0010}K_{1100}\omega_d(\sigma^2$$

$$+ (1 - 4e^{2\sigma t})\omega_d{}^2)(\sigma^3 + 9\sigma\omega_d{}^2)\sin(2\varphi_1) - 2A_1A_2A_4K_{1100}L_{0001}L_{0010}\sigma\omega_d(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 - \varphi_2) + 2A_1{}^2A_4(-1$$

$$+ e^{2\sigma t})K_{0001}K_{1100}L_{0010}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(2\varphi_1 - \varphi_2)$$

$$+ 2A_4L_{0010}\sigma\omega_d(\sigma^2 + 9\omega_d{}^2)(4A_2{}^2e^{2\sigma t}L_{0001}L_{1100}\omega_d{}^2 + A_1{}^2(-1$$

$$+ 2e^{2\sigma t})K_{0001}K_{1100}(\sigma^2 + \omega_d{}^2))\sin(\varphi_2) - 2A_1A_2A_4L_{0010}\omega_d(\sigma^3$$

$$+ 9\sigma\omega_d{}^2)(-4e^{2\sigma t}K_{0001}L_{1100}\omega_d{}^2 + K_{1100}L_{0001}(\sigma^2 + (1 - 4e^{2\sigma t})\omega_d{}^2))\sin(\varphi_1$$

$$+ \varphi_2) - 2A_1{}^2A_4e^{2\sigma t}K_{0001}K_{1100}L_{0010}\omega_d(\sigma^5 + 6\sigma^3\omega_d{}^2 - 27\sigma\omega_d{}^4)\sin(2\varphi_1$$

$$+ \varphi_2) - 2A_3e^{\sigma t}K_{0010}\omega_d(\sigma^2 + \omega_d{}^2)(4A_2{}^2L_{0001}L_{1100}(\sigma^3 + 5\sigma\omega_d{}^2 + \sigma^2t\omega_d{}^2$$

$$+ 9t\omega_d{}^4) + A_1{}^2K_{0001}K_{1100}(4\sigma^3 + \sigma^4t + 20\sigma\omega_d{}^2 + 10\sigma^2t\omega_d{}^2$$

$$+ 9t\omega_d{}^4))\sin(t\omega_d) + 2A_3K_{0010}\sigma\omega_d(4A_2{}^2L_{0001}L_{1100}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2)$$

$$+ A_1{}^2K_{0001}K_{1100}(-\sigma^4 + 6\sigma^2\omega_d{}^2 + 7\omega_d{}^4))\sin(2t\omega_d)$$

$$+ 2A_1A_2A_3K_{0010}K_{1100}L_{0001}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_1 - 2t\omega_d)$$

$$+ 2A_1A_2A_4K_{1100}L_{0001}L_{0010}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_1 - \varphi_2 - 2t\omega_d)$$

$$+ 2A_1A_2A_3e^{\sigma t}K_{0010}\omega_d(\sigma^2 + \omega_d{}^2)(2K_{1100}L_{0001}\sigma(\sigma^2 + 9\omega_d{}^2)$$

$$+ K_{0001}L_{1100}(\sigma^2 + \omega_d{}^2)(2\sigma + \sigma^2t + 9t\omega_d{}^2))\sin(\varphi_1 - t\omega_d)$$

$$+ 2A_1A_2A_4e^{\sigma t}L_{0010}\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(2K_{1100}L_{0001}\sigma$$

$$+ K_{0001}L_{1100}t(\sigma^2 + \omega_d{}^2))\sin(\varphi_1 - \varphi_2 - t\omega_d)$$

$$+ 2A_4e^{\sigma t}L_{0010}\omega_d(\sigma^2 + \omega_d{}^2)^2(2A_1{}^2K_{0001}K_{1100}\sigma + A_2{}^2L_{0001}L_{1100}(2\sigma + \sigma^2t$$

$$+ 9t\omega_d{}^2))\sin(\varphi_2 - t\omega_d)$$

$$+ 4A_1A_2A_4e^{\sigma t}K_{0001}L_{0010}L_{1100}\sigma\omega_d(\sigma^2 + \omega_d{}^2)^2\sin(\varphi_1 + \varphi_2 - t\omega_d)$$

$$- 2A_1A_2A_3e^{\sigma t}K_{0010}\omega_d(\sigma^2 + \omega_d{}^2)(-K_{0001}L_{1100}(\sigma^2 + 9\omega_d{}^2)(-2\sigma + \sigma^2t$$

$$- 3t{\omega_d}^2) + 2K_{1100}L_{0001}(\sigma^3 + \sigma{\omega_d}^2 + 2\sigma^2 t{\omega_d}^2 + 18t{\omega_d}^4))\sin(\varphi_1 + t\omega_d)$$

$$+ 2{A_1}^2 A_3 K_{0001}K_{0010}K_{1100}\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4)\sin(2(\varphi_1 + t\omega_d))$$

$$+ 2{A_1}^2 A_3 e^{\sigma t}K_{0001}K_{0010}K_{1100}t\omega_d(\sigma^6 + 7\sigma^4{\omega_d}^2 - 21\sigma^2{\omega_d}^4 - 27{\omega_d}^6)\sin(2\varphi_1$$

$$+ t\omega_d) - 2A_1A_2A_4e^{\sigma t}L_{0010}\omega_d(\sigma^2 + {\omega_d}^2)^2(K_{0001}L_{1100}t(\sigma^2 + 9{\omega_d}^2)$$

$$+ K_{1100}L_{0001}(2\sigma + \sigma^2 t + 9t{\omega_d}^2))\sin(\varphi_1 - \varphi_2 + t\omega_d)$$

$$- 2{A_1}^2 A_4 e^{\sigma t}K_{0001}K_{1100}L_{0010}t\omega_d(\sigma^2 + {\omega_d}^2)^2(\sigma^2 + 9{\omega_d}^2)\sin(2\varphi_1 - \varphi_2$$

$$+ t\omega_d) - 2A_4e^{\sigma t}L_{0010}\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)({A_1}^2K_{0001}K_{1100}(2\sigma + \sigma^2 t$$

$$+ t{\omega_d}^2) + {A_2}^2L_{0001}L_{1100}(2\sigma - \sigma^2 t + 3t{\omega_d}^2))\sin(\varphi_2 + t\omega_d)$$

$$- 2A_1A_2A_4e^{\sigma t}L_{0010}\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)(-K_{1100}L_{0001}t(\sigma^2 - 3{\omega_d}^2)$$

$$+ 2K_{0001}L_{1100}(\sigma - \sigma^2 t + t{\omega_d}^2))\sin(\varphi_1 + \varphi_2 + t\omega_d)$$

$$+ 4{A_1}^2A_4e^{\sigma t}K_{0001}K_{1100}L_{0010}t\omega_d(\sigma^6 + 9\sigma^4{\omega_d}^2 - \sigma^2{\omega_d}^4 - 9{\omega_d}^6)\sin(2\varphi_1$$

$$+ \varphi_2 + t\omega_d) + 2A_1A_2A_3K_{0010}\sigma\omega_d(4K_{0001}L_{1100}{\omega_d}^2(3\sigma^2 - 5{\omega_d}^2)$$

$$+ K_{1100}L_{0001}(\sigma^4 + 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4))\sin(\varphi_1 + 2t\omega_d)$$

$$+ 2A_4L_{0010}\sigma\omega_d(4{A_2}^2L_{0001}L_{1100}{\omega_d}^2(3\sigma^2 - 5{\omega_d}^2) + {A_1}^2K_{0001}K_{1100}(-\sigma^4$$

$$+ 6\sigma^2{\omega_d}^2 + 7{\omega_d}^4))\sin(\varphi_2 + 2t\omega_d)$$

$$+ 2A_1A_2A_4L_{0010}\sigma\omega_d(4K_{0001}L_{1100}{\omega_d}^2(3\sigma^2 - 5{\omega_d}^2) + K_{1100}L_{0001}(\sigma^4$$

$$+ 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4))\sin(\varphi_1 + \varphi_2 + 2t\omega_d) + 2{A_1}^2A_4K_{0001}K_{1100}L_{0010}\sigma\omega_d(\sigma^4$$

$$+ 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4)\sin(2\varphi_1 + \varphi_2 + 2t\omega_d)\}$$

(C. 10)

$$x_{1,2}^{bs1s2,u_2,u_2}(t) = \frac{1}{8\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{2A_3K_{0001}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_1{}^2(1$$

$$+ 2e^{2\sigma t})K_{0001}K_{1100}(\sigma^2+\omega_d{}^2) + 2A_2{}^2L_{0001}L_{1100}(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)$$

$$+ 2A_1A_2(K_{1100}L_{0001}+K_{0001}L_{1100})(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_1))$$

$$+ 2A_1{}^2A_3K_{0001}{}^2K_{1100}\omega_d{}^2(\sigma^2+9\omega_d{}^2)((1-2e^{2\sigma t})\sigma^2+(1$$

$$+ 2e^{2\sigma t})\omega_d{}^2)\cos(2\varphi_1) + 2A_1A_2A_4L_{0001}((1+2e^{2\sigma t})K_{1100}L_{0001}+2(1$$

$$+ e^{2\sigma t})K_{0001}L_{1100})\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1-\varphi_2) + 2A_1{}^2A_4(1$$

$$+ e^{2\sigma t})K_{0001}K_{1100}L_{0001}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(2\varphi_1-\varphi_2)$$

$$+ 2A_4L_{0001}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_1{}^2(1+2e^{2\sigma t})K_{0001}K_{1100}(\sigma^2+\omega_d{}^2)$$

$$+ 2A_2{}^2L_{0001}L_{1100}(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2))\cos(\varphi_2) + 2A_1A_2A_4L_{0001}\omega_d{}^2(\sigma^2$$

$$+ 9\omega_d{}^2)(-2e^{2\sigma t}K_{0001}L_{1100}(\sigma^2-\omega_d{}^2) + K_{1100}L_{0001}((1-2e^{2\sigma t})\sigma^2+(1$$

$$+ 2e^{2\sigma t})\omega_d{}^2))\cos(\varphi_1+\varphi_2) + 2A_1{}^2A_4e^{2\sigma t}K_{0001}K_{1100}L_{0001}\omega_d{}^2(-3\sigma^4$$

$$- 26\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(2\varphi_1+\varphi_2) + 4A_3e^{\sigma t}K_{0001}\omega_d{}^2(\sigma^2$$

$$+ \omega_d{}^2)(A_1{}^2K_{0001}K_{1100}(\sigma^2-15\omega_d{}^2) + 2A_2{}^2L_{0001}L_{1100}(\sigma^3t-12\omega_d{}^2$$

$$+ 9\sigma t\omega_d{}^2))\cos(t\omega_d) - 2A_3K_{0001}\omega_d{}^2(A_1{}^2K_{0001}K_{1100}(5\sigma^4+2\sigma^2\omega_d{}^2-3\omega_d{}^4)$$

$$+ 2A_2{}^2L_{0001}L_{1100}(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(2t\omega_d)$$

$$+ 2A_1A_2A_3K_{0001}K_{1100}L_{0001}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1-2t\omega_d)$$

$$+ 2A_1A_2A_4K_{1100}L_{0001}{}^2\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1-\varphi_2-2t\omega_d)$$

$$- 2A_1A_2A_3e^{\sigma t}K_{0001}(\sigma^2+\omega_d{}^2)(2K_{1100}L_{0001}\omega_d{}^2(\sigma^2+9\omega_d{}^2) + K_{0001}L_{1100}(\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2+21\omega_d{}^4))\cos(\varphi_1-t\omega_d) - A_1{}^2A_3e^{\sigma t}K_{0001}{}^2K_{1100}(\sigma^2+\omega_d{}^2)^2(\sigma^2$$

$$+ 9\omega_d{}^2)\cos(2\varphi_1-t\omega_d) + A_1A_2A_4e^{\sigma t}L_{0001}(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)(-4K_{0001}L_{1100}\omega_d{}^2 + K_{1100}L_{0001}(\sigma^2-3\omega_d{}^2))\cos(\varphi_1-\varphi_2-t\omega_d)$$

$$- A_1{}^2A_4e^{\sigma t}K_{0001}K_{1100}L_{0001}(\sigma^2+\omega_d{}^2)^2(\sigma^2+9\omega_d{}^2)\cos(2\varphi_1-\varphi_2-t\omega_d)$$

$$- A_4 e^{\sigma t} L_{0001}(\sigma^2 + \omega_d{}^2)(3A_1{}^2 K_{0001} K_{1100}(\sigma^4 + 6\sigma^2\omega_d{}^2 + 5\omega_d{}^4)$$

$$+ 2A_2{}^2 L_{0001} L_{1100}(\sigma^4 + 6\sigma^2\omega_d{}^2 + 21\omega_d{}^4))\cos(\varphi_2 - t\omega_d)$$

$$- A_1 A_2 A_4 e^{\sigma t} L_{0001}(\sigma^2 + \omega_d{}^2)^2(2K_{0001} L_{1100}(\sigma^2 + 3\omega_d{}^2) + K_{1100} L_{0001}(\sigma^2$$

$$+ 9\omega_d{}^2))\cos(\varphi_1 + \varphi_2 - t\omega_d) + 2A_1 A_2 A_3 e^{\sigma t} K_{0001}(\sigma^2 + \omega_d{}^2)(K_{0001} L_{1100}(\sigma^2$$

$$+ 9\omega_d{}^2)(\sigma^2 - 3\omega_d{}^2 + 4\sigma t\omega_d{}^2) + 2K_{1100} L_{0001}\omega_d{}^2(\sigma^2 + 2\sigma^3 t - 15\omega_d{}^2$$

$$+ 18\sigma t\omega_d{}^2))\cos(\varphi_1 + t\omega_d) + A_1{}^2 A_3 K_{0001}{}^2 K_{1100}(\sigma^2 + 9\omega_d{}^2)((6\sigma^2\omega_d{}^2$$

$$- 2\omega_d{}^4)\cos(2(\varphi_1 + t\omega_d)) + e^{\sigma t}(\sigma^2 + \omega_d{}^2)(\sigma^2 - 3\omega_d{}^2 + 8\sigma t\omega_d{}^2)\cos(2\varphi_1$$

$$+ t\omega_d)) - A_1 A_2 A_4 e^{\sigma t} L_{0001}(\sigma^2 + \omega_d{}^2)(4K_{0001} L_{1100}\omega_d{}^2(\sigma^2 + 9\omega_d{}^2)$$

$$+ K_{1100} L_{0001}(\sigma^4 + 2\sigma^2\omega_d{}^2 + 33\omega_d{}^4))\cos(\varphi_1 - \varphi_2 + t\omega_d)$$

$$+ A_1{}^2 A_4 e^{\sigma t} K_{0001} K_{1100} L_{0001}(\sigma^6 + 7\sigma^4\omega_d{}^2 - 21\sigma^2\omega_d{}^4 - 27\omega_d{}^6)\cos(2\varphi_1$$

$$- \varphi_2 + t\omega_d) + A_4 e^{\sigma t} L_{0001}(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(A_1{}^2 K_{0001} K_{1100}(3\sigma^2$$

$$- 5\omega_d{}^2) + 2A_2{}^2 L_{0001} L_{1100}(\sigma^2 - 3\omega_d{}^2 + 4\sigma t\omega_d{}^2))\cos(\varphi_2 + t\omega_d)$$

$$+ A_1 A_2 A_4 e^{\sigma t} L_{0001}(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(2K_{0001} L_{1100}(\sigma^2 - \omega_d{}^2 + 4\sigma t\omega_d{}^2)$$

$$+ K_{1100} L_{0001}(\sigma^2 - 3\omega_d{}^2 + 8\sigma t\omega_d{}^2))\cos(\varphi_1 + \varphi_2 + t\omega_d)$$

$$+ 8A_1{}^2 A_4 e^{\sigma t} K_{0001} K_{1100} L_{0001}\sigma t\omega_d{}^2(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\cos(2\varphi_1 + \varphi_2$$

$$+ t\omega_d) + 2A_1 A_2 A_3 K_{0001}\omega_d{}^2(K_{1100} L_{0001}(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4)$$

$$- 2K_{0001} L_{1100}(\sigma^4 - 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))\cos(\varphi_1 + 2t\omega_d)$$

$$- 2A_4 L_{0001}\omega_d{}^2(A_1{}^2 K_{0001} K_{1100}(5\sigma^4 + 2\sigma^2\omega_d{}^2 - 3\omega_d{}^4) + 2A_2{}^2 L_{0001} L_{1100}(\sigma^4$$

$$- 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))\cos(\varphi_2 + 2t\omega_d) + 2A_1 A_2 A_4 L_{0001}\omega_d{}^2(K_{1100} L_{0001}(3\sigma^4$$

$$+ 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4) - 2K_{0001} L_{1100}(\sigma^4 - 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))\cos(\varphi_1 + \varphi_2$$

$$+ 2t\omega_d) + 2A_1{}^2 A_4 K_{0001} K_{1100} L_{0001}\omega_d{}^2(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4)\cos(2\varphi_1$$

$$+ \varphi_2 + 2t\omega_d) + 4A_1 A_2 A_3 K_{0001}\sigma\omega_d(\sigma^2 + 9\omega_d{}^2)(2e^{2\sigma t} K_{0001} L_{1100}\omega_d{}^2$$

$$- K_{1100} L_{0001}(\sigma^2 + (1 - 2e^{2\sigma t})\omega_d{}^2))\sin(\varphi_1) - 2A_1{}^2 A_3 K_{0001}{}^2 K_{1100}\omega_d(\sigma^2$$

$$+ (1 - 4e^{2\sigma t})\omega_d{}^2)(\sigma^3 + 9\sigma\omega_d{}^2)\sin(2\varphi_1) - 2A_1A_2A_4K_{1100}L_{0001}{}^2\sigma\omega_d(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 - \varphi_2) + 2A_1{}^2A_4(-1$$

$$+ e^{2\sigma t})K_{0001}K_{1100}L_{0001}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(2\varphi_1 - \varphi_2)$$

$$+ 2A_4L_{0001}\sigma\omega_d(\sigma^2 + 9\omega_d{}^2)(4A_2{}^2e^{2\sigma t}L_{0001}L_{1100}\omega_d{}^2 + A_1{}^2(-1$$

$$+ 2e^{2\sigma t})K_{0001}K_{1100}(\sigma^2 + \omega_d{}^2))\sin(\varphi_2) - 2A_1A_2A_4L_{0001}\omega_d(\sigma^3$$

$$+ 9\sigma\omega_d{}^2)(-4e^{2\sigma t}K_{0001}L_{1100}\omega_d{}^2 + K_{1100}L_{0001}(\sigma^2 + (1 - 4e^{2\sigma t})\omega_d{}^2))\sin(\varphi_1$$

$$+ \varphi_2) - 2A_1{}^2A_4e^{2\sigma t}K_{0001}K_{1100}L_{0001}\omega_d(\sigma^5 + 6\sigma^3\omega_d{}^2 - 27\sigma\omega_d{}^4)\sin(2\varphi_1$$

$$+ \varphi_2) - 2A_3e^{\sigma t}K_{0001}\omega_d(\sigma^2 + \omega_d{}^2)(4A_2{}^2L_{0001}L_{1100}(\sigma^3 + 5\sigma\omega_d{}^2 + \sigma^2t\omega_d{}^2$$

$$+ 9t\omega_d{}^4) + A_1{}^2K_{0001}K_{1100}(4\sigma^3 + \sigma^4t + 20\sigma\omega_d{}^2 + 10\sigma^2t\omega_d{}^2$$

$$+ 9t\omega_d{}^4))\sin(t\omega_d) + 2A_3K_{0001}\sigma\omega_d(4A_2{}^2L_{0001}L_{1100}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2)$$

$$+ A_1{}^2K_{0001}K_{1100}(-\sigma^4 + 6\sigma^2\omega_d{}^2 + 7\omega_d{}^4))\sin(2t\omega_d)$$

$$+ 2A_1A_2A_3K_{0001}K_{1100}L_{0001}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_1 - 2t\omega_d)$$

$$+ 2A_1A_2A_4K_{1100}L_{0001}{}^2\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_1 - \varphi_2 - 2t\omega_d)$$

$$+ 2A_1A_2A_3e^{\sigma t}K_{0001}\omega_d(\sigma^2 + \omega_d{}^2)(2K_{1100}L_{0001}\sigma(\sigma^2 + 9\omega_d{}^2)$$

$$+ K_{0001}L_{1100}(\sigma^2 + \omega_d{}^2)(2\sigma + \sigma^2t + 9t\omega_d{}^2))\sin(\varphi_1 - t\omega_d)$$

$$+ 2A_1A_2A_4e^{\sigma t}L_{0001}\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(2K_{1100}L_{0001}\sigma$$

$$+ K_{0001}L_{1100}t(\sigma^2 + \omega_d{}^2))\sin(\varphi_1 - \varphi_2 - t\omega_d)$$

$$+ 2A_4e^{\sigma t}L_{0001}\omega_d(\sigma^2 + \omega_d{}^2)^2(2A_1{}^2K_{0001}K_{1100}\sigma + A_2{}^2L_{0001}L_{1100}(2\sigma + \sigma^2t$$

$$+ 9t\omega_d{}^2))\sin(\varphi_2 - t\omega_d)$$

$$+ 4A_1A_2A_4e^{\sigma t}K_{0001}L_{0001}L_{1100}\sigma\omega_d(\sigma^2 + \omega_d{}^2)^2\sin(\varphi_1 + \varphi_2 - t\omega_d)$$

$$- 2A_1A_2A_3e^{\sigma t}K_{0001}\omega_d(\sigma^2 + \omega_d{}^2)(-K_{0001}L_{1100}(\sigma^2 + 9\omega_d{}^2)(-2\sigma + \sigma^2t$$

$$- 3t\omega_d{}^2) + 2K_{1100}L_{0001}(\sigma^3 + \sigma\omega_d{}^2 + 2\sigma^2t\omega_d{}^2 + 18t\omega_d{}^4))\sin(\varphi_1 + t\omega_d)$$

$$+ 2A_1{}^2A_3K_{0001}{}^2K_{1100}\sigma\omega_d(\sigma^4 + 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4)\sin(2(\varphi_1 + t\omega_d))$$

$$
\begin{aligned}
&+ 2A_1{}^2A_3e^{\sigma t}K_{0001}{}^2K_{1100}t\omega_d(\sigma^6 + 7\sigma^4\omega_d{}^2 - 21\sigma^2\omega_d{}^4 - 27\omega_d{}^6)\sin(2\varphi_1 \\
&+ t\omega_d) - 2A_1A_2A_4e^{\sigma t}L_{0001}\omega_d(\sigma^2 + \omega_d{}^2)^2(K_{0001}L_{1100}t(\sigma^2 + 9\omega_d{}^2) \\
&+ K_{1100}L_{0001}(2\sigma + \sigma^2 t + 9t\omega_d{}^2))\sin(\varphi_1 - \varphi_2 + t\omega_d) \\
&- 2A_1{}^2A_4e^{\sigma t}K_{0001}K_{1100}L_{0001}t\omega_d(\sigma^2 + \omega_d{}^2)^2(\sigma^2 + 9\omega_d{}^2)\sin(2\varphi_1 - \varphi_2 \\
&+ t\omega_d) - 2A_4e^{\sigma t}L_{0001}\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(A_1{}^2K_{0001}K_{1100}(2\sigma + \sigma^2 t \\
&+ t\omega_d{}^2) + A_2{}^2L_{0001}L_{1100}(2\sigma - \sigma^2 t + 3t\omega_d{}^2))\sin(\varphi_2 + t\omega_d) \\
&- 2A_1A_2A_4e^{\sigma t}L_{0001}\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(-K_{1100}L_{0001}t(\sigma^2 - 3\omega_d{}^2) \\
&+ 2K_{0001}L_{1100}(\sigma - \sigma^2 t + t\omega_d{}^2))\sin(\varphi_1 + \varphi_2 + t\omega_d) \\
&+ 4A_1{}^2A_4e^{\sigma t}K_{0001}K_{1100}L_{0001}t\omega_d(\sigma^6 + 9\sigma^4\omega_d{}^2 - \sigma^2\omega_d{}^4 - 9\omega_d{}^6)\sin(2\varphi_1 \\
&+ \varphi_2 + t\omega_d) + 2A_1A_2A_3K_{0001}\sigma\omega_d(4K_{0001}L_{1100}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) \\
&+ K_{1100}L_{0001}(\sigma^4 + 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4))\sin(\varphi_1 + 2t\omega_d) \\
&+ 2A_4L_{0001}\sigma\omega_d(4A_2{}^2L_{0001}L_{1100}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + A_1{}^2K_{0001}K_{1100}(-\sigma^4 \\
&+ 6\sigma^2\omega_d{}^2 + 7\omega_d{}^4))\sin(\varphi_2 + 2t\omega_d) \\
&+ 2A_1A_2A_4L_{0001}\sigma\omega_d(4K_{0001}L_{1100}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + K_{1100}L_{0001}(\sigma^4 \\
&+ 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4))\sin(\varphi_1 + \varphi_2 + 2t\omega_d) + 2A_1{}^2A_4K_{0001}K_{1100}L_{0001}\sigma\omega_d(\sigma^4 \\
&+ 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4)\sin(2\varphi_1 + \varphi_2 + 2t\omega_d)\}
\end{aligned}
\tag{C.11}
$$

$$x_{1,2}^{qs2}(t) = x_{1,2}^{qs2,u_1,u_1}(t) \times u_1 u_1 + x_{1,2}^{qs2,u_1,u_2}(t) \times u_1 u_2 + x_{1,2}^{qs2,u_2,u_1}(t) \times u_2 u_1 + x_{1,2}^{qs2,u_2,u_2}(t) \times u_2 u_2 \quad (C.12)$$

$$
\begin{aligned}
\boldsymbol{x_{1,2}^{qs2,u_1,u_1}}(\boldsymbol{t}) = {} & \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{2\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_4{}^2(1 \\
& + e^{2\sigma t})L_{0010}{}^2(\sigma^2+\omega_d{}^2) + A_3{}^2K_{0010}{}^2(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2))(A_2L_{0200} \\
& + A_1K_{0200}\cos(\varphi_1)) + A_1A_4{}^2e^{2\sigma t}K_{0200}L_{0010}{}^2\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2 \\
& + 9\omega_d{}^4)\cos(\varphi_1-2\varphi_2) + 2A_1A_3A_4(1+2e^{2\sigma t})K_{0010}K_{0200}L_{0010}\omega_d{}^2(\sigma^4 \\
& + 10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1-\varphi_2) + 4A_2A_3A_4K_{0010}L_{0010}L_{0200}\omega_d{}^2(\sigma^2 \\
& + 9\omega_d{}^2)(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_2) - 2A_2A_4{}^2e^{2\sigma t}L_{0010}{}^2L_{0200}\omega_d{}^2(\sigma^4 \\
& + 8\sigma^2\omega_d{}^2-9\omega_d{}^4)\cos(2\varphi_2) + 2A_1A_3A_4K_{0010}K_{0200}L_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2)((1 \\
& - 2e^{2\sigma t})\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_1+\varphi_2) \\
& + A_1A_4{}^2e^{2\sigma t}K_{0200}L_{0010}{}^2\omega_d{}^2(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1+2\varphi_2) \\
& + 4A_2e^{\sigma t}L_{0200}\omega_d{}^2(\sigma^2+\omega_d{}^2)(-A_4{}^2L_{0010}{}^2(\sigma^2+9\omega_d{}^2)+A_3{}^2K_{0010}{}^2(\sigma^3t \\
& - 12\omega_d{}^2+9\sigma t\omega_d{}^2))\cos(t\omega_d) - 2A_2A_3{}^2K_{0010}{}^2L_{0200}\omega_d{}^2(\sigma^4-12\sigma^2\omega_d{}^2 \\
& + 3\omega_d{}^4)\cos(2t\omega_d) + A_1A_3{}^2K_{0010}{}^2K_{0200}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1 \\
& - 2t\omega_d) + 2A_1A_3A_4K_{0010}K_{0200}L_{0010}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1 \\
& - \varphi_2-2t\omega_d) - A_1e^{\sigma t}K_{0200}(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)(2A_3{}^2K_{0010}{}^2\omega_d{}^2 \\
& + A_4{}^2L_{0010}{}^2(\sigma^2+\omega_d{}^2))\cos(\varphi_1-t\omega_d) + A_1A_4{}^2e^{\sigma t}K_{0200}L_{0010}{}^2(\sigma^6+9\sigma^4\omega_d{}^2 \\
& - \sigma^2\omega_d{}^4-9\omega_d{}^6)\cos(\varphi_1-2\varphi_2-t\omega_d) + A_1A_3A_4e^{\sigma t}K_{0010}K_{0200}L_{0010}(\sigma^6 \\
& + 7\sigma^4\omega_d{}^2-21\sigma^2\omega_d{}^4-27\omega_d{}^6)\cos(\varphi_1-\varphi_2-t\omega_d) \\
& - 2A_2A_3A_4e^{\sigma t}K_{0010}L_{0010}L_{0200}(\sigma^6+7\sigma^4\omega_d{}^2+27\sigma^2\omega_d{}^4+21\omega_d{}^6)\cos(\varphi_2 \\
& - t\omega_d) - A_1A_3A_4e^{\sigma t}K_{0010}K_{0200}L_{0010}(\sigma^2+\omega_d{}^2)^2(\sigma^2+9\omega_d{}^2)\cos(\varphi_1+\varphi_2 \\
& - t\omega_d) - A_2A_4{}^2e^{\sigma t}L_{0010}{}^2L_{0200}(\sigma^2+\omega_d{}^2)^2(\sigma^2+3\omega_d{}^2)\cos(2\varphi_2-t\omega_d) \\
& + A_1e^{\sigma t}K_{0200}(\sigma^2+\omega_d{}^2)(2A_3{}^2K_{0010}{}^2\omega_d{}^2(\sigma^2+2\sigma^3t-15\omega_d{}^2+18\sigma t\omega_d{}^2) \\
& + A_4{}^2L_{0010}{}^2(\sigma^4+6\sigma^2\omega_d{}^2-27\omega_d{}^4))\cos(\varphi_1+t\omega_d)
\end{aligned}
$$

$$- A_1 A_4{}^2 e^{\sigma t} K_{0200} L_{0010}{}^2 (\sigma^2 + \omega_d{}^2)^2 (\sigma^2 + 3\omega_d{}^2) \cos(\varphi_1 - 2\varphi_2 + t\omega_d)$$

$$- A_1 A_3 A_4 e^{\sigma t} K_{0010} K_{0200} L_{0010} (\sigma^6 + 3\sigma^4 \omega_d{}^2 + 35\sigma^2 \omega_d{}^4 + 33\omega_d{}^6) \cos(\varphi_1$$

$$- \varphi_2 + t\omega_d) + 2A_2 A_3 A_4 e^{\sigma t} K_{0010} L_{0010} L_{0200} (\sigma^2 - 3\omega_d{}^2 + 4\sigma t \omega_d{}^2)(\sigma^4$$

$$+ 10\sigma^2 \omega_d{}^2 + 9\omega_d{}^4) \cos(\varphi_2 + t\omega_d) - 2A_2 A_4{}^2 L_{0010}{}^2 L_{0200} \omega_d{}^2 (\sigma^4 - 12\sigma^2 \omega_d{}^2$$

$$+ 3\omega_d{}^4) \cos(2(\varphi_2 + t\omega_d)) + A_1 A_3 A_4 e^{\sigma t} K_{0010} K_{0200} L_{0010} (\sigma^2 - 3\omega_d{}^2$$

$$+ 8\sigma t \omega_d{}^2)(\sigma^4 + 10\sigma^2 \omega_d{}^2 + 9\omega_d{}^4) \cos(\varphi_1 + \varphi_2 + t\omega_d)$$

$$+ A_2 A_4{}^2 e^{\sigma t} L_{0010}{}^2 L_{0200} (\sigma^2 - \omega_d{}^2 + 4\sigma t \omega_d{}^2)(\sigma^4 + 10\sigma^2 \omega_d{}^2$$

$$+ 9\omega_d{}^4) \cos(2\varphi_2 + t\omega_d) + 4A_1 A_4{}^2 e^{\sigma t} K_{0200} L_{0010}{}^2 \sigma t \omega_d{}^2 (\sigma^4 + 10\sigma^2 \omega_d{}^2$$

$$+ 9\omega_d{}^4) \cos(\varphi_1 + 2\varphi_2 + t\omega_d) + A_1 A_3{}^2 K_{0010}{}^2 K_{0200} \omega_d{}^2 (3\sigma^4 + 26\sigma^2 \omega_d{}^2$$

$$- 9\omega_d{}^4) \cos(\varphi_1 + 2t\omega_d) - 4A_2 A_3 A_4 K_{0010} L_{0010} L_{0200} \omega_d{}^2 (\sigma^4 - 12\sigma^2 \omega_d{}^2$$

$$+ 3\omega_d{}^4) \cos(\varphi_2 + 2t\omega_d) + A_1 A_4{}^2 K_{0200} L_{0010}{}^2 \omega_d{}^2 (-5\sigma^4 - 2\sigma^2 \omega_d{}^2$$

$$+ 3\omega_d{}^4) \cos(\varphi_1 - 2(\varphi_2 + t\omega_d)) - A_1 A_4 K_{0200} L_{0010} \omega_d{}^2 (-3\sigma^4 - 26\sigma^2 \omega_d{}^2$$

$$+ 9\omega_d{}^4)(2A_3 K_{0010} \cos(\varphi_1 + \varphi_2 + 2t\omega_d) + A_4 L_{0010} \cos(\varphi_1 + 2(\varphi_2 + t\omega_d)))$$

$$- 2A_1 K_{0200} \omega_d (\sigma^3 + 9\sigma \omega_d{}^2)(-A_4{}^2 (-1 + e^{2\sigma t}) L_{0010}{}^2 (\sigma^2 + \omega_d{}^2)$$

$$+ A_3{}^2 K_{0010}{}^2 (\sigma^2 + (1 - 2e^{2\sigma t}) \omega_d{}^2)) \sin(\varphi_1) - A_1 A_4{}^2 e^{2\sigma t} K_{0200} L_{0010}{}^2 \sigma \omega_d (\sigma^4$$

$$+ 10\sigma^2 \omega_d{}^2 + 9\omega_d{}^4) \sin(\varphi_1 - 2\varphi_2) - 2A_1 A_3 A_4 K_{0010} K_{0200} L_{0010} \sigma \omega_d (\sigma^4$$

$$+ 10\sigma^2 \omega_d{}^2 + 9\omega_d{}^4) \sin(\varphi_1 - \varphi_2) + 8A_2 A_3 A_4 e^{2\sigma t} K_{0010} L_{0010} L_{0200} \sigma \omega_d{}^3 (\sigma^2$$

$$+ 9\omega_d{}^2) \sin(\varphi_2) + 4A_2 A_4{}^2 e^{2\sigma t} L_{0010}{}^2 L_{0200} \sigma \omega_d{}^3 (\sigma^2 + 9\omega_d{}^2) \sin(2\varphi_2)$$

$$- 2A_1 A_3 A_4 K_{0010} K_{0200} L_{0010} \omega_d (\sigma^2 + (1 - 4e^{2\sigma t}) \omega_d{}^2)(\sigma^3 + 9\sigma \omega_d{}^2) \sin(\varphi_1$$

$$+ \varphi_2) - A_1 A_4{}^2 e^{2\sigma t} K_{0200} L_{0010}{}^2 \omega_d (\sigma^5 + 6\sigma^3 \omega_d{}^2 - 27\sigma \omega_d{}^4) \sin(\varphi_1 + 2\varphi_2)$$

$$- 2A_2 e^{\sigma t} L_{0200} \omega_d (\sigma^2 + \omega_d{}^2)(A_4{}^2 L_{0010}{}^2 t (\sigma^4 + 10\sigma^2 \omega_d{}^2 + 9\omega_d{}^4)$$

$$+ 2A_3{}^2 K_{0010}{}^2 (\sigma^3 + 5\sigma \omega_d{}^2 + \sigma^2 t \omega_d{}^2 + 9t\omega_d{}^4)) \sin(t\omega_d)$$

$$+ 4A_2A_3^{\,2}K_{0010}^{\;\;2}L_{0200}\sigma\omega_d^{\;3}(3\sigma^2 - 5\omega_d^{\;2})\sin(2t\omega_d)$$

$$+ A_1A_3^{\,2}K_{0010}^{\;\;2}K_{0200}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d^{\;2} - 7\omega_d^{\;4})\sin(\varphi_1 - 2t\omega_d)$$

$$+ 2A_1A_3A_4K_{0010}K_{0200}L_{0010}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d^{\;2} - 7\omega_d^{\;4})\sin(\varphi_1 - \varphi_2 - 2t\omega_d)$$

$$+ 2A_1A_3^{\,2}e^{\sigma t}K_{0010}^{\;\;2}K_{0200}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d^{\;2} + 9\omega_d^{\;4})\sin(\varphi_1 - t\omega_d)$$

$$+ 2A_1A_4^{\,2}e^{\sigma t}K_{0200}L_{0010}^{\;\;2}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d^{\;2} + 9\omega_d^{\;4})\sin(\varphi_1 - 2\varphi_2 - t\omega_d)$$

$$+ 4A_1A_3A_4e^{\sigma t}K_{0010}K_{0200}L_{0010}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d^{\;2} + 9\omega_d^{\;4})\sin(\varphi_1 - \varphi_2$$

$$- t\omega_d) + 2A_2A_3A_4e^{\sigma t}K_{0010}L_{0010}L_{0200}\omega_d(\sigma^2 + \omega_d^{\;2})^2(2\sigma + \sigma^2 t$$

$$+ 9t\omega_d^{\;2})\sin(\varphi_2 - t\omega_d) + 2A_2A_4^{\,2}e^{\sigma t}L_{0010}^{\;\;2}L_{0200}\sigma\omega_d(\sigma^2 + \omega_d^{\;2})^2\sin(2\varphi_2$$

$$- t\omega_d) - 2A_1e^{\sigma t}K_{0200}\omega_d(\sigma^2 + \omega_d^{\;2})(A_4^{\,2}L_{0010}^{\;\;2}t(\sigma^4 + 10\sigma^2\omega_d^{\;2} + 9\omega_d^{\;4})$$

$$+ A_3^{\,2}K_{0010}^{\;\;2}(\sigma^3 + \sigma\omega_d^{\;2} + 2\sigma^2 t\omega_d^{\;2} + 18t\omega_d^{\;4}))\sin(\varphi_1 + t\omega_d)$$

$$- 2A_1A_4^{\,2}e^{\sigma t}K_{0200}L_{0010}^{\;\;2}\sigma\omega_d(\sigma^2 + \omega_d^{\;2})^2\sin(\varphi_1 - 2\varphi_2 + t\omega_d)$$

$$- 2A_1A_3A_4e^{\sigma t}K_{0010}K_{0200}L_{0010}\omega_d(\sigma^2 + \omega_d^{\;2})^2(2\sigma + \sigma^2 t + 9t\omega_d^{\;2})\sin(\varphi_1$$

$$- \varphi_2 + t\omega_d) + 2A_2A_3A_4e^{\sigma t}K_{0010}L_{0010}L_{0200}\omega_d(-2\sigma + \sigma^2 t - 3t\omega_d^{\;2})(\sigma^4$$

$$+ 10\sigma^2\omega_d^{\;2} + 9\omega_d^{\;4})\sin(\varphi_2 + t\omega_d) + 4A_2A_4^{\,2}L_{0010}^{\;\;2}L_{0200}\sigma\omega_d^{\;3}(3\sigma^2$$

$$- 5\omega_d^{\;2})\sin(2(\varphi_2 + t\omega_d)) + 2A_1A_3A_4e^{\sigma t}K_{0010}K_{0200}L_{0010}t\omega_d(\sigma^6 + 7\sigma^4\omega_d^{\;2}$$

$$- 21\sigma^2\omega_d^{\;4} - 27\omega_d^{\;6})\sin(\varphi_1 + \varphi_2 + t\omega_d) - 2A_2A_4^{\,2}e^{\sigma t}L_{0010}^{\;\;2}L_{0200}\omega_d(\sigma$$

$$- \sigma^2 t + t\omega_d^{\;2})(\sigma^4 + 10\sigma^2\omega_d^{\;2} + 9\omega_d^{\;4})\sin(2\varphi_2 + t\omega_d)$$

$$+ 2A_1A_4^{\,2}e^{\sigma t}K_{0200}L_{0010}^{\;\;2}t\omega_d(\sigma^6 + 9\sigma^4\omega_d^{\;2} - \sigma^2\omega_d^{\;4} - 9\omega_d^{\;6})\sin(\varphi_1 + 2\varphi_2$$

$$+ t\omega_d) + A_1A_3^{\,2}K_{0010}^{\;\;2}K_{0200}\sigma\omega_d(\sigma^4 + 6\sigma^2\omega_d^{\;2} - 27\omega_d^{\;4})\sin(\varphi_1 + 2t\omega_d)$$

$$+ 8A_2A_3A_4K_{0010}L_{0010}L_{0200}\sigma\omega_d^{\;3}(3\sigma^2 - 5\omega_d^{\;2})\sin(\varphi_2 + 2t\omega_d)$$

$$+ 2A_1A_3A_4K_{0010}K_{0200}L_{0010}\sigma\omega_d(\sigma^4 + 6\sigma^2\omega_d^{\;2} - 27\omega_d^{\;4})\sin(\varphi_1 + \varphi_2 + 2t\omega_d)$$

$$+ A_1A_4^{\,2}K_{0200}L_{0010}^{\;\;2}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d^{\;2} - 7\omega_d^{\;4})\sin(\varphi_1 - 2(\varphi_2 + t\omega_d))$$

$$+ A_1 {A_4}^2 K_{0200} {L_{0010}}^2 \sigma \omega_d (\sigma^4 + 6\sigma^2 {\omega_d}^2 - 27{\omega_d}^4)\sin(\varphi_1 + 2(\varphi_2 + t\omega_d))\}$$

(C. 13)

$$x_{1,2}^{qs2,u_1,u_2}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{2\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_4{}^2(1$$

$$+ e^{2\sigma t})L_{0001}L_{0010}(\sigma^2+\omega_d{}^2) + A_3{}^2K_{0001}K_{0010}(\sigma^2+(1$$

$$+ 2e^{2\sigma t})\omega_d{}^2))(A_2L_{0200} + A_1K_{0200}\cos(\varphi_1))$$

$$+ A_1A_4{}^2e^{2\sigma t}K_{0200}L_{0001}L_{0010}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1-2\varphi_2)$$

$$+ A_1A_3A_4(1+2e^{2\sigma t})K_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(\varphi_1-\varphi_2) + 2A_2A_3A_4(K_{0010}L_{0001}+K_{0001}L_{0010})L_{0200}\omega_d{}^2(\sigma^2$$

$$+ 9\omega_d{}^2)(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_2) - 2A_2A_4{}^2e^{2\sigma t}L_{0001}L_{0010}L_{0200}\omega_d{}^2(\sigma^4$$

$$+ 8\sigma^2\omega_d{}^2 - 9\omega_d{}^4)\cos(2\varphi_2) + A_1A_3A_4K_{0200}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})\omega_d{}^2(\sigma^2+9\omega_d{}^2)((1-2e^{2\sigma t})\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_1$$

$$+ \varphi_2) + A_1A_4{}^2e^{2\sigma t}K_{0200}L_{0001}L_{0010}\omega_d{}^2(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1$$

$$+ 2\varphi_2) + 4A_2e^{\sigma t}L_{0200}\omega_d{}^2(\sigma^2+\omega_d{}^2)(-A_4{}^2L_{0001}L_{0010}(\sigma^2+9\omega_d{}^2)$$

$$+ A_3{}^2K_{0001}K_{0010}(\sigma^3t-12\omega_d{}^2+9\sigma t\omega_d{}^2))\cos(t\omega_d)$$

$$- 2A_2A_3{}^2K_{0001}K_{0010}L_{0200}\omega_d{}^2(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(2t\omega_d)$$

$$+ A_1A_3{}^2K_{0001}K_{0010}K_{0200}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1-2t\omega_d)$$

$$+ A_1A_3A_4K_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(\varphi_1-\varphi_2-2t\omega_d) - A_1e^{\sigma t}K_{0200}(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)(2A_3{}^2K_{0001}K_{0010}\omega_d{}^2 + A_4{}^2L_{0001}L_{0010}(\sigma^2+\omega_d{}^2))\cos(\varphi_1-t\omega_d)$$

$$+ A_1A_4{}^2e^{\sigma t}K_{0200}L_{0001}L_{0010}(\sigma^6+9\sigma^4\omega_d{}^2-\sigma^2\omega_d{}^4-9\omega_d{}^6)\cos(\varphi_1-2\varphi_2$$

$$- t\omega_d) + \frac{1}{2}A_1A_3A_4e^{\sigma t}K_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^6+7\sigma^4\omega_d{}^2$$

$$- 21\sigma^2\omega_d{}^4 - 27\omega_d{}^6)\cos(\varphi_1-\varphi_2-t\omega_d) - A_2A_3A_4e^{\sigma t}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200}(\sigma^6+7\sigma^4\omega_d{}^2+27\sigma^2\omega_d{}^4+21\omega_d{}^6)\cos(\varphi_2-t\omega_d)$$

$$-\frac{1}{2}A_1A_3A_4e^{\sigma t}K_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^2+\omega_d{}^2)^2(\sigma^2$$

$$+9\omega_d{}^2)\cos(\varphi_1+\varphi_2-t\omega_d)-A_2A_4{}^2e^{\sigma t}L_{0001}L_{0010}L_{0200}(\sigma^2+\omega_d{}^2)^2(\sigma^2$$

$$+3\omega_d{}^2)\cos(2\varphi_2-t\omega_d)+A_1e^{\sigma t}K_{0200}(\sigma^2+\omega_d{}^2)(2A_3{}^2K_{0001}K_{0010}\omega_d{}^2(\sigma^2$$

$$+2\sigma^3t-15\omega_d{}^2+18\sigma t\omega_d{}^2)+A_4{}^2L_{0001}L_{0010}(\sigma^4+6\sigma^2\omega_d{}^2$$

$$-27\omega_d{}^4))\cos(\varphi_1+t\omega_d)-A_1A_4{}^2e^{\sigma t}K_{0200}L_{0001}L_{0010}(\sigma^2+\omega_d{}^2)^2(\sigma^2$$

$$+3\omega_d{}^2)\cos(\varphi_1-2\varphi_2+t\omega_d)-\frac{1}{2}A_1A_3A_4e^{\sigma t}K_{0200}(K_{0010}L_{0001}$$

$$+K_{0001}L_{0010})(\sigma^6+3\sigma^4\omega_d{}^2+35\sigma^2\omega_d{}^4+33\omega_d{}^6)\cos(\varphi_1-\varphi_2+t\omega_d)$$

$$+A_2A_3A_4e^{\sigma t}(K_{0010}L_{0001}+K_{0001}L_{0010})L_{0200}(\sigma^2-3\omega_d{}^2+4\sigma t\omega_d{}^2)(\sigma^4$$

$$+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_2+t\omega_d)-2A_2A_4{}^2L_{0001}L_{0010}L_{0200}\omega_d{}^2(\sigma^4$$

$$-12\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(2(\varphi_2+t\omega_d))+\frac{1}{2}A_1A_3A_4e^{\sigma t}K_{0200}(K_{0010}L_{0001}$$

$$+K_{0001}L_{0010})(\sigma^2-3\omega_d{}^2+8\sigma t\omega_d{}^2)(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1+\varphi_2$$

$$+t\omega_d)+A_2A_4{}^2e^{\sigma t}L_{0001}L_{0010}L_{0200}(\sigma^2-\omega_d{}^2+4\sigma t\omega_d{}^2)(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+9\omega_d{}^4)\cos(2\varphi_2+t\omega_d)+4A_1A_4{}^2e^{\sigma t}K_{0200}L_{0001}L_{0010}\sigma t\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+9\omega_d{}^4)\cos(\varphi_1+2\varphi_2+t\omega_d)+A_1A_3{}^2K_{0001}K_{0010}K_{0200}\omega_d{}^2(3\sigma^4+26\sigma^2\omega_d{}^2$$

$$-9\omega_d{}^4)\cos(\varphi_1+2t\omega_d)-2A_2A_3A_4(K_{0010}L_{0001}+K_{0001}L_{0010})L_{0200}\omega_d{}^2(\sigma^4$$

$$-12\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_2+2t\omega_d)-A_1A_3A_4K_{0200}(K_{0010}L_{0001}$$

$$+K_{0001}L_{0010})\omega_d{}^2(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1+\varphi_2+2t\omega_d)$$

$$+A_1A_4{}^2K_{0200}L_{0001}L_{0010}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1-2(\varphi_2$$

$$+t\omega_d))+A_1A_4{}^2K_{0200}L_{0001}L_{0010}\omega_d{}^2(3\sigma^4+26\sigma^2\omega_d{}^2-9\omega_d{}^4)\cos(\varphi_1+2(\varphi_2$$

$$+t\omega_d))-2A_1K_{0200}\sigma\omega_d(\sigma^2+9\omega_d{}^2)(-A_4{}^2(-1+e^{2\sigma t})L_{0001}L_{0010}(\sigma^2+\omega_d{}^2)$$

$$+A_3{}^2K_{0001}K_{0010}(\sigma^2+(1-2e^{2\sigma t})\omega_d{}^2))\sin(\varphi_1)$$

$$- A_1A_4{}^2e^{2\sigma t}K_{0200}L_{0001}L_{0010}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 - 2\varphi_2)$$

$$- A_1A_3A_4K_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1$$

$$- \varphi_2) + 4A_2A_3A_4e^{2\sigma t}(K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}\sigma\omega_d{}^3(\sigma^2$$

$$+ 9\omega_d{}^2)\sin(\varphi_2) + 4A_2A_4{}^2e^{2\sigma t}L_{0001}L_{0010}L_{0200}\sigma\omega_d{}^3(\sigma^2 + 9\omega_d{}^2)\sin(2\varphi_2)$$

$$- A_1A_3A_4K_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma\omega_d(\sigma^2 + 9\omega_d{}^2)(\sigma^2 + (1$$

$$- 4e^{2\sigma t})\omega_d{}^2)\sin(\varphi_1 + \varphi_2) - A_1A_4{}^2e^{2\sigma t}K_{0200}L_{0001}L_{0010}\omega_d(\sigma^5 + 6\sigma^3\omega_d{}^2$$

$$- 27\sigma\omega_d{}^4)\sin(\varphi_1 + 2\varphi_2) - 2A_2e^{\sigma t}L_{0200}\omega_d(\sigma^2 + \omega_d{}^2)(A_4{}^2L_{0001}L_{0010}t(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4) + 2A_3{}^2K_{0001}K_{0010}(\sigma^3 + 5\sigma\omega_d{}^2 + \sigma^2t\omega_d{}^2$$

$$+ 9t\omega_d{}^4))\sin(t\omega_d) + 4A_2A_3{}^2K_{0001}K_{0010}L_{0200}\sigma\omega_d{}^3(3\sigma^2 - 5\omega_d{}^2)\sin(2t\omega_d)$$

$$+ A_1A_3{}^2K_{0001}K_{0010}K_{0200}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_1 - 2t\omega_d)$$

$$+ A_1A_3A_4K_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_1$$

$$- \varphi_2 - 2t\omega_d) + 2A_1A_3{}^2e^{\sigma t}K_{0001}K_{0010}K_{0200}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\sin(\varphi_1 - t\omega_d) + 2A_1A_4{}^2e^{\sigma t}K_{0200}L_{0001}L_{0010}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\sin(\varphi_1 - 2\varphi_2 - t\omega_d) + 2A_1A_3A_4e^{\sigma t}K_{0200}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 - \varphi_2 - t\omega_d)$$

$$+ A_2A_3A_4e^{\sigma t}(K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}\omega_d(\sigma^2 + \omega_d{}^2)^2(2\sigma + \sigma^2t$$

$$+ 9t\omega_d{}^2)\sin(\varphi_2 - t\omega_d) + 2A_2A_4{}^2e^{\sigma t}L_{0001}L_{0010}L_{0200}\sigma\omega_d(\sigma^2 + \omega_d{}^2)^2\sin(2\varphi_2$$

$$- t\omega_d) - 2A_1e^{\sigma t}K_{0200}\omega_d(\sigma^2 + \omega_d{}^2)(A_4{}^2L_{0001}L_{0010}t(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)$$

$$+ A_3{}^2K_{0001}K_{0010}(\sigma^3 + \sigma\omega_d{}^2 + 2\sigma^2t\omega_d{}^2 + 18t\omega_d{}^4))\sin(\varphi_1 + t\omega_d)$$

$$- 2A_1A_4{}^2e^{\sigma t}K_{0200}L_{0001}L_{0010}\sigma\omega_d(\sigma^2 + \omega_d{}^2)^2\sin(\varphi_1 - 2\varphi_2 + t\omega_d)$$

$$- A_1A_3A_4e^{\sigma t}K_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})\omega_d(\sigma^2 + \omega_d{}^2)^2(2\sigma + \sigma^2t$$

$$+ 9t\omega_d{}^2)\sin(\varphi_1 - \varphi_2 + t\omega_d) - A_2A_3A_4e^{\sigma t}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200}\omega_d(2\sigma - \sigma^2 t + 3t{\omega_d}^2)(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_2$$

$$+ t\omega_d) + 4A_2{A_4}^2 L_{0001}L_{0010}L_{0200}\sigma{\omega_d}^3(3\sigma^2 - 5{\omega_d}^2)\sin(2(\varphi_2 + t\omega_d))$$

$$+ A_1A_3A_4e^{\sigma t}K_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})t\omega_d(\sigma^6 + 7\sigma^4{\omega_d}^2 - 21\sigma^2{\omega_d}^4$$

$$- 27{\omega_d}^6)\sin(\varphi_1 + \varphi_2 + t\omega_d) - 2A_2{A_4}^2 e^{\sigma t}L_{0001}L_{0010}L_{0200}\omega_d(\sigma - \sigma^2 t$$

$$+ t{\omega_d}^2)(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(2\varphi_2 + t\omega_d)$$

$$+ 2A_1{A_4}^2 e^{\sigma t}K_{0200}L_{0001}L_{0010}t\omega_d(\sigma^6 + 9\sigma^4{\omega_d}^2 - \sigma^2{\omega_d}^4 - 9{\omega_d}^6)\sin(\varphi_1$$

$$+ 2\varphi_2 + t\omega_d) + A_1{A_3}^2 K_{0001}K_{0010}K_{0200}\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4)\sin(\varphi_1$$

$$+ 2t\omega_d) + 4A_2A_3A_4(K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}\sigma{\omega_d}^3(3\sigma^2 - 5{\omega_d}^2)\sin(\varphi_2$$

$$+ 2t\omega_d) + A_1A_3A_4K_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2$$

$$- 27{\omega_d}^4)\sin(\varphi_1 + \varphi_2 + 2t\omega_d) + A_1{A_4}^2 K_{0200}L_{0001}L_{0010}\sigma\omega_d(\sigma^4 - 6\sigma^2{\omega_d}^2$$

$$- 7{\omega_d}^4)\sin(\varphi_1 - 2(\varphi_2 + t\omega_d)) + A_1{A_4}^2 K_{0200}L_{0001}L_{0010}\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2$$

$$- 27{\omega_d}^4)\sin(\varphi_1 + 2(\varphi_2 + t\omega_d))\}$$

(C. 14)

$$x_{1,2}^{qs2,u_2,u_1}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{2\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_4{}^2(1$$

$$+ e^{2\sigma t})L_{0001}L_{0010}(\sigma^2+\omega_d{}^2) + A_3{}^2K_{0001}K_{0010}(\sigma^2+(1$$

$$+ 2e^{2\sigma t})\omega_d{}^2))(A_2L_{0200}+A_1K_{0200}\cos(\varphi_1))$$

$$+ A_1A_4{}^2e^{2\sigma t}K_{0200}L_{0001}L_{0010}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1-2\varphi_2)$$

$$+ A_1A_3A_4(1+2e^{2\sigma t})K_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(\varphi_1-\varphi_2) + 2A_2A_3A_4(K_{0010}L_{0001}+K_{0001}L_{0010})L_{0200}\omega_d{}^2(\sigma^2$$

$$+ 9\omega_d{}^2)(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_2) - 2A_2A_4{}^2e^{2\sigma t}L_{0001}L_{0010}L_{0200}\omega_d{}^2(\sigma^4$$

$$+ 8\sigma^2\omega_d{}^2 - 9\omega_d{}^4)\cos(2\varphi_2) + A_1A_3A_4K_{0200}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})\omega_d{}^2(\sigma^2+9\omega_d{}^2)((1-2e^{2\sigma t})\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_1$$

$$+ \varphi_2) + A_1A_4{}^2e^{2\sigma t}K_{0200}L_{0001}L_{0010}\omega_d{}^2(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1$$

$$+ 2\varphi_2) + 4A_2e^{\sigma t}L_{0200}\omega_d{}^2(\sigma^2+\omega_d{}^2)(-A_4{}^2L_{0001}L_{0010}(\sigma^2+9\omega_d{}^2)$$

$$+ A_3{}^2K_{0001}K_{0010}(\sigma^3t-12\omega_d{}^2+9\sigma t\omega_d{}^2))\cos(t\omega_d)$$

$$- 2A_2A_3{}^2K_{0001}K_{0010}L_{0200}\omega_d{}^2(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(2t\omega_d)$$

$$+ A_1A_3{}^2K_{0001}K_{0010}K_{0200}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1-2t\omega_d)$$

$$+ A_1A_3A_4K_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(\varphi_1-\varphi_2-2t\omega_d) - A_1e^{\sigma t}K_{0200}(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)(2A_3{}^2K_{0001}K_{0010}\omega_d{}^2 + A_4{}^2L_{0001}L_{0010}(\sigma^2+\omega_d{}^2))\cos(\varphi_1-t\omega_d)$$

$$+ A_1A_4{}^2e^{\sigma t}K_{0200}L_{0001}L_{0010}(\sigma^6+9\sigma^4\omega_d{}^2-\sigma^2\omega_d{}^4-9\omega_d{}^6)\cos(\varphi_1-2\varphi_2$$

$$- t\omega_d) + \frac{1}{2}A_1A_3A_4e^{\sigma t}K_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^6+7\sigma^4\omega_d{}^2$$

$$- 21\sigma^2\omega_d{}^4 - 27\omega_d{}^6)\cos(\varphi_1-\varphi_2-t\omega_d) - A_2A_3A_4e^{\sigma t}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200}(\sigma^6+7\sigma^4\omega_d{}^2+27\sigma^2\omega_d{}^4+21\omega_d{}^6)\cos(\varphi_2-t\omega_d)$$

$$-\frac{1}{2}A_1A_3A_4e^{\sigma t}K_{0200}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^2+\omega_d{}^2)^2(\sigma^2$$

$$+9\omega_d{}^2)\cos(\varphi_1+\varphi_2-t\omega_d)-A_2A_4{}^2e^{\sigma t}L_{0001}L_{0010}L_{0200}(\sigma^2+\omega_d{}^2)^2(\sigma^2$$

$$+3\omega_d{}^2)\cos(2\varphi_2-t\omega_d)+A_1e^{\sigma t}K_{0200}(\sigma^2+\omega_d{}^2)(2A_3{}^2K_{0001}K_{0010}\omega_d{}^2(\sigma^2$$

$$+2\sigma^3t-15\omega_d{}^2+18\sigma t\omega_d{}^2)+A_4{}^2L_{0001}L_{0010}(\sigma^4+6\sigma^2\omega_d{}^2$$

$$-27\omega_d{}^4))\cos(\varphi_1+t\omega_d)-A_1A_4{}^2e^{\sigma t}K_{0200}L_{0001}L_{0010}(\sigma^2+\omega_d{}^2)^2(\sigma^2$$

$$+3\omega_d{}^2)\cos(\varphi_1-2\varphi_2+t\omega_d)-\frac{1}{2}A_1A_3A_4e^{\sigma t}K_{0200}(K_{0010}L_{0001}$$

$$+K_{0001}L_{0010})(\sigma^6+3\sigma^4\omega_d{}^2+35\sigma^2\omega_d{}^4+33\omega_d{}^6)\cos(\varphi_1-\varphi_2+t\omega_d)$$

$$+A_2A_3A_4e^{\sigma t}(K_{0010}L_{0001}+K_{0001}L_{0010})L_{0200}(\sigma^2-3\omega_d{}^2+4\sigma t\omega_d{}^2)(\sigma^4$$

$$+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_2+t\omega_d)-2A_2A_4{}^2L_{0001}L_{0010}L_{0200}\omega_d{}^2(\sigma^4$$

$$-12\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(2(\varphi_2+t\omega_d))+\frac{1}{2}A_1A_3A_4e^{\sigma t}K_{0200}(K_{0010}L_{0001}$$

$$+K_{0001}L_{0010})(\sigma^2-3\omega_d{}^2+8\sigma t\omega_d{}^2)(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1+\varphi_2$$

$$+t\omega_d)+A_2A_4{}^2e^{\sigma t}L_{0001}L_{0010}L_{0200}(\sigma^2-\omega_d{}^2+4\sigma t\omega_d{}^2)(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+9\omega_d{}^4)\cos(2\varphi_2+t\omega_d)+4A_1A_4{}^2e^{\sigma t}K_{0200}L_{0001}L_{0010}\sigma t\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+9\omega_d{}^4)\cos(\varphi_1+2\varphi_2+t\omega_d)+A_1A_3{}^2K_{0001}K_{0010}K_{0200}\omega_d{}^2(3\sigma^4+26\sigma^2\omega_d{}^2$$

$$-9\omega_d{}^4)\cos(\varphi_1+2t\omega_d)-2A_2A_3A_4(K_{0010}L_{0001}+K_{0001}L_{0010})L_{0200}\omega_d{}^2(\sigma^4$$

$$-12\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_2+2t\omega_d)-A_1A_3A_4K_{0200}(K_{0010}L_{0001}$$

$$+K_{0001}L_{0010})\omega_d{}^2(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1+\varphi_2+2t\omega_d)$$

$$+A_1A_4{}^2K_{0200}L_{0001}L_{0010}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1-2(\varphi_2$$

$$+t\omega_d))+A_1A_4{}^2K_{0200}L_{0001}L_{0010}\omega_d{}^2(3\sigma^4+26\sigma^2\omega_d{}^2-9\omega_d{}^4)\cos(\varphi_1+2(\varphi_2$$

$$+t\omega_d))-2A_1K_{0200}\sigma\omega_d(\sigma^2+9\omega_d{}^2)(-A_4{}^2(-1+e^{2\sigma t})L_{0001}L_{0010}(\sigma^2+\omega_d{}^2)$$

$$+A_3{}^2K_{0001}K_{0010}(\sigma^2+(1-2e^{2\sigma t})\omega_d{}^2))\sin(\varphi_1)$$

$$- A_1 A_4{}^2 e^{2\sigma t} K_{0200} L_{0001} L_{0010} \sigma \omega_d (\sigma^4 + 10\sigma^2 \omega_d{}^2 + 9\omega_d{}^4) \sin(\varphi_1 - 2\varphi_2)$$

$$- A_1 A_3 A_4 K_{0200} (K_{0010} L_{0001} + K_{0001} L_{0010}) \sigma \omega_d (\sigma^4 + 10\sigma^2 \omega_d{}^2 + 9\omega_d{}^4) \sin(\varphi_1$$

$$- \varphi_2) + 4A_2 A_3 A_4 e^{2\sigma t} (K_{0010} L_{0001} + K_{0001} L_{0010}) L_{0200} \sigma \omega_d{}^3 (\sigma^2$$

$$+ 9\omega_d{}^2) \sin(\varphi_2) + 4A_2 A_4{}^2 e^{2\sigma t} L_{0001} L_{0010} L_{0200} \sigma \omega_d{}^3 (\sigma^2 + 9\omega_d{}^2) \sin(2\varphi_2)$$

$$- A_1 A_3 A_4 K_{0200} (K_{0010} L_{0001} + K_{0001} L_{0010}) \sigma \omega_d (\sigma^2 + 9\omega_d{}^2)(\sigma^2 + (1$$

$$- 4e^{2\sigma t}) \omega_d{}^2) \sin(\varphi_1 + \varphi_2) - A_1 A_4{}^2 e^{2\sigma t} K_{0200} L_{0001} L_{0010} \omega_d (\sigma^5 + 6\sigma^3 \omega_d{}^2$$

$$- 27\sigma \omega_d{}^4) \sin(\varphi_1 + 2\varphi_2) - 2A_2 e^{\sigma t} L_{0200} \omega_d (\sigma^2 + \omega_d{}^2)(A_4{}^2 L_{0001} L_{0010} t (\sigma^4$$

$$+ 10\sigma^2 \omega_d{}^2 + 9\omega_d{}^4) + 2A_3{}^2 K_{0001} K_{0010} (\sigma^3 + 5\sigma \omega_d{}^2 + \sigma^2 t \omega_d{}^2$$

$$+ 9t\omega_d{}^4)) \sin(t\omega_d) + 4A_2 A_3{}^2 K_{0001} K_{0010} L_{0200} \sigma \omega_d{}^3 (3\sigma^2 - 5\omega_d{}^2) \sin(2t\omega_d)$$

$$+ A_1 A_3{}^2 K_{0001} K_{0010} K_{0200} \sigma \omega_d (\sigma^4 - 6\sigma^2 \omega_d{}^2 - 7\omega_d{}^4) \sin(\varphi_1 - 2t\omega_d)$$

$$+ A_1 A_3 A_4 K_{0200} (K_{0010} L_{0001} + K_{0001} L_{0010}) \sigma \omega_d (\sigma^4 - 6\sigma^2 \omega_d{}^2 - 7\omega_d{}^4) \sin(\varphi_1$$

$$- \varphi_2 - 2t\omega_d) + 2A_1 A_3{}^2 e^{\sigma t} K_{0001} K_{0010} K_{0200} \sigma \omega_d (\sigma^4 + 10\sigma^2 \omega_d{}^2$$

$$+ 9\omega_d{}^4) \sin(\varphi_1 - t\omega_d) + 2A_1 A_4{}^2 e^{\sigma t} K_{0200} L_{0001} L_{0010} \sigma \omega_d (\sigma^4 + 10\sigma^2 \omega_d{}^2$$

$$+ 9\omega_d{}^4) \sin(\varphi_1 - 2\varphi_2 - t\omega_d) + 2A_1 A_3 A_4 e^{\sigma t} K_{0200} (K_{0010} L_{0001}$$

$$+ K_{0001} L_{0010}) \sigma \omega_d (\sigma^4 + 10\sigma^2 \omega_d{}^2 + 9\omega_d{}^4) \sin(\varphi_1 - \varphi_2 - t\omega_d)$$

$$+ A_2 A_3 A_4 e^{\sigma t} (K_{0010} L_{0001} + K_{0001} L_{0010}) L_{0200} \omega_d (\sigma^2 + \omega_d{}^2)^2 (2\sigma + \sigma^2 t$$

$$+ 9t\omega_d{}^2) \sin(\varphi_2 - t\omega_d) + 2A_2 A_4{}^2 e^{\sigma t} L_{0001} L_{0010} L_{0200} \sigma \omega_d (\sigma^2 + \omega_d{}^2)^2 \sin(2\varphi_2$$

$$- t\omega_d) - 2A_1 e^{\sigma t} K_{0200} \omega_d (\sigma^2 + \omega_d{}^2)(A_4{}^2 L_{0001} L_{0010} t (\sigma^4 + 10\sigma^2 \omega_d{}^2 + 9\omega_d{}^4)$$

$$+ A_3{}^2 K_{0001} K_{0010} (\sigma^3 + \sigma \omega_d{}^2 + 2\sigma^2 t \omega_d{}^2 + 18t\omega_d{}^4)) \sin(\varphi_1 + t\omega_d)$$

$$- 2A_1 A_4{}^2 e^{\sigma t} K_{0200} L_{0001} L_{0010} \sigma \omega_d (\sigma^2 + \omega_d{}^2)^2 \sin(\varphi_1 - 2\varphi_2 + t\omega_d)$$

$$- A_1 A_3 A_4 e^{\sigma t} K_{0200} (K_{0010} L_{0001} + K_{0001} L_{0010}) \omega_d (\sigma^2 + \omega_d{}^2)^2 (2\sigma + \sigma^2 t$$

$$+ 9t\omega_d{}^2) \sin(\varphi_1 - \varphi_2 + t\omega_d) - A_2 A_3 A_4 e^{\sigma t} (K_{0010} L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200}\omega_d(2\sigma - \sigma^2 t + 3t{\omega_d}^2)(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_2$$

$$+ t\omega_d) + 4A_2{A_4}^2 L_{0001}L_{0010}L_{0200}\sigma{\omega_d}^3(3\sigma^2 - 5{\omega_d}^2)\sin(2(\varphi_2 + t\omega_d))$$

$$+ A_1A_3A_4e^{\sigma t}K_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})t\omega_d(\sigma^6 + 7\sigma^4{\omega_d}^2 - 21\sigma^2{\omega_d}^4$$

$$- 27{\omega_d}^6)\sin(\varphi_1 + \varphi_2 + t\omega_d) - 2A_2{A_4}^2e^{\sigma t}L_{0001}L_{0010}L_{0200}\omega_d(\sigma - \sigma^2 t$$

$$+ t{\omega_d}^2)(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(2\varphi_2 + t\omega_d)$$

$$+ 2A_1{A_4}^2e^{\sigma t}K_{0200}L_{0001}L_{0010}t\omega_d(\sigma^6 + 9\sigma^4{\omega_d}^2 - \sigma^2{\omega_d}^4 - 9{\omega_d}^6)\sin(\varphi_1$$

$$+ 2\varphi_2 + t\omega_d) + A_1{A_3}^2K_{0001}K_{0010}K_{0200}\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4)\sin(\varphi_1$$

$$+ 2t\omega_d) + 4A_2A_3A_4(K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}\sigma{\omega_d}^3(3\sigma^2 - 5{\omega_d}^2)\sin(\varphi_2$$

$$+ 2t\omega_d) + A_1A_3A_4K_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2$$

$$- 27{\omega_d}^4)\sin(\varphi_1 + \varphi_2 + 2t\omega_d) + A_1{A_4}^2K_{0200}L_{0001}L_{0010}\sigma\omega_d(\sigma^4 - 6\sigma^2{\omega_d}^2$$

$$- 7{\omega_d}^4)\sin(\varphi_1 - 2(\varphi_2 + t\omega_d)) + A_1{A_4}^2K_{0200}L_{0001}L_{0010}\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2$$

$$- 27{\omega_d}^4)\sin(\varphi_1 + 2(\varphi_2 + t\omega_d))\}$$

(C. 15)

$$x_{1,2}^{qs2,u_2,u_2}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{2\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_4{}^2(1$$

$$+ e^{2\sigma t})L_{0001}{}^2(\sigma^2+\omega_d{}^2) + A_3{}^2K_{0001}{}^2(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2))(A_2L_{0200}$$

$$+ A_1K_{0200}\cos(\varphi_1)) + A_1A_4{}^2e^{2\sigma t}K_{0200}L_{0001}{}^2\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(\varphi_1-2\varphi_2) + 2A_1A_3A_4(1+2e^{2\sigma t})K_{0001}K_{0200}L_{0001}\omega_d{}^2(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1-\varphi_2) + 4A_2A_3A_4K_{0001}L_{0001}L_{0200}\omega_d{}^2(\sigma^2$$

$$+ 9\omega_d{}^2)(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_2) - 2A_2A_4{}^2e^{2\sigma t}L_{0001}{}^2L_{0200}\omega_d{}^2(\sigma^4$$

$$+ 8\sigma^2\omega_d{}^2-9\omega_d{}^4)\cos(2\varphi_2) + 2A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d{}^2(\sigma^2+9\omega_d{}^2)((1$$

$$- 2e^{2\sigma t})\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_1+\varphi_2)$$

$$+ A_1A_4{}^2e^{2\sigma t}K_{0200}L_{0001}{}^2\omega_d{}^2(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1+2\varphi_2)$$

$$+ 4A_2e^{\sigma t}L_{0200}\omega_d{}^2(\sigma^2+\omega_d{}^2)(-A_4{}^2L_{0001}{}^2(\sigma^2+9\omega_d{}^2) + A_3{}^2K_{0001}{}^2(\sigma^3t$$

$$- 12\omega_d{}^2+9\sigma t\omega_d{}^2))\cos(t\omega_d) - 2A_2A_3{}^2K_{0001}{}^2L_{0200}\omega_d{}^2(\sigma^4-12\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(2t\omega_d) + A_1A_3{}^2K_{0001}{}^2K_{0200}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1$$

$$- 2t\omega_d) + 2A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_1$$

$$- \varphi_2-2t\omega_d) - A_1e^{\sigma t}K_{0200}(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)(2A_3{}^2K_{0001}{}^2\omega_d{}^2$$

$$+ A_4{}^2L_{0001}{}^2(\sigma^2+\omega_d{}^2))\cos(\varphi_1-t\omega_d) + A_1A_4{}^2e^{\sigma t}K_{0200}L_{0001}{}^2(\sigma^6+9\sigma^4\omega_d{}^2$$

$$- \sigma^2\omega_d{}^4-9\omega_d{}^6)\cos(\varphi_1-2\varphi_2-t\omega_d) + A_1A_3A_4e^{\sigma t}K_{0001}K_{0200}L_{0001}(\sigma^6$$

$$+ 7\sigma^4\omega_d{}^2-21\sigma^2\omega_d{}^4-27\omega_d{}^6)\cos(\varphi_1-\varphi_2-t\omega_d)$$

$$- 2A_2A_3A_4e^{\sigma t}K_{0001}L_{0001}L_{0200}(\sigma^6+7\sigma^4\omega_d{}^2+27\sigma^2\omega_d{}^4+21\omega_d{}^6)\cos(\varphi_2$$

$$- t\omega_d) - A_1A_3A_4e^{\sigma t}K_{0001}K_{0200}L_{0001}(\sigma^2+\omega_d{}^2)^2(\sigma^2+9\omega_d{}^2)\cos(\varphi_1+\varphi_2$$

$$- t\omega_d) - A_2A_4{}^2e^{\sigma t}L_{0001}{}^2L_{0200}(\sigma^2+\omega_d{}^2)^2(\sigma^2+3\omega_d{}^2)\cos(2\varphi_2-t\omega_d)$$

$$+ A_1e^{\sigma t}K_{0200}(\sigma^2+\omega_d{}^2)(2A_3{}^2K_{0001}{}^2\omega_d{}^2(\sigma^2+2\sigma^3t-15\omega_d{}^2+18\sigma t\omega_d{}^2)$$

$$+ A_4{}^2L_{0001}{}^2(\sigma^4+6\sigma^2\omega_d{}^2-27\omega_d{}^4))\cos(\varphi_1+t\omega_d)$$

$$
\begin{aligned}
&- A_1A_4{}^2e^{\sigma t}K_{0200}L_{0001}{}^2(\sigma^2+\omega_d{}^2)^2(\sigma^2+3\omega_d{}^2)\cos(\varphi_1-2\varphi_2+t\omega_d)\\
&- A_1A_3A_4e^{\sigma t}K_{0001}K_{0200}L_{0001}(\sigma^6+3\sigma^4\omega_d{}^2+35\sigma^2\omega_d{}^4+33\omega_d{}^6)\cos(\varphi_1\\
&- \varphi_2+t\omega_d)+2A_2A_3A_4e^{\sigma t}K_{0001}L_{0001}L_{0200}(\sigma^2-3\omega_d{}^2+4\sigma t\omega_d{}^2)(\sigma^4\\
&+ 10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_2+t\omega_d)-2A_2A_4{}^2L_{0001}{}^2L_{0200}\omega_d{}^2(\sigma^4-12\sigma^2\omega_d{}^2\\
&+ 3\omega_d{}^4)\cos(2(\varphi_2+t\omega_d))+A_1A_3A_4e^{\sigma t}K_{0001}K_{0200}L_{0001}(\sigma^2-3\omega_d{}^2\\
&+ 8\sigma t\omega_d{}^2)(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1+\varphi_2+t\omega_d)\\
&+ A_2A_4{}^2e^{\sigma t}L_{0001}{}^2L_{0200}(\sigma^2-\omega_d{}^2+4\sigma t\omega_d{}^2)(\sigma^4+10\sigma^2\omega_d{}^2\\
&+ 9\omega_d{}^4)\cos(2\varphi_2+t\omega_d)+4A_1A_4{}^2e^{\sigma t}K_{0200}L_{0001}{}^2\sigma t\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2\\
&+ 9\omega_d{}^4)\cos(\varphi_1+2\varphi_2+t\omega_d)+A_1A_3{}^2K_{0001}{}^2K_{0200}\omega_d{}^2(3\sigma^4+26\sigma^2\omega_d{}^2\\
&- 9\omega_d{}^4)\cos(\varphi_1+2t\omega_d)-4A_2A_3A_4K_{0001}L_{0001}L_{0200}\omega_d{}^2(\sigma^4-12\sigma^2\omega_d{}^2\\
&+ 3\omega_d{}^4)\cos(\varphi_2+2t\omega_d)+A_1A_4{}^2K_{0200}L_{0001}{}^2\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2\\
&+ 3\omega_d{}^4)\cos(\varphi_1-2(\varphi_2+t\omega_d))-A_1A_4K_{0200}L_{0001}\omega_d{}^2(-3\sigma^4-26\sigma^2\omega_d{}^2\\
&+ 9\omega_d{}^4)(2A_3K_{0001}\cos(\varphi_1+\varphi_2+2t\omega_d)+A_4L_{0001}\cos(\varphi_1+2(\varphi_2+t\omega_d)))\\
&- 2A_1K_{0200}\omega_d(\sigma^3+9\sigma\omega_d{}^2)(-A_4{}^2(-1+e^{2\sigma t})L_{0001}{}^2(\sigma^2+\omega_d{}^2)\\
&+ A_3{}^2K_{0001}{}^2(\sigma^2+(1-2e^{2\sigma t})\omega_d{}^2))\sin(\varphi_1)-A_1A_4{}^2e^{2\sigma t}K_{0200}L_{0001}{}^2\sigma\omega_d(\sigma^4\\
&+ 10\sigma^2\omega_d{}^2+9\omega_d{}^4)\sin(\varphi_1-2\varphi_2)-2A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma\omega_d(\sigma^4\\
&+ 10\sigma^2\omega_d{}^2+9\omega_d{}^4)\sin(\varphi_1-\varphi_2)+8A_2A_3A_4e^{2\sigma t}K_{0001}L_{0001}L_{0200}\sigma\omega_d{}^3(\sigma^2\\
&+ 9\omega_d{}^2)\sin(\varphi_2)+4A_2A_4{}^2e^{2\sigma t}L_{0001}{}^2L_{0200}\sigma\omega_d{}^3(\sigma^2+9\omega_d{}^2)\sin(2\varphi_2)\\
&- 2A_1A_3A_4K_{0001}K_{0200}L_{0001}\omega_d(\sigma^2+(1-4e^{2\sigma t})\omega_d{}^2)(\sigma^3+9\sigma\omega_d{}^2)\sin(\varphi_1\\
&+ \varphi_2)-A_1A_4{}^2e^{2\sigma t}K_{0200}L_{0001}{}^2\omega_d(\sigma^5+6\sigma^3\omega_d{}^2-27\sigma\omega_d{}^4)\sin(\varphi_1+2\varphi_2)\\
&- 2A_2e^{\sigma t}L_{0200}\omega_d(\sigma^2+\omega_d{}^2)(A_4{}^2L_{0001}{}^2t(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\\
&+ 2A_3{}^2K_{0001}{}^2(\sigma^3+5\sigma\omega_d{}^2+\sigma^2t\omega_d{}^2+9t\omega_d{}^4))\sin(t\omega_d)
\end{aligned}
$$

$$+ 4A_2A_3{}^2K_{0001}{}^2L_{0200}\sigma\omega_d{}^3(3\sigma^2 - 5\omega_d{}^2)\sin(2t\omega_d)$$

$$+ A_1A_3{}^2K_{0001}{}^2K_{0200}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_1 - 2t\omega_d)$$

$$+ 2A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_1 - \varphi_2 - 2t\omega_d)$$

$$+ 2A_1A_3{}^2e^{\sigma t}K_{0001}{}^2K_{0200}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 - t\omega_d)$$

$$+ 2A_1A_4{}^2e^{\sigma t}K_{0200}L_{0001}{}^2\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 - 2\varphi_2 - t\omega_d)$$

$$+ 4A_1A_3A_4e^{\sigma t}K_{0001}K_{0200}L_{0001}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 - \varphi_2$$

$$- t\omega_d) + 2A_2A_3A_4e^{\sigma t}K_{0001}L_{0001}L_{0200}\omega_d(\sigma^2 + \omega_d{}^2)^2(2\sigma + \sigma^2t$$

$$+ 9t\omega_d{}^2)\sin(\varphi_2 - t\omega_d) + 2A_2A_4{}^2e^{\sigma t}L_{0001}{}^2L_{0200}\sigma\omega_d(\sigma^2 + \omega_d{}^2)^2\sin(2\varphi_2$$

$$- t\omega_d) - 2A_1e^{\sigma t}K_{0200}\omega_d(\sigma^2 + \omega_d{}^2)(A_4{}^2L_{0001}{}^2t(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)$$

$$+ A_3{}^2K_{0001}{}^2(\sigma^3 + \sigma\omega_d{}^2 + 2\sigma^2t\omega_d{}^2 + 18t\omega_d{}^4))\sin(\varphi_1 + t\omega_d)$$

$$- 2A_1A_4{}^2e^{\sigma t}K_{0200}L_{0001}{}^2\sigma\omega_d(\sigma^2 + \omega_d{}^2)^2\sin(\varphi_1 - 2\varphi_2 + t\omega_d)$$

$$- 2A_1A_3A_4e^{\sigma t}K_{0001}K_{0200}L_{0001}\omega_d(\sigma^2 + \omega_d{}^2)^2(2\sigma + \sigma^2t + 9t\omega_d{}^2)\sin(\varphi_1$$

$$- \varphi_2 + t\omega_d) + 2A_2A_3A_4e^{\sigma t}K_{0001}L_{0001}L_{0200}\omega_d(-2\sigma + \sigma^2t - 3t\omega_d{}^2)(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_2 + t\omega_d) + 4A_2A_4{}^2L_{0001}{}^2L_{0200}\sigma\omega_d{}^3(3\sigma^2$$

$$- 5\omega_d{}^2)\sin(2(\varphi_2 + t\omega_d)) + 2A_1A_3A_4e^{\sigma t}K_{0001}K_{0200}L_{0001}t\omega_d(\sigma^6 + 7\sigma^4\omega_d{}^2$$

$$- 21\sigma^2\omega_d{}^4 - 27\omega_d{}^6)\sin(\varphi_1 + \varphi_2 + t\omega_d) - 2A_2A_4{}^2e^{\sigma t}L_{0001}{}^2L_{0200}\omega_d(\sigma$$

$$- \sigma^2t + t\omega_d{}^2)(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(2\varphi_2 + t\omega_d)$$

$$+ 2A_1A_4{}^2e^{\sigma t}K_{0200}L_{0001}{}^2t\omega_d(\sigma^6 + 9\sigma^4\omega_d{}^2 - \sigma^2\omega_d{}^4 - 9\omega_d{}^6)\sin(\varphi_1 + 2\varphi_2$$

$$+ t\omega_d) + A_1A_3{}^2K_{0001}{}^2K_{0200}\sigma\omega_d(\sigma^4 + 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4)\sin(\varphi_1 + 2t\omega_d)$$

$$+ 8A_2A_3A_4K_{0001}L_{0001}L_{0200}\sigma\omega_d{}^3(3\sigma^2 - 5\omega_d{}^2)\sin(\varphi_2 + 2t\omega_d)$$

$$+ 2A_1A_3A_4K_{0001}K_{0200}L_{0001}\sigma\omega_d(\sigma^4 + 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4)\sin(\varphi_1 + \varphi_2 + 2t\omega_d)$$

$$+ A_1A_4{}^2K_{0200}L_{0001}{}^2\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_1 - 2(\varphi_2 + t\omega_d))$$

$$+ A_1 A_4{}^2 K_{0200} L_{0001}{}^2 \sigma\omega_d(\sigma^4 + 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4)\sin(\varphi_1 + 2(\varphi_2 + t\omega_d))\}$$

$$(C.16)$$

$$x_{1,2}^{bs1i1}(t) = x_{1,2}^{bs1i1,u_1,u_1}(t) \times u_1 u_1 + x_{1,2}^{bs1i1,u_2,u_1}(t) \times u_2 u_1 \qquad (C.17)$$

$$x_{1,2}^{bs1i1,u_1,u_1}(t) = \frac{1}{4(\sigma^2+{\omega_d}^2)^2}\{4{A_2}^2L_{0010}L_{1010}{\omega_d}^2 + 2{A_1}^2K_{0010}K_{1010}(\sigma^2+{\omega_d}^2)$$

$$+ 4A_1A_2(K_{1010}L_{0010} + K_{0010}L_{1010}){\omega_d}^2\cos(\varphi_1) + 2{A_1}^2K_{0010}K_{1010}(-\sigma^2$$

$$+ {\omega_d}^2)\cos(2\varphi_1) + 2e^{-\sigma t}(-{A_1}^2K_{0010}K_{1010}(\sigma^2+{\omega_d}^2) + {A_2}^2L_{0010}L_{1010}(\sigma^3 t$$

$$- 2{\omega_d}^2 + \sigma t{\omega_d}^2))\cos(t\omega_d) - A_1A_2e^{-\sigma t}(K_{1010}L_{0010} + K_{0010}L_{1010})(\sigma^2$$

$$+ {\omega_d}^2)\cos(\varphi_1 - t\omega_d) + A_1A_2e^{-\sigma t}(K_{1010}L_{0010} + K_{0010}L_{1010})(\sigma^2 + 2\sigma^3 t$$

$$- 3{\omega_d}^2 + 2\sigma t{\omega_d}^2)\cos(\varphi_1 + t\omega_d) + 2{A_1}^2e^{-\sigma t}K_{0010}K_{1010}(\sigma^2 + \sigma^3 t - {\omega_d}^2$$

$$+ \sigma t{\omega_d}^2)\cos(2\varphi_1 + t\omega_d) + 4A_1A_2(K_{1010}L_{0010} + K_{0010}L_{1010})\sigma\omega_d\sin(\varphi_1)$$

$$+ 4{A_1}^2K_{0010}K_{1010}\sigma\omega_d\sin(2\varphi_1) - \frac{1}{\omega_d}2e^{-\sigma t}({A_1}^2K_{0010}K_{1010}\sigma(\sigma^2+{\omega_d}^2)$$

$$+ {A_2}^2L_{0010}L_{1010}(\sigma^3 + 3\sigma{\omega_d}^2 + \sigma^2 t{\omega_d}^2 + t{\omega_d}^4))\sin(t\omega_d)$$

$$+ \frac{1}{\omega_d}A_1A_2e^{-\sigma t}(K_{1010}L_{0010} + K_{0010}L_{1010})\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_1 - t\omega_d)$$

$$- \frac{1}{\omega_d}A_1A_2e^{-\sigma t}(K_{1010}L_{0010} + K_{0010}L_{1010})(\sigma^3 + 5\sigma{\omega_d}^2 + 2\sigma^2 t{\omega_d}^2$$

$$+ 2t{\omega_d}^4)\sin(\varphi_1 + t\omega_d) - 2{A_1}^2e^{-\sigma t}K_{0010}K_{1010}\omega_d(2\sigma + \sigma^2 t + t{\omega_d}^2)\sin(2\varphi_1$$

$$+ t\omega_d))\}$$

(C. 18)

$$
\begin{aligned}
\boldsymbol{x}_{1,2}^{bs1i1,u_2,u_1}(\boldsymbol{t}) = {} & \frac{1}{4(\sigma^2+{\omega_d}^2)^2}\{4{A_2}^2L_{0001}L_{1010}{\omega_d}^2 + 2{A_1}^2K_{0001}K_{1010}(\sigma^2+{\omega_d}^2) \\
& + 4A_1A_2(K_{1010}L_{0001}+K_{0001}L_{1010}){\omega_d}^2\cos(\varphi_1) + 2{A_1}^2K_{0001}K_{1010}(-\sigma^2 \\
& + {\omega_d}^2)\cos(2\varphi_1) + 2e^{-\sigma t}(-{A_1}^2K_{0001}K_{1010}(\sigma^2+{\omega_d}^2) + {A_2}^2L_{0001}L_{1010}(\sigma^3 t \\
& - 2{\omega_d}^2 + \sigma t{\omega_d}^2))\cos(t\omega_d) - A_1A_2e^{-\sigma t}(K_{1010}L_{0001}+K_{0001}L_{1010})(\sigma^2 \\
& + {\omega_d}^2)\cos(\varphi_1 - t\omega_d) + A_1A_2e^{-\sigma t}(K_{1010}L_{0001}+K_{0001}L_{1010})(\sigma^2 + 2\sigma^3 t \\
& - 3{\omega_d}^2 + 2\sigma t{\omega_d}^2)\cos(\varphi_1 + t\omega_d) + 2{A_1}^2e^{-\sigma t}K_{0001}K_{1010}(\sigma^2 + \sigma^3 t - {\omega_d}^2 \\
& + \sigma t{\omega_d}^2)\cos(2\varphi_1 + t\omega_d) + 4A_1A_2(K_{1010}L_{0001}+K_{0001}L_{1010})\sigma\omega_d\sin(\varphi_1) \\
& + 4{A_1}^2K_{0001}K_{1010}\sigma\omega_d\sin(2\varphi_1) - \frac{1}{\omega_d}2e^{-\sigma t}({A_1}^2K_{0001}K_{1010}\sigma(\sigma^2+{\omega_d}^2) \\
& + {A_2}^2L_{0001}L_{1010}(\sigma^3 + 3\sigma{\omega_d}^2 + \sigma^2 t{\omega_d}^2 + t{\omega_d}^4))\sin(t\omega_d) \\
& + \frac{1}{\omega_d}A_1A_2e^{-\sigma t}(K_{1010}L_{0001}+K_{0001}L_{1010})\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_1 - t\omega_d) \\
& - \frac{1}{\omega_d}A_1A_2e^{-\sigma t}(K_{1010}L_{0001}+K_{0001}L_{1010})(\sigma^3 + 5\sigma{\omega_d}^2 + 2\sigma^2 t{\omega_d}^2 \\
& + 2t{\omega_d}^4)\sin(\varphi_1 + t\omega_d) - 2{A_1}^2e^{-\sigma t}K_{0001}K_{1010}\omega_d(2\sigma + \sigma^2 t + t{\omega_d}^2)\sin(2\varphi_1 \\
& + t\omega_d))\}
\end{aligned}
$$

(C. 19)

$$x_{1,2}^{bs2i1}(t) = x_{1,2}^{bs2i1,u_1,u_1}(t) \times u_1 u_1 + x_{1,2}^{bs2i1,u_2,u_1}(t) \times u_2 u_1 \quad (C.20)$$

$$
\begin{aligned}
\boldsymbol{x}_{1,2}^{bs2i1,u_1,u_1}(\boldsymbol{t}) = {} & \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^2}\{4A_3K_{0010}\omega_d{}^3(A_2L_{0110}+A_1K_{0110}\cos(\varphi_1)) \\
& + 2A_1A_4K_{0110}L_{0010}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-\varphi_2) \\
& + 4A_2A_4L_{0010}L_{0110}\omega_d{}^3\cos(\varphi_2) + 2A_1A_4K_{0110}L_{0010}\omega_d(-\sigma^2+\omega_d{}^2)\cos(\varphi_1 \\
& + \varphi_2) + 2A_2A_3e^{-\sigma t}K_{0010}L_{0110}\omega_d(\sigma^3 t - 2\omega_d{}^2 + \sigma t\omega_d{}^2)\cos(t\omega_d) \\
& - A_1A_3e^{-\sigma t}K_{0010}K_{0110}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1 - t\omega_d) \\
& - A_1A_4e^{-\sigma t}K_{0110}L_{0010}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-\varphi_2-t\omega_d) \\
& - A_2A_4e^{-\sigma t}L_{0010}L_{0110}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_2 - t\omega_d) \\
& + A_1A_3e^{-\sigma t}K_{0010}K_{0110}\omega_d(\sigma^2+2\sigma^3 t-3\omega_d{}^2+2\sigma t\omega_d{}^2)\cos(\varphi_1+t\omega_d) \\
& - A_1A_4e^{-\sigma t}K_{0110}L_{0010}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-\varphi_2+t\omega_d) \\
& + A_2A_4e^{-\sigma t}L_{0010}L_{0110}\omega_d(\sigma^2+2\sigma^3 t-3\omega_d{}^2+2\sigma t\omega_d{}^2)\cos(\varphi_2+t\omega_d) \\
& + 2A_1A_4e^{-\sigma t}K_{0110}L_{0010}\omega_d(\sigma^2+\sigma^3 t-\omega_d{}^2+\sigma t\omega_d{}^2)\cos(\varphi_1+\varphi_2+t\omega_d) \\
& + 4A_1A_3K_{0010}K_{0110}\sigma\omega_d{}^2\sin(\varphi_1) + 4A_2A_4L_{0010}L_{0110}\sigma\omega_d{}^2\sin(\varphi_2) \\
& + 4A_1A_4K_{0110}L_{0010}\sigma\omega_d{}^2\sin(\varphi_1+\varphi_2) - 2A_2A_3e^{-\sigma t}K_{0010}L_{0110}(\sigma^3+3\sigma\omega_d{}^2 \\
& + \sigma^2 t\omega_d{}^2 + t\omega_d{}^4)\sin(t\omega_d) + A_1A_3e^{-\sigma t}K_{0010}K_{0110}\sigma(\sigma^2+\omega_d{}^2)\sin(\varphi_1-t\omega_d) \\
& + A_1A_4e^{-\sigma t}K_{0110}L_{0010}\sigma(\sigma^2+\omega_d{}^2)\sin(\varphi_1-\varphi_2-t\omega_d) \\
& + A_2A_4e^{-\sigma t}L_{0010}L_{0110}\sigma(\sigma^2+\omega_d{}^2)\sin(\varphi_2-t\omega_d) - A_1A_3e^{-\sigma t}K_{0010}K_{0110}(\sigma^3 \\
& + 5\sigma\omega_d{}^2 + 2\sigma^2 t\omega_d{}^2 + 2t\omega_d{}^4)\sin(\varphi_1+t\omega_d) - A_1A_4e^{-\sigma t}K_{0110}L_{0010}\sigma(\sigma^2 \\
& + \omega_d{}^2)\sin(\varphi_1-\varphi_2+t\omega_d) - A_2A_4e^{-\sigma t}L_{0010}L_{0110}(\sigma^3+5\sigma\omega_d{}^2+2\sigma^2 t\omega_d{}^2 \\
& + 2t\omega_d{}^4)\sin(\varphi_2+t\omega_d) - 2A_1A_4e^{-\sigma t}K_{0110}L_{0010}\omega_d{}^2(2\sigma+\sigma^2 t+t\omega_d{}^2)\sin(\varphi_1 \\
& + \varphi_2 + t\omega_d)\}
\end{aligned}
\tag{C.21}
$$

$$\boldsymbol{x_{1,2}^{bs2i1,u_2,u_1}(t)} = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^2}\{4A_3K_{0001}\omega_d{}^3(A_2L_{0110}+A_1K_{0110}\cos(\varphi_1))$$
$$+2A_1A_4K_{0110}L_{0001}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-\varphi_2)$$
$$+4A_2A_4L_{0001}L_{0110}\omega_d{}^3\cos(\varphi_2)+2A_1A_4K_{0110}L_{0001}\omega_d(-\sigma^2+\omega_d{}^2)\cos(\varphi_1$$
$$+\varphi_2)+2A_2A_3e^{-\sigma t}K_{0001}L_{0110}\omega_d(\sigma^3 t-2\omega_d{}^2+\sigma t\omega_d{}^2)\cos(t\omega_d)$$
$$-A_1A_3e^{-\sigma t}K_{0001}K_{0110}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-t\omega_d)$$
$$-A_1A_4e^{-\sigma t}K_{0110}L_{0001}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-\varphi_2-t\omega_d)$$
$$-A_2A_4e^{-\sigma t}L_{0001}L_{0110}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_2-t\omega_d)$$
$$+A_1A_3e^{-\sigma t}K_{0001}K_{0110}\omega_d(\sigma^2+2\sigma^3 t-3\omega_d{}^2+2\sigma t\omega_d{}^2)\cos(\varphi_1+t\omega_d)$$
$$-A_1A_4e^{-\sigma t}K_{0110}L_{0001}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-\varphi_2+t\omega_d)$$
$$+A_2A_4e^{-\sigma t}L_{0001}L_{0110}\omega_d(\sigma^2+2\sigma^3 t-3\omega_d{}^2+2\sigma t\omega_d{}^2)\cos(\varphi_2+t\omega_d)$$
$$+2A_1A_4e^{-\sigma t}K_{0110}L_{0001}\omega_d(\sigma^2+\sigma^3 t-\omega_d{}^2+\sigma t\omega_d{}^2)\cos(\varphi_1+\varphi_2+t\omega_d)$$
$$+4A_1A_3K_{0001}K_{0110}\sigma\omega_d{}^2\sin(\varphi_1)+4A_2A_4L_{0001}L_{0110}\sigma\omega_d{}^2\sin(\varphi_2)$$
$$+4A_1A_4K_{0110}L_{0001}\sigma\omega_d{}^2\sin(\varphi_1+\varphi_2)-2A_2A_3e^{-\sigma t}K_{0001}L_{0110}(\sigma^3+3\sigma\omega_d{}^2$$
$$+\sigma^2 t\omega_d{}^2+t\omega_d{}^4)\sin(t\omega_d)+A_1A_3e^{-\sigma t}K_{0001}K_{0110}\sigma(\sigma^2+\omega_d{}^2)\sin(\varphi_1-t\omega_d)$$
$$+A_1A_4e^{-\sigma t}K_{0110}L_{0001}\sigma(\sigma^2+\omega_d{}^2)\sin(\varphi_1-\varphi_2-t\omega_d)$$
$$+A_2A_4e^{-\sigma t}L_{0001}L_{0110}\sigma(\sigma^2+\omega_d{}^2)\sin(\varphi_2-t\omega_d)-A_1A_3e^{-\sigma t}K_{0001}K_{0110}(\sigma^3$$
$$+5\sigma\omega_d{}^2+2\sigma^2 t\omega_d{}^2+2t\omega_d{}^4)\sin(\varphi_1+t\omega_d)-A_1A_4e^{-\sigma t}K_{0110}L_{0001}\sigma(\sigma^2$$
$$+\omega_d{}^2)\sin(\varphi_1-\varphi_2+t\omega_d)-A_2A_4e^{-\sigma t}L_{0001}L_{0110}(\sigma^3+5\sigma\omega_d{}^2+2\sigma^2 t\omega_d{}^2$$
$$+2t\omega_d{}^4)\sin(\varphi_2+t\omega_d)-2A_1A_4e^{-\sigma t}K_{0110}L_{0001}\omega_d{}^2(2\sigma+\sigma^2 t+t\omega_d{}^2)\sin(\varphi_1$$
$$+\varphi_2+t\omega_d)\}$$

(C. 22)

$$x_{1,2}^{bs1i2}(t) = x_{1,2}^{bs1i2,u_1,u_2}(t) \times u_1u_2 + x_{1,2}^{bs1i2,u_2,u_2}(t) \times u_2u_2 \qquad (C.23)$$

$$x_{1,2}^{bs1i2,u_1,u_2}(t) = \frac{1}{4(\sigma^2+{\omega_d}^2)^2}\{4{A_2}^2L_{0010}L_{1001}{\omega_d}^2 + 2{A_1}^2K_{0010}K_{1001}(\sigma^2+{\omega_d}^2)$$
$$+ 4A_1A_2(K_{1001}L_{0010} + K_{0010}L_{1001}){\omega_d}^2\cos(\varphi_1) + 2{A_1}^2K_{0010}K_{1001}(-\sigma^2$$
$$+ {\omega_d}^2)\cos(2\varphi_1) + 2e^{-\sigma t}(-{A_1}^2K_{0010}K_{1001}(\sigma^2+{\omega_d}^2) + {A_2}^2L_{0010}L_{1001}(\sigma^3 t$$
$$- 2{\omega_d}^2 + \sigma t{\omega_d}^2))\cos(t\omega_d) - A_1A_2e^{-\sigma t}(K_{1001}L_{0010} + K_{0010}L_{1001})(\sigma^2$$
$$+ {\omega_d}^2)\cos(\varphi_1 - t\omega_d) + A_1A_2e^{-\sigma t}(K_{1001}L_{0010} + K_{0010}L_{1001})(\sigma^2 + 2\sigma^3 t$$
$$- 3{\omega_d}^2 + 2\sigma t{\omega_d}^2)\cos(\varphi_1 + t\omega_d) + 2{A_1}^2e^{-\sigma t}K_{0010}K_{1001}(\sigma^2 + \sigma^3 t - {\omega_d}^2$$
$$+ \sigma t{\omega_d}^2)\cos(2\varphi_1 + t\omega_d) + 4A_1A_2(K_{1001}L_{0010} + K_{0010}L_{1001})\sigma\omega_d\sin(\varphi_1)$$
$$+ 4{A_1}^2K_{0010}K_{1001}\sigma\omega_d\sin(2\varphi_1) - \frac{1}{\omega_d}2e^{-\sigma t}({A_1}^2K_{0010}K_{1001}\sigma(\sigma^2+{\omega_d}^2)$$
$$+ {A_2}^2L_{0010}L_{1001}(\sigma^3 + 3\sigma{\omega_d}^2 + \sigma^2t{\omega_d}^2 + t{\omega_d}^4))\sin(t\omega_d)$$
$$+ \frac{1}{\omega_d}A_1A_2e^{-\sigma t}(K_{1001}L_{0010} + K_{0010}L_{1001})\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_1 - t\omega_d)$$
$$- \frac{1}{\omega_d}A_1A_2e^{-\sigma t}(K_{1001}L_{0010} + K_{0010}L_{1001})(\sigma^3 + 5\sigma{\omega_d}^2 + 2\sigma^2t{\omega_d}^2$$
$$+ 2t{\omega_d}^4)\sin(\varphi_1 + t\omega_d) - 2{A_1}^2e^{-\sigma t}K_{0010}K_{1001}\omega_d(2\sigma + \sigma^2t + t{\omega_d}^2)\sin(2\varphi_1$$
$$+ t\omega_d))\}$$

(C. 24)

$$
\begin{aligned}
\boldsymbol{x}_{\mathbf{1,2}}^{\boldsymbol{bs1i2,u_2,u_2}}(\boldsymbol{t}) = \frac{1}{4(\sigma^2+{\omega_d}^2)^2}\{&4{A_2}^2L_{0001}L_{1001}{\omega_d}^2 + 2{A_1}^2K_{0001}K_{1001}(\sigma^2+{\omega_d}^2) \\
&+ 4A_1A_2(K_{1001}L_{0001}+K_{0001}L_{1001}){\omega_d}^2\cos(\varphi_1) + 2{A_1}^2K_{0001}K_{1001}(-\sigma^2 \\
&+ {\omega_d}^2)\cos(2\varphi_1) + 2e^{-\sigma t}(-{A_1}^2K_{0001}K_{1001}(\sigma^2+{\omega_d}^2) + {A_2}^2L_{0001}L_{1001}(\sigma^3 t \\
&- 2{\omega_d}^2 + \sigma t{\omega_d}^2))\cos(t\omega_d) - A_1A_2e^{-\sigma t}(K_{1001}L_{0001}+K_{0001}L_{1001})(\sigma^2 \\
&+ {\omega_d}^2)\cos(\varphi_1 - t\omega_d) + A_1A_2e^{-\sigma t}(K_{1001}L_{0001}+K_{0001}L_{1001})(\sigma^2 + 2\sigma^3 t \\
&- 3{\omega_d}^2 + 2\sigma t{\omega_d}^2)\cos(\varphi_1 + t\omega_d) + 2{A_1}^2e^{-\sigma t}K_{0001}K_{1001}(\sigma^2 + \sigma^3 t - {\omega_d}^2 \\
&+ \sigma t{\omega_d}^2)\cos(2\varphi_1 + t\omega_d) + 4A_1A_2(K_{1001}L_{0001}+K_{0001}L_{1001})\sigma\omega_d\sin(\varphi_1) \\
&+ 4{A_1}^2K_{0001}K_{1001}\sigma\omega_d\sin(2\varphi_1) - \frac{1}{\omega_d}2e^{-\sigma t}({A_1}^2K_{0001}K_{1001}\sigma(\sigma^2+{\omega_d}^2) \\
&+ {A_2}^2L_{0001}L_{1001}(\sigma^3 + 3\sigma{\omega_d}^2 + \sigma^2 t{\omega_d}^2 + t{\omega_d}^4))\sin(t\omega_d) \\
&+ \frac{1}{\omega_d}A_1A_2e^{-\sigma t}(K_{1001}L_{0001}+K_{0001}L_{1001})\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_1 - t\omega_d) \\
&- \frac{1}{\omega_d}A_1A_2e^{-\sigma t}(K_{1001}L_{0001}+K_{0001}L_{1001})(\sigma^3 + 5\sigma{\omega_d}^2 + 2\sigma^2 t{\omega_d}^2 \\
&+ 2t{\omega_d}^4)\sin(\varphi_1 + t\omega_d) - 2{A_1}^2e^{-\sigma t}K_{0001}K_{1001}\omega_d(2\sigma + \sigma^2 t + t{\omega_d}^2)\sin(2\varphi_1 \\
&+ t\omega_d)\}
\end{aligned}
\tag{C.25}
$$

$$x_{1,2}^{bs2i2}(t) = x_{1,2}^{bs2i2,u_1,u_2}(t) \times u_1 u_2 + x_{1,2}^{bs2i2,u_2,u_2}(t) \times u_2 u_2 \quad (C.26)$$

$$x_{1,2}^{bs2i2,u_1,u_2}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^2}\{4A_3K_{0010}\omega_d{}^3(A_2L_{0101}+A_1K_{0101}\cos(\varphi_1))$$

$$+2A_1A_4K_{0101}L_{0010}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-\varphi_2)$$

$$+4A_2A_4L_{0010}L_{0101}\omega_d{}^3\cos(\varphi_2)+2A_1A_4K_{0101}L_{0010}\omega_d(-\sigma^2+\omega_d{}^2)\cos(\varphi_1$$

$$+\varphi_2)+2A_2A_3e^{-\sigma t}K_{0010}L_{0101}\omega_d(\sigma^3t-2\omega_d{}^2+\sigma t\omega_d{}^2)\cos(t\omega_d)$$

$$-A_1A_3e^{-\sigma t}K_{0010}K_{0101}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-t\omega_d)$$

$$-A_1A_4e^{-\sigma t}K_{0101}L_{0010}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-\varphi_2-t\omega_d)$$

$$-A_2A_4e^{-\sigma t}L_{0010}L_{0101}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_2-t\omega_d)$$

$$+A_1A_3e^{-\sigma t}K_{0010}K_{0101}\omega_d(\sigma^2+2\sigma^3t-3\omega_d{}^2+2\sigma t\omega_d{}^2)\cos(\varphi_1+t\omega_d)$$

$$-A_1A_4e^{-\sigma t}K_{0101}L_{0010}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-\varphi_2+t\omega_d)$$

$$+A_2A_4e^{-\sigma t}L_{0010}L_{0101}\omega_d(\sigma^2+2\sigma^3t-3\omega_d{}^2+2\sigma t\omega_d{}^2)\cos(\varphi_2+t\omega_d)$$

$$+2A_1A_4e^{-\sigma t}K_{0101}L_{0010}\omega_d(\sigma^2+\sigma^3t-\omega_d{}^2+\sigma t\omega_d{}^2)\cos(\varphi_1+\varphi_2+t\omega_d)$$

$$+4A_1A_3K_{0010}K_{0101}\sigma\omega_d{}^2\sin(\varphi_1)+4A_2A_4L_{0010}L_{0101}\sigma\omega_d{}^2\sin(\varphi_2)$$

$$+4A_1A_4K_{0101}L_{0010}\sigma\omega_d{}^2\sin(\varphi_1+\varphi_2)-2A_2A_3e^{-\sigma t}K_{0010}L_{0101}(\sigma^3+3\sigma\omega_d{}^2$$

$$+\sigma^2t\omega_d{}^2+t\omega_d{}^4)\sin(t\omega_d)+A_1A_3e^{-\sigma t}K_{0010}K_{0101}\sigma(\sigma^2+\omega_d{}^2)\sin(\varphi_1-t\omega_d)$$

$$+A_1A_4e^{-\sigma t}K_{0101}L_{0010}\sigma(\sigma^2+\omega_d{}^2)\sin(\varphi_1-\varphi_2-t\omega_d)$$

$$+A_2A_4e^{-\sigma t}L_{0010}L_{0101}\sigma(\sigma^2+\omega_d{}^2)\sin(\varphi_2-t\omega_d)-A_1A_3e^{-\sigma t}K_{0010}K_{0101}(\sigma^3$$

$$+5\sigma\omega_d{}^2+2\sigma^2t\omega_d{}^2+2t\omega_d{}^4)\sin(\varphi_1+t\omega_d)-A_1A_4e^{-\sigma t}K_{0101}L_{0010}\sigma(\sigma^2$$

$$+\omega_d{}^2)\sin(\varphi_1-\varphi_2+t\omega_d)-A_2A_4e^{-\sigma t}L_{0010}L_{0101}(\sigma^3+5\sigma\omega_d{}^2+2\sigma^2t\omega_d{}^2$$

$$+2t\omega_d{}^4)\sin(\varphi_2+t\omega_d)-2A_1A_4e^{-\sigma t}K_{0101}L_{0010}\omega_d{}^2(2\sigma+\sigma^2t+t\omega_d{}^2)\sin(\varphi_1$$

$$+\varphi_2+t\omega_d)\}$$

(C. 27)

$$x_{1,2}^{bs2i2,u_2,u_2}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^2}\{4A_3K_{0001}\omega_d{}^3(A_2L_{0101}+A_1K_{0101}\cos(\varphi_1))$$

$$+2A_1A_4K_{0101}L_{0001}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-\varphi_2)$$

$$+4A_2A_4L_{0001}L_{0101}\omega_d{}^3\cos(\varphi_2)+2A_1A_4K_{0101}L_{0001}\omega_d(-\sigma^2+\omega_d{}^2)\cos(\varphi_1$$

$$+\varphi_2)+2A_2A_3e^{-\sigma t}K_{0001}L_{0101}\omega_d(\sigma^3 t-2\omega_d{}^2+\sigma t\omega_d{}^2)\cos(t\omega_d)$$

$$-A_1A_3e^{-\sigma t}K_{0001}K_{0101}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-t\omega_d)$$

$$-A_1A_4e^{-\sigma t}K_{0101}L_{0001}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-\varphi_2-t\omega_d)$$

$$-A_2A_4e^{-\sigma t}L_{0001}L_{0101}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_2-t\omega_d)$$

$$+A_1A_3e^{-\sigma t}K_{0001}K_{0101}\omega_d(\sigma^2+2\sigma^3 t-3\omega_d{}^2+2\sigma t\omega_d{}^2)\cos(\varphi_1+t\omega_d)$$

$$-A_1A_4e^{-\sigma t}K_{0101}L_{0001}\omega_d(\sigma^2+\omega_d{}^2)\cos(\varphi_1-\varphi_2+t\omega_d)$$

$$+A_2A_4e^{-\sigma t}L_{0001}L_{0101}\omega_d(\sigma^2+2\sigma^3 t-3\omega_d{}^2+2\sigma t\omega_d{}^2)\cos(\varphi_2+t\omega_d)$$

$$+2A_1A_4e^{-\sigma t}K_{0101}L_{0001}\omega_d(\sigma^2+\sigma^3 t-\omega_d{}^2+\sigma t\omega_d{}^2)\cos(\varphi_1+\varphi_2+t\omega_d)$$

$$+4A_1A_3K_{0001}K_{0101}\sigma\omega_d{}^2\sin(\varphi_1)+4A_2A_4L_{0001}L_{0101}\sigma\omega_d{}^2\sin(\varphi_2)$$

$$+4A_1A_4K_{0101}L_{0001}\sigma\omega_d{}^2\sin(\varphi_1+\varphi_2)-2A_2A_3e^{-\sigma t}K_{0001}L_{0101}(\sigma^3+3\sigma\omega_d{}^2$$

$$+\sigma^2 t\omega_d{}^2+t\omega_d{}^4)\sin(t\omega_d)+A_1A_3e^{-\sigma t}K_{0001}K_{0101}\sigma(\sigma^2+\omega_d{}^2)\sin(\varphi_1-t\omega_d)$$

$$+A_1A_4e^{-\sigma t}K_{0101}L_{0001}\sigma(\sigma^2+\omega_d{}^2)\sin(\varphi_1-\varphi_2-t\omega_d)$$

$$+A_2A_4e^{-\sigma t}L_{0001}L_{0101}\sigma(\sigma^2+\omega_d{}^2)\sin(\varphi_2-t\omega_d)-A_1A_3e^{-\sigma t}K_{0001}K_{0101}(\sigma^3$$

$$+5\sigma\omega_d{}^2+2\sigma^2 t\omega_d{}^2+2t\omega_d{}^4)\sin(\varphi_1+t\omega_d)-A_1A_4e^{-\sigma t}K_{0101}L_{0001}\sigma(\sigma^2$$

$$+\omega_d{}^2)\sin(\varphi_1-\varphi_2+t\omega_d)-A_2A_4e^{-\sigma t}L_{0001}L_{0101}(\sigma^3+5\sigma\omega_d{}^2+2\sigma^2 t\omega_d{}^2$$

$$+2t\omega_d{}^4)\sin(\varphi_2+t\omega_d)-2A_1A_4e^{-\sigma t}K_{0101}L_{0001}\omega_d{}^2(2\sigma+\sigma^2 t+t\omega_d{}^2)\sin(\varphi_1$$

$$+\varphi_2+t\omega_d)\}$$

(C. 28)

$$x_{1,2}^{qi1}(t) = x_{1,2}^{qi1,u_1,u_1}(t) \times u_1 u_1 \quad \text{(C.29)}$$

$$\boldsymbol{x_{1,2}^{qi1,u_1,u_1}(t)} = \frac{1}{\sigma^2 + {\omega_d}^2} e^{-\sigma t}\{A_2 e^{\sigma t} L_{0020}\omega_d + A_1 e^{\sigma t} K_{0020}\omega_d \cos(\varphi_1) - A_2 L_{0020}\omega_d \cos(t\omega_d) - A_1 K_{0020}\omega_d \cos(\varphi_1 + t\omega_d) + A_1 e^{\sigma t} K_{0020}\sigma \sin(\varphi_1) - A_2 L_{0020}\sigma \sin(t\omega_d) - A_1 K_{0020}\sigma \sin(\varphi_1 + t\omega_d)\}$$

(C.30)

$$x_{1,2}^{bi1i2}(t) = x_{1,2}^{bi1i2,u_1,u_2}(t) \times u_1 u_2 \tag{C.31}$$

$$\boldsymbol{x_{1,2}^{bi1i2,u_1,u_2}(t)} = \frac{1}{\sigma^2 + {\omega_d}^2} e^{-\sigma t}\{A_2 e^{\sigma t} L_{0011}\omega_d + A_1 e^{\sigma t} K_{0011}\omega_d \cos(\varphi_1)$$
$$- A_2 L_{0011}\omega_d \cos(t\omega_d) - A_1 K_{0011}\omega_d \cos(\varphi_1 + t\omega_d) + A_1 e^{\sigma t} K_{0011}\sigma \sin(\varphi_1)$$
$$- A_2 L_{0011}\sigma \sin(t\omega_d) - A_1 K_{0011}\sigma \sin(\varphi_1 + t\omega_d)\}$$

(C.32)

$$x_{1,2}^{qi2}(t) = x_{1,2}^{qi2,u_2,u_2}(t) \times u_2 u_2 \qquad (C.33)$$

$$\boldsymbol{x_{1,2}^{qi2,u_2,u_2}(t)} = \frac{1}{\sigma^2 + \omega_d{}^2} e^{-\sigma t}\{A_2 e^{\sigma t} L_{0002}\omega_d + A_1 e^{\sigma t} K_{0002}\omega_d \cos(\varphi_1) - A_2 L_{0002}\omega_d \cos(t\omega_d)$$
$$- A_1 K_{0002}\omega_d \cos(\varphi_1 + t\omega_d) + A_1 e^{\sigma t} K_{0002}\sigma \sin(\varphi_1) - A_2 L_{0002}\sigma \sin(t\omega_d)$$
$$- A_1 K_{0002}\sigma \sin(\varphi_1 + t\omega_d)\}$$

$$(C.34)$$

$$x_{2,2} = x_{2,2}^{qs1} + x_{2,2}^{bs1s2} + x_{2,2}^{qs2} + x_{2,2}^{bs1i1} + x_{2,2}^{bs2i1} + x_{2,2}^{bs1i2} + x_{2,2}^{bs2i2} + x_{2,2}^{qi1} + x_{2,2}^{bi1i2} + x_{2,2}^{qi2} \quad (C.35)$$

$$x_{2,2}^{qs1}(t) = x_{2,2}^{qs1,u_1,u_1}(t) \times u_1u_1 + x_{2,2}^{qs1,u_1,u_2}(t) \times u_1u_2 + x_{2,2}^{qs1,u_2,u_1}(t) \times u_2u_1 + x_{2,2}^{qs1,u_2,u_2}(t) \times u_2u_2$$

(C.36)

$$\boldsymbol{x_{2,2}^{qs1,u_1,u_1}(t)} = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{2A_3K_{2000}(A_1{}^2K_{0010}{}^2$$

$$+ A_2{}^2L_{0010}{}^2)\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4) + 4A_1A_2A_3K_{0010}K_{2000}L_{0010}\omega_d{}^2(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1) + 2A_1A_2A_4K_{0010}L_{0010}L_{2000}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(\varphi_1-\varphi_2) + 2A_4(A_1{}^2K_{0010}{}^2 + A_2{}^2L_{0010}{}^2)L_{2000}\omega_d{}^2(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_2) + 2A_1A_2A_4K_{0010}L_{0010}L_{2000}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(\varphi_1+\varphi_2) + 4A_3e^{\sigma t}K_{2000}\omega_d{}^2(\sigma^2+\omega_d{}^2)(-A_1{}^2K_{0010}{}^2(\sigma^2$$

$$+ 9\omega_d{}^2) + A_2{}^2L_{0010}{}^2(\sigma^3t-12\omega_d{}^2+9\sigma t\omega_d{}^2))\cos(t\omega_d)$$

$$- 2A_2{}^2A_3K_{2000}L_{0010}{}^2\omega_d{}^2(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(2t\omega_d)$$

$$+ A_2{}^2A_4L_{0010}{}^2L_{2000}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_2-2t\omega_d)$$

$$- 2A_1A_2A_3e^{\sigma t}K_{0010}K_{2000}L_{0010}(\sigma^6+7\sigma^4\omega_d{}^2+27\sigma^2\omega_d{}^4+21\omega_d{}^6)\cos(\varphi_1$$

$$- t\omega_d) - A_1{}^2A_3e^{\sigma t}K_{0010}{}^2K_{2000}(\sigma^2+\omega_d{}^2)^2(\sigma^2+3\omega_d{}^2)\cos(2\varphi_1-t\omega_d)$$

$$- A_1A_2A_4e^{\sigma t}K_{0010}L_{0010}L_{2000}(\sigma^6+3\sigma^4\omega_d{}^2+35\sigma^2\omega_d{}^4+33\omega_d{}^6)\cos(\varphi_1$$

$$- \varphi_2 - t\omega_d) - A_1{}^2A_4e^{\sigma t}K_{0010}{}^2L_{2000}(\sigma^2+\omega_d{}^2)^2(\sigma^2+3\omega_d{}^2)\cos(2\varphi_1-\varphi_2$$

$$- t\omega_d) - A_4e^{\sigma t}L_{2000}(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)(2A_2{}^2L_{0010}{}^2\omega_d{}^2$$

$$+ A_1{}^2K_{0010}{}^2(\sigma^2+\omega_d{}^2))\cos(\varphi_2-t\omega_d)$$

$$- A_1A_2A_4e^{\sigma t}K_{0010}L_{0010}L_{2000}(\sigma^2+\omega_d{}^2)^2(\sigma^2+9\omega_d{}^2)\cos(\varphi_1+\varphi_2-t\omega_d)$$

$$+ 2A_1A_2A_3e^{\sigma t}K_{0010}K_{2000}L_{0010}(\sigma^2-3\omega_d{}^2+4\sigma t\omega_d{}^2)(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(\varphi_1+t\omega_d) - 2A_1{}^2A_3K_{0010}{}^2K_{2000}\omega_d{}^2(\sigma^4-12\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(2(\varphi_1+t\omega_d)) + A_1{}^2A_3e^{\sigma t}K_{0010}{}^2K_{2000}(\sigma^2-\omega_d{}^2+4\sigma t\omega_d{}^2)(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(2\varphi_1+t\omega_d) + A_1A_2A_4e^{\sigma t}K_{0010}L_{0010}L_{2000}(\sigma^6$$

$$+ 7\sigma^4\omega_d{}^2-21\sigma^2\omega_d{}^4-27\omega_d{}^6)\cos(\varphi_1-\varphi_2+t\omega_d)$$

$$+ A_1{}^2A_4e^{\sigma t}K_{0010}{}^2L_{2000}(\sigma^6+9\sigma^4\omega_d{}^2-\sigma^2\omega_d{}^4-9\omega_d{}^6)\cos(2\varphi_1-\varphi_2+t\omega_d)$$

$$+ A_4 e^{\sigma t} L_{2000}(\sigma^2 + \omega_d{}^2)(2A_2{}^2 L_{0010}{}^2 \omega_d{}^2(\sigma^2 + 2\sigma^3 t - 15\omega_d{}^2 + 18\sigma t\omega_d{}^2)$$

$$+ A_1{}^2 K_{0010}{}^2(\sigma^4 + 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4))\cos(\varphi_2 + t\omega_d)$$

$$+ A_1 A_2 A_4 e^{\sigma t} K_{0010} L_{0010} L_{2000}(\sigma^2 - 3\omega_d{}^2 + 8\sigma t\omega_d{}^2)(\sigma^4 + 10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(\varphi_1 + \varphi_2 + t\omega_d) + 4A_1{}^2 A_4 e^{\sigma t} K_{0010}{}^2 L_{2000}\sigma t\omega_d{}^2(\sigma^4 + 10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(2\varphi_1 + \varphi_2 + t\omega_d) - 4A_1 A_2 A_3 K_{0010} K_{2000} L_{0010}\omega_d{}^2(\sigma^4 - 12\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(\varphi_1 + 2t\omega_d) + 2A_1 A_2 A_4 K_{0010} L_{0010} L_{2000}\omega_d{}^2(-5\sigma^4 - 2\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(\varphi_1 - \varphi_2 + 2t\omega_d) + A_1{}^2 A_4 K_{0010}{}^2 L_{2000}\omega_d{}^2(-5\sigma^4 - 2\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(2\varphi_1 - \varphi_2 + 2t\omega_d) + A_2{}^2 A_4 L_{0010}{}^2 L_{2000}\omega_d{}^2(3\sigma^4 + 26\sigma^2\omega_d{}^2$$

$$- 9\omega_d{}^4)\cos(\varphi_2 + 2t\omega_d) + 2A_1 A_2 A_4 K_{0010} L_{0010} L_{2000}\omega_d{}^2(3\sigma^4 + 26\sigma^2\omega_d{}^2$$

$$- 9\omega_d{}^4)\cos(\varphi_1 + \varphi_2 + 2t\omega_d) + A_1{}^2 A_4 K_{0010}{}^2 L_{2000}\omega_d{}^2(3\sigma^4 + 26\sigma^2\omega_d{}^2$$

$$- 9\omega_d{}^4)\cos(2\varphi_1 + \varphi_2 + 2t\omega_d) + 2A_1 A_2 A_4 K_{0010} L_{0010} L_{2000}\sigma\omega_d(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 - \varphi_2) - 2A_4(A_1{}^2 K_{0010}{}^2$$

$$+ A_2{}^2 L_{0010}{}^2) L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_2) + 4e^{2\sigma t}\omega_d(\sigma^2$$

$$+ 9\omega_d{}^2)(A_2 L_{0010}\omega_d + A_1 K_{0010}\omega_d\cos(\varphi_1) + A_1 K_{0010}\sigma\sin(\varphi_1))^2(A_3 K_{2000}\omega_d$$

$$+ A_4 L_{2000}\omega_d\cos(\varphi_2) + A_4 L_{2000}\sigma\sin(\varphi_2)) - 2A_1 A_2 A_4 K_{0010} L_{0010} L_{2000}\sigma\omega_d(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 + \varphi_2) - 2A_3 e^{\sigma t} K_{2000}\omega_d(\sigma^2$$

$$+ \omega_d{}^2)(A_1{}^2 K_{0010}{}^2 t(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4) + 2A_2{}^2 L_{0010}{}^2(\sigma^3 + 5\sigma\omega_d{}^2$$

$$+ \sigma^2 t\omega_d{}^2 + 9t\omega_d{}^4))\sin(t\omega_d) + 4A_2{}^2 A_3 K_{2000} L_{0010}{}^2\sigma\omega_d{}^3(3\sigma^2$$

$$- 5\omega_d{}^2)\sin(2t\omega_d) + A_2{}^2 A_4 L_{0010}{}^2 L_{2000}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_2$$

$$- 2t\omega_d) + 2A_1 A_2 A_3 e^{\sigma t} K_{0010} K_{2000} L_{0010}\omega_d(\sigma^2 + \omega_d{}^2)^2(2\sigma + \sigma^2 t$$

$$+ 9t\omega_d{}^2)\sin(\varphi_1 - t\omega_d) + 2A_1{}^2 A_3 e^{\sigma t} K_{0010}{}^2 K_{2000}\sigma\omega_d(\sigma^2 + \omega_d{}^2)^2\sin(2\varphi_1$$

$$- t\omega_d) + 2A_1 A_2 A_4 e^{\sigma t} K_{0010} L_{0010} L_{2000}\omega_d(\sigma^2 + \omega_d{}^2)^2(2\sigma + \sigma^2 t$$

$$+ 9t\omega_d{}^2)\sin(\varphi_1 - \varphi_2 - t\omega_d)$$

$$
\begin{aligned}
&+ 2{A_1}^2A_4e^{\sigma t}{K_{0010}}^2L_{2000}\sigma\omega_d(\sigma^2 + {\omega_d}^2)^2\sin(2\varphi_1 - \varphi_2 - t\omega_d) \\
&+ 2{A_2}^2A_4e^{\sigma t}{L_{0010}}^2L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_2 - t\omega_d) \\
&+ 2A_1A_2A_3e^{\sigma t}K_{0010}K_{2000}L_{0010}\omega_d(-2\sigma + \sigma^2t - 3t{\omega_d}^2)(\sigma^4 + 10\sigma^2{\omega_d}^2 \\
&+ 9{\omega_d}^4)\sin(\varphi_1 + t\omega_d) + 4{A_1}^2A_3{K_{0010}}^2K_{2000}\sigma{\omega_d}^3(3\sigma^2 - 5{\omega_d}^2)\sin(2(\varphi_1 \\
&+ t\omega_d)) - 2{A_1}^2A_3e^{\sigma t}{K_{0010}}^2K_{2000}\omega_d(\sigma - \sigma^2t + t{\omega_d}^2)(\sigma^4 + 10\sigma^2{\omega_d}^2 \\
&+ 9{\omega_d}^4)\sin(2\varphi_1 + t\omega_d) - 4A_1A_2A_4e^{\sigma t}K_{0010}L_{0010}L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 \\
&+ 9{\omega_d}^4)\sin(\varphi_1 - \varphi_2 + t\omega_d) - 2{A_1}^2A_4e^{\sigma t}{K_{0010}}^2L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 \\
&+ 9{\omega_d}^4)\sin(2\varphi_1 - \varphi_2 + t\omega_d) - 2A_4e^{\sigma t}L_{2000}\omega_d(\sigma^2 + {\omega_d}^2)({A_1}^2{K_{0010}}^2t(\sigma^4 \\
&+ 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4) + {A_2}^2{L_{0010}}^2(\sigma^3 + \sigma{\omega_d}^2 + 2\sigma^2t{\omega_d}^2 + 18t{\omega_d}^4))\sin(\varphi_2 \\
&+ t\omega_d) + 2A_1A_2A_4e^{\sigma t}K_{0010}L_{0010}L_{2000}t\omega_d(\sigma^6 + 7\sigma^4{\omega_d}^2 - 21\sigma^2{\omega_d}^4 \\
&- 27{\omega_d}^6)\sin(\varphi_1 + \varphi_2 + t\omega_d) + 2{A_1}^2A_4e^{\sigma t}{K_{0010}}^2L_{2000}t\omega_d(\sigma^6 + 9\sigma^4{\omega_d}^2 \\
&- \sigma^2{\omega_d}^4 - 9{\omega_d}^6)\sin(2\varphi_1 + \varphi_2 + t\omega_d) + 8A_1A_2A_3K_{0010}K_{2000}L_{0010}\sigma{\omega_d}^3(3\sigma^2 \\
&- 5{\omega_d}^2)\sin(\varphi_1 + 2t\omega_d) - 2A_1A_2A_4K_{0010}L_{0010}L_{2000}\sigma\omega_d(\sigma^4 - 6\sigma^2{\omega_d}^2 \\
&- 7{\omega_d}^4)\sin(\varphi_1 - \varphi_2 + 2t\omega_d) + {A_1}^2A_4{K_{0010}}^2L_{2000}\sigma\omega_d(-\sigma^4 + 6\sigma^2{\omega_d}^2 \\
&+ 7{\omega_d}^4)\sin(2\varphi_1 - \varphi_2 + 2t\omega_d) + {A_2}^2A_4{L_{0010}}^2L_{2000}\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2 \\
&- 27{\omega_d}^4)\sin(\varphi_2 + 2t\omega_d) + 2A_1A_2A_4K_{0010}L_{0010}L_{2000}\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2 \\
&- 27{\omega_d}^4)\sin(\varphi_1 + \varphi_2 + 2t\omega_d) + {A_1}^2A_4{K_{0010}}^2L_{2000}\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2 \\
&- 27{\omega_d}^4)\sin(2\varphi_1 + \varphi_2 + 2t\omega_d)\}
\end{aligned}
\tag{C.37}
$$

$$
\begin{aligned}
\boldsymbol{x}_{\boldsymbol{2,2}}^{\boldsymbol{qs1,u_1,u_2}}(\boldsymbol{t}) = {} & \frac{1}{8\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{4A_3K_{2000}(A_1{}^2K_{0001}K_{0010} \\
& + A_2{}^2L_{0001}L_{0010})\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4) + 4A_1A_2A_3K_{2000}(K_{0010}L_{0001} \\
& + K_{0001}L_{0010})\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1) + 2A_1A_2A_4(K_{0010}L_{0001} \\
& + K_{0001}L_{0010})L_{2000}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1-\varphi_2) \\
& + 4A_4(A_1{}^2K_{0001}K_{0010}+A_2{}^2L_{0001}L_{0010})L_{2000}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2 \\
& + 9\omega_d{}^4)\cos(\varphi_2) + 2A_1A_2A_4(K_{0010}L_{0001}+K_{0001}L_{0010})L_{2000}\omega_d{}^2(\sigma^4 \\
& + 10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1+\varphi_2) + 8A_3e^{\sigma t}K_{2000}\omega_d{}^2(\sigma^2 \\
& + \omega_d{}^2)(-A_1{}^2K_{0001}K_{0010}(\sigma^2+9\omega_d{}^2) + A_2{}^2L_{0001}L_{0010}(\sigma^3t-12\omega_d{}^2 \\
& + 9\sigma t\omega_d{}^2))\cos(t\omega_d) - 4A_2{}^2A_3K_{2000}L_{0001}L_{0010}\omega_d{}^2(\sigma^4-12\sigma^2\omega_d{}^2 \\
& + 3\omega_d{}^4)\cos(2t\omega_d) + 2A_2{}^2A_4L_{0001}L_{0010}L_{2000}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2 \\
& + 3\omega_d{}^4)\cos(\varphi_2-2t\omega_d) - 2A_1A_2A_3e^{\sigma t}K_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^6 \\
& + 7\sigma^4\omega_d{}^2+27\sigma^2\omega_d{}^4+21\omega_d{}^6)\cos(\varphi_1-t\omega_d) \\
& - 2A_1{}^2A_3e^{\sigma t}K_{0001}K_{0010}K_{2000}(\sigma^2+\omega_d{}^2)^2(\sigma^2+3\omega_d{}^2)\cos(2\varphi_1-t\omega_d) \\
& - A_1A_2A_4e^{\sigma t}(K_{0010}L_{0001}+K_{0001}L_{0010})L_{2000}(\sigma^6+3\sigma^4\omega_d{}^2+35\sigma^2\omega_d{}^4 \\
& + 33\omega_d{}^6)\cos(\varphi_1-\varphi_2-t\omega_d) - 2A_1{}^2A_4e^{\sigma t}K_{0001}K_{0010}L_{2000}(\sigma^2+\omega_d{}^2)^2(\sigma^2 \\
& + 3\omega_d{}^2)\cos(2\varphi_1-\varphi_2-t\omega_d) - 2A_4e^{\sigma t}L_{2000}(\sigma^4+10\sigma^2\omega_d{}^2 \\
& + 9\omega_d{}^4)(2A_2{}^2L_{0001}L_{0010}\omega_d{}^2 + A_1{}^2K_{0001}K_{0010}(\sigma^2+\omega_d{}^2))\cos(\varphi_2-t\omega_d) \\
& - A_1A_2A_4e^{\sigma t}(K_{0010}L_{0001}+K_{0001}L_{0010})L_{2000}(\sigma^2+\omega_d{}^2)^2(\sigma^2+9\omega_d{}^2)\cos(\varphi_1 \\
& + \varphi_2-t\omega_d) + 2A_1A_2A_3e^{\sigma t}K_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^2-3\omega_d{}^2 \\
& + 4\sigma t\omega_d{}^2)(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1+t\omega_d) \\
& - 4A_1{}^2A_3K_{0001}K_{0010}K_{2000}\omega_d{}^2(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(2(\varphi_1+t\omega_d)) \\
& + 2A_1{}^2A_3e^{\sigma t}K_{0001}K_{0010}K_{2000}(\sigma^2-\omega_d{}^2+4\sigma t\omega_d{}^2)(\sigma^4+10\sigma^2\omega_d{}^2
\end{aligned}
$$

$$+ 9{\omega_d}^4)\cos(2\varphi_1 + t\omega_d) + A_1A_2A_4e^{\sigma t}(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}(\sigma^6$$

$$+ 7\sigma^4{\omega_d}^2 - 21\sigma^2{\omega_d}^4 - 27{\omega_d}^6)\cos(\varphi_1 - \varphi_2 + t\omega_d)$$

$$+ 2{A_1}^2A_4e^{\sigma t}K_{0001}K_{0010}L_{2000}(\sigma^6 + 9\sigma^4{\omega_d}^2 - \sigma^2{\omega_d}^4 - 9{\omega_d}^6)\cos(2\varphi_1 - \varphi_2$$

$$+ t\omega_d) + 2A_4e^{\sigma t}L_{2000}(\sigma^2 + {\omega_d}^2)(2{A_2}^2L_{0001}L_{0010}{\omega_d}^2(\sigma^2 + 2\sigma^3t - 15{\omega_d}^2$$

$$+ 18\sigma t{\omega_d}^2) + {A_1}^2K_{0001}K_{0010}(\sigma^4 + 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4))\cos(\varphi_2 + t\omega_d)$$

$$+ A_1A_2A_4e^{\sigma t}(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}(\sigma^2 - 3{\omega_d}^2 + 8\sigma t{\omega_d}^2)(\sigma^4$$

$$+ 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\cos(\varphi_1 + \varphi_2 + t\omega_d)$$

$$+ 8{A_1}^2A_4e^{\sigma t}K_{0001}K_{0010}L_{2000}\sigma t{\omega_d}^2(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\cos(2\varphi_1 + \varphi_2$$

$$+ t\omega_d) - 4A_1A_2A_3K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010}){\omega_d}^2(\sigma^4 - 12\sigma^2{\omega_d}^2$$

$$+ 3{\omega_d}^4)\cos(\varphi_1 + 2t\omega_d) + 2A_1A_2A_4(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{2000}{\omega_d}^2(-5\sigma^4 - 2\sigma^2{\omega_d}^2 + 3{\omega_d}^4)\cos(\varphi_1 - \varphi_2 + 2t\omega_d)$$

$$+ 2{A_1}^2A_4K_{0001}K_{0010}L_{2000}{\omega_d}^2(-5\sigma^4 - 2\sigma^2{\omega_d}^2 + 3{\omega_d}^4)\cos(2\varphi_1 - \varphi_2$$

$$+ 2t\omega_d) + 2{A_2}^2A_4L_{0001}L_{0010}L_{2000}{\omega_d}^2(3\sigma^4 + 26\sigma^2{\omega_d}^2 - 9{\omega_d}^4)\cos(\varphi_2$$

$$+ 2t\omega_d) - 2A_1A_2A_4(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}{\omega_d}^2(-3\sigma^4 - 26\sigma^2{\omega_d}^2$$

$$+ 9{\omega_d}^4)\cos(\varphi_1 + \varphi_2 + 2t\omega_d) + 2{A_1}^2A_4K_{0001}K_{0010}L_{2000}{\omega_d}^2(3\sigma^4 + 26\sigma^2{\omega_d}^2$$

$$- 9{\omega_d}^4)\cos(2\varphi_1 + \varphi_2 + 2t\omega_d) + 2A_1A_2A_4(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_1 - \varphi_2)$$

$$- 4A_4({A_1}^2K_{0001}K_{0010} + {A_2}^2L_{0001}L_{0010})L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2$$

$$+ 9{\omega_d}^4)\sin(\varphi_2) + 8e^{2\sigma t}\omega_d(\sigma^2 + 9{\omega_d}^2)(A_2L_{0001}\omega_d + A_1K_{0001}\omega_d\cos(\varphi_1)$$

$$+ A_1K_{0001}\sigma\sin(\varphi_1))(A_2L_{0010}\omega_d + A_1K_{0010}\omega_d\cos(\varphi_1)$$

$$+ A_1K_{0010}\sigma\sin(\varphi_1))(A_3K_{2000}\omega_d + A_4L_{2000}\omega_d\cos(\varphi_2) + A_4L_{2000}\sigma\sin(\varphi_2))$$

$$- 2A_1A_2A_4(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2$$

$$+ 9{\omega_d}^4)\sin(\varphi_1 + \varphi_2) - 4A_3e^{\sigma t}K_{2000}\omega_d(\sigma^2 + {\omega_d}^2)({A_1}^2K_{0001}K_{0010}t(\sigma^4$$

$$+ 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4) + 2{A_2}^2 L_{0001}L_{0010}(\sigma^3 + 5\sigma{\omega_d}^2 + \sigma^2 t{\omega_d}^2$$

$$+ 9t{\omega_d}^4))\sin(t\omega_d) + 8{A_2}^2 A_3 K_{2000}L_{0001}L_{0010}\sigma{\omega_d}^3(3\sigma^2 - 5{\omega_d}^2)\sin(2t\omega_d)$$

$$+ 2{A_2}^2 A_4 L_{0001}L_{0010}L_{2000}\sigma\omega_d(\sigma^4 - 6\sigma^2{\omega_d}^2 - 7{\omega_d}^4)\sin(\varphi_2 - 2t\omega_d)$$

$$+ 2A_1A_2A_3e^{\sigma t}K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010})\omega_d(\sigma^2 + {\omega_d}^2)^2(2\sigma + \sigma^2 t$$

$$+ 9t{\omega_d}^2)\sin(\varphi_1 - t\omega_d)$$

$$+ 4{A_1}^2 A_3 e^{\sigma t}K_{0001}K_{0010}K_{2000}\sigma\omega_d(\sigma^2 + {\omega_d}^2)^2\sin(2\varphi_1 - t\omega_d)$$

$$+ 2A_1A_2A_4e^{\sigma t}(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}\omega_d(\sigma^2 + {\omega_d}^2)^2(2\sigma + \sigma^2 t$$

$$+ 9t{\omega_d}^2)\sin(\varphi_1 - \varphi_2 - t\omega_d)$$

$$+ 4{A_1}^2 A_4 e^{\sigma t}K_{0001}K_{0010}L_{2000}\sigma\omega_d(\sigma^2 + {\omega_d}^2)^2\sin(2\varphi_1 - \varphi_2 - t\omega_d)$$

$$+ 4{A_2}^2 A_4 e^{\sigma t}L_{0001}L_{0010}L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_2 - t\omega_d)$$

$$+ 2A_1A_2A_3e^{\sigma t}K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010})\omega_d(-2\sigma + \sigma^2 t - 3t{\omega_d}^2)(\sigma^4$$

$$+ 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_1 + t\omega_d) + 8{A_1}^2 A_3 K_{0001}K_{0010}K_{2000}\sigma{\omega_d}^3(3\sigma^2$$

$$- 5{\omega_d}^2)\sin(2(\varphi_1 + t\omega_d)) - 4{A_1}^2 A_3 e^{\sigma t}K_{0001}K_{0010}K_{2000}\omega_d(\sigma - \sigma^2 t$$

$$+ t{\omega_d}^2)(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(2\varphi_1 + t\omega_d) - 4A_1A_2A_4e^{\sigma t}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_1 - \varphi_2 + t\omega_d)$$

$$- 4{A_1}^2 A_4 e^{\sigma t}K_{0001}K_{0010}L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(2\varphi_1 - \varphi_2$$

$$+ t\omega_d) - 4A_4e^{\sigma t}L_{2000}\omega_d(\sigma^2 + {\omega_d}^2)({A_1}^2 K_{0001}K_{0010}t(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)$$

$$+ {A_2}^2 L_{0001}L_{0010}(\sigma^3 + \sigma{\omega_d}^2 + 2\sigma^2 t{\omega_d}^2 + 18t{\omega_d}^4))\sin(\varphi_2 + t\omega_d)$$

$$+ 2A_1A_2A_4e^{\sigma t}(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}t\omega_d(\sigma^6 + 7\sigma^4{\omega_d}^2 - 21\sigma^2{\omega_d}^4$$

$$- 27{\omega_d}^6)\sin(\varphi_1 + \varphi_2 + t\omega_d) + 4{A_1}^2 A_4 e^{\sigma t}K_{0001}K_{0010}L_{2000}t\omega_d(\sigma^6 + 9\sigma^4{\omega_d}^2$$

$$- \sigma^2{\omega_d}^4 - 9{\omega_d}^6)\sin(2\varphi_1 + \varphi_2 + t\omega_d) + 8A_1A_2A_3K_{2000}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})\sigma{\omega_d}^3(3\sigma^2 - 5{\omega_d}^2)\sin(\varphi_1 + 2t\omega_d) - 2A_1A_2A_4(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{2000}\sigma\omega_d(\sigma^4 - 6\sigma^2{\omega_d}^2 - 7{\omega_d}^4)\sin(\varphi_1 - \varphi_2 + 2t\omega_d)$$

$$- 2{A_1}^2 A_4 K_{0001} K_{0010} L_{2000} \sigma\omega_d (\sigma^4 - 6\sigma^2 {\omega_d}^2 - 7{\omega_d}^4)\sin(2\varphi_1 - \varphi_2 + 2t\omega_d)$$

$$+ 2{A_2}^2 A_4 L_{0001} L_{0010} L_{2000} \sigma\omega_d (\sigma^4 + 6\sigma^2 {\omega_d}^2 - 27{\omega_d}^4)\sin(\varphi_2 + 2t\omega_d)$$

$$+ 2A_1 A_2 A_4 (K_{0010} L_{0001} + K_{0001} L_{0010}) L_{2000} \sigma\omega_d (\sigma^4 + 6\sigma^2 {\omega_d}^2$$

$$- 27{\omega_d}^4)\sin(\varphi_1 + \varphi_2 + 2t\omega_d) + 2{A_1}^2 A_4 K_{0001} K_{0010} L_{2000} \sigma\omega_d (\sigma^4 + 6\sigma^2 {\omega_d}^2$$

$$- 27{\omega_d}^4)\sin(2\varphi_1 + \varphi_2 + 2t\omega_d)\}$$

$$(C.38)$$

$$x_{2,2}^{qs1,u_2,u_1}(t) = \frac{1}{8\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{4A_3K_{2000}(A_1{}^2K_{0001}K_{0010}$$

$$+ A_2{}^2L_{0001}L_{0010})\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4) + 4A_1A_2A_3K_{2000}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1) + 2A_1A_2A_4(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{2000}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1-\varphi_2)$$

$$+ 4A_4(A_1{}^2K_{0001}K_{0010}+A_2{}^2L_{0001}L_{0010})L_{2000}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(\varphi_2) + 2A_1A_2A_4(K_{0010}L_{0001}+K_{0001}L_{0010})L_{2000}\omega_d{}^2(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1+\varphi_2) + 8A_3e^{\sigma t}K_{2000}\omega_d{}^2(\sigma^2$$

$$+ \omega_d{}^2)(-A_1{}^2K_{0001}K_{0010}(\sigma^2+9\omega_d{}^2) + A_2{}^2L_{0001}L_{0010}(\sigma^3 t - 12\omega_d{}^2$$

$$+ 9\sigma t\omega_d{}^2))\cos(t\omega_d) - 4A_2{}^2A_3K_{2000}L_{0001}L_{0010}\omega_d{}^2(\sigma^4-12\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(2t\omega_d) + 2A_2{}^2A_4L_{0001}L_{0010}L_{2000}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(\varphi_2-2t\omega_d) - 2A_1A_2A_3e^{\sigma t}K_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^6$$

$$+ 7\sigma^4\omega_d{}^2+27\sigma^2\omega_d{}^4+21\omega_d{}^6)\cos(\varphi_1-t\omega_d)$$

$$- 2A_1{}^2A_3e^{\sigma t}K_{0001}K_{0010}K_{2000}(\sigma^2+\omega_d{}^2)^2(\sigma^2+3\omega_d{}^2)\cos(2\varphi_1-t\omega_d)$$

$$- A_1A_2A_4e^{\sigma t}(K_{0010}L_{0001}+K_{0001}L_{0010})L_{2000}(\sigma^6+3\sigma^4\omega_d{}^2+35\sigma^2\omega_d{}^4$$

$$+ 33\omega_d{}^6)\cos(\varphi_1-\varphi_2-t\omega_d) - 2A_1{}^2A_4e^{\sigma t}K_{0001}K_{0010}L_{2000}(\sigma^2+\omega_d{}^2)^2(\sigma^2$$

$$+ 3\omega_d{}^2)\cos(2\varphi_1-\varphi_2-t\omega_d) - 2A_4e^{\sigma t}L_{2000}(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)(2A_2{}^2L_{0001}L_{0010}\omega_d{}^2 + A_1{}^2K_{0001}K_{0010}(\sigma^2+\omega_d{}^2))\cos(\varphi_2-t\omega_d)$$

$$- A_1A_2A_4e^{\sigma t}(K_{0010}L_{0001}+K_{0001}L_{0010})L_{2000}(\sigma^2+\omega_d{}^2)^2(\sigma^2+9\omega_d{}^2)\cos(\varphi_1$$

$$+ \varphi_2-t\omega_d) + 2A_1A_2A_3e^{\sigma t}K_{2000}(K_{0010}L_{0001}+K_{0001}L_{0010})(\sigma^2-3\omega_d{}^2$$

$$+ 4\sigma t\omega_d{}^2)(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1+t\omega_d)$$

$$- 4A_1{}^2A_3K_{0001}K_{0010}K_{2000}\omega_d{}^2(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(2(\varphi_1+t\omega_d))$$

$$+ 2A_1{}^2A_3e^{\sigma t}K_{0001}K_{0010}K_{2000}(\sigma^2-\omega_d{}^2+4\sigma t\omega_d{}^2)(\sigma^4+10\sigma^2\omega_d{}^2$$

$+ 9{\omega_d}^4)\cos(2\varphi_1 + t\omega_d) + A_1A_2A_4e^{\sigma t}(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}(\sigma^6$

$+ 7\sigma^4{\omega_d}^2 - 21\sigma^2{\omega_d}^4 - 27{\omega_d}^6)\cos(\varphi_1 - \varphi_2 + t\omega_d)$

$+ 2{A_1}^2A_4e^{\sigma t}K_{0001}K_{0010}L_{2000}(\sigma^6 + 9\sigma^4{\omega_d}^2 - \sigma^2{\omega_d}^4 - 9{\omega_d}^6)\cos(2\varphi_1 - \varphi_2$

$+ t\omega_d) + 2A_4e^{\sigma t}L_{2000}(\sigma^2 + {\omega_d}^2)(2{A_2}^2L_{0001}L_{0010}{\omega_d}^2(\sigma^2 + 2\sigma^3t - 15{\omega_d}^2$

$+ 18\sigma t{\omega_d}^2) + {A_1}^2K_{0001}K_{0010}(\sigma^4 + 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4))\cos(\varphi_2 + t\omega_d)$

$+ A_1A_2A_4e^{\sigma t}(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}(\sigma^2 - 3{\omega_d}^2 + 8\sigma t{\omega_d}^2)(\sigma^4$

$+ 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\cos(\varphi_1 + \varphi_2 + t\omega_d)$

$+ 8{A_1}^2A_4e^{\sigma t}K_{0001}K_{0010}L_{2000}\sigma t{\omega_d}^2(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\cos(2\varphi_1 + \varphi_2$

$+ t\omega_d) - 4A_1A_2A_3K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010}){\omega_d}^2(\sigma^4 - 12\sigma^2{\omega_d}^2$

$+ 3{\omega_d}^4)\cos(\varphi_1 + 2t\omega_d) + 2A_1A_2A_4(K_{0010}L_{0001}$

$+ K_{0001}L_{0010})L_{2000}{\omega_d}^2(-5\sigma^4 - 2\sigma^2{\omega_d}^2 + 3{\omega_d}^4)\cos(\varphi_1 - \varphi_2 + 2t\omega_d)$

$+ 2{A_1}^2A_4K_{0001}K_{0010}L_{2000}{\omega_d}^2(-5\sigma^4 - 2\sigma^2{\omega_d}^2 + 3{\omega_d}^4)\cos(2\varphi_1 - \varphi_2$

$+ 2t\omega_d) + 2{A_2}^2A_4L_{0001}L_{0010}L_{2000}{\omega_d}^2(3\sigma^4 + 26\sigma^2{\omega_d}^2 - 9{\omega_d}^4)\cos(\varphi_2$

$+ 2t\omega_d) - 2A_1A_2A_4(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}{\omega_d}^2(-3\sigma^4 - 26\sigma^2{\omega_d}^2$

$+ 9{\omega_d}^4)\cos(\varphi_1 + \varphi_2 + 2t\omega_d) + 2{A_1}^2A_4K_{0001}K_{0010}L_{2000}{\omega_d}^2(3\sigma^4 + 26\sigma^2{\omega_d}^2$

$- 9{\omega_d}^4)\cos(2\varphi_1 + \varphi_2 + 2t\omega_d) + 2A_1A_2A_4(K_{0010}L_{0001}$

$+ K_{0001}L_{0010})L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_1 - \varphi_2)$

$- 4A_4({A_1}^2K_{0001}K_{0010} + {A_2}^2L_{0001}L_{0010})L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2$

$+ 9{\omega_d}^4)\sin(\varphi_2) + 8e^{2\sigma t}\omega_d(\sigma^2 + 9{\omega_d}^2)(A_2L_{0001}\omega_d + A_1K_{0001}\omega_d\cos(\varphi_1)$

$+ A_1K_{0001}\sigma\sin(\varphi_1))(A_2L_{0010}\omega_d + A_1K_{0010}\omega_d\cos(\varphi_1)$

$+ A_1K_{0010}\sigma\sin(\varphi_1))(A_3K_{2000}\omega_d + A_4L_{2000}\omega_d\cos(\varphi_2) + A_4L_{2000}\sigma\sin(\varphi_2))$

$- 2A_1A_2A_4(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2$

$+ 9{\omega_d}^4)\sin(\varphi_1 + \varphi_2) - 4A_3e^{\sigma t}K_{2000}\omega_d(\sigma^2 + {\omega_d}^2)({A_1}^2K_{0001}K_{0010}t(\sigma^4$

$$+ 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4) + 2{A_2}^2L_{0001}L_{0010}(\sigma^3 + 5\sigma{\omega_d}^2 + \sigma^2t{\omega_d}^2$$

$$+ 9t{\omega_d}^4))\sin(t\omega_d) + 8{A_2}^2A_3K_{2000}L_{0001}L_{0010}\sigma{\omega_d}^3(3\sigma^2 - 5{\omega_d}^2)\sin(2t\omega_d)$$

$$+ 2{A_2}^2A_4L_{0001}L_{0010}L_{2000}\sigma\omega_d(\sigma^4 - 6\sigma^2{\omega_d}^2 - 7{\omega_d}^4)\sin(\varphi_2 - 2t\omega_d)$$

$$+ 2A_1A_2A_3e^{\sigma t}K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010})\omega_d(\sigma^2 + {\omega_d}^2)^2(2\sigma + \sigma^2t$$

$$+ 9t{\omega_d}^2)\sin(\varphi_1 - t\omega_d)$$

$$+ 4{A_1}^2A_3e^{\sigma t}K_{0001}K_{0010}K_{2000}\sigma\omega_d(\sigma^2 + {\omega_d}^2)^2\sin(2\varphi_1 - t\omega_d)$$

$$+ 2A_1A_2A_4e^{\sigma t}(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}\omega_d(\sigma^2 + {\omega_d}^2)^2(2\sigma + \sigma^2t$$

$$+ 9t{\omega_d}^2)\sin(\varphi_1 - \varphi_2 - t\omega_d)$$

$$+ 4{A_1}^2A_4e^{\sigma t}K_{0001}K_{0010}L_{2000}\sigma\omega_d(\sigma^2 + {\omega_d}^2)^2\sin(2\varphi_1 - \varphi_2 - t\omega_d)$$

$$+ 4{A_2}^2A_4e^{\sigma t}L_{0001}L_{0010}L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_2 - t\omega_d)$$

$$+ 2A_1A_2A_3e^{\sigma t}K_{2000}(K_{0010}L_{0001} + K_{0001}L_{0010})\omega_d(-2\sigma + \sigma^2t - 3t{\omega_d}^2)(\sigma^4$$

$$+ 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_1 + t\omega_d) + 8{A_1}^2A_3K_{0001}K_{0010}K_{2000}\sigma{\omega_d}^3(3\sigma^2$$

$$- 5{\omega_d}^2)\sin(2(\varphi_1 + t\omega_d)) - 4{A_1}^2A_3e^{\sigma t}K_{0001}K_{0010}K_{2000}\omega_d(\sigma - \sigma^2t$$

$$+ t{\omega_d}^2)(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(2\varphi_1 + t\omega_d) - 4A_1A_2A_4e^{\sigma t}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_1 - \varphi_2 + t\omega_d)$$

$$- 4{A_1}^2A_4e^{\sigma t}K_{0001}K_{0010}L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(2\varphi_1 - \varphi_2$$

$$+ t\omega_d) - 4A_4e^{\sigma t}L_{2000}\omega_d(\sigma^2 + {\omega_d}^2)({A_1}^2K_{0001}K_{0010}t(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)$$

$$+ {A_2}^2L_{0001}L_{0010}(\sigma^3 + \sigma{\omega_d}^2 + 2\sigma^2t{\omega_d}^2 + 18t{\omega_d}^4))\sin(\varphi_2 + t\omega_d)$$

$$+ 2A_1A_2A_4e^{\sigma t}(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}t\omega_d(\sigma^6 + 7\sigma^4{\omega_d}^2 - 21\sigma^2{\omega_d}^4$$

$$- 27{\omega_d}^6)\sin(\varphi_1 + \varphi_2 + t\omega_d) + 4{A_1}^2A_4e^{\sigma t}K_{0001}K_{0010}L_{2000}t\omega_d(\sigma^6 + 9\sigma^4{\omega_d}^2$$

$$- \sigma^2{\omega_d}^4 - 9{\omega_d}^6)\sin(2\varphi_1 + \varphi_2 + t\omega_d) + 8A_1A_2A_3K_{2000}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})\sigma{\omega_d}^3(3\sigma^2 - 5{\omega_d}^2)\sin(\varphi_1 + 2t\omega_d) - 2A_1A_2A_4(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{2000}\sigma\omega_d(\sigma^4 - 6\sigma^2{\omega_d}^2 - 7{\omega_d}^4)\sin(\varphi_1 - \varphi_2 + 2t\omega_d)$$

$$- 2{A_1}^2A_4K_{0001}K_{0010}L_{2000}\sigma\omega_d(\sigma^4 - 6\sigma^2{\omega_d}^2 - 7{\omega_d}^4)\sin(2\varphi_1 - \varphi_2 + 2t\omega_d)$$

$$+ 2{A_2}^2A_4L_{0001}L_{0010}L_{2000}\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4)\sin(\varphi_2 + 2t\omega_d)$$

$$+ 2A_1A_2A_4(K_{0010}L_{0001} + K_{0001}L_{0010})L_{2000}\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2$$

$$- 27{\omega_d}^4)\sin(\varphi_1 + \varphi_2 + 2t\omega_d) + 2{A_1}^2A_4K_{0001}K_{0010}L_{2000}\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2$$

$$- 27{\omega_d}^4)\sin(2\varphi_1 + \varphi_2 + 2t\omega_d)\}$$

(C. 39)

$$x_{2,2}^{qs1,u_2,u_2}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{2A_3K_{2000}(A_1{}^2K_{0001}{}^2$$

$$+ A_2{}^2L_{0001}{}^2)\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4) + 4A_1A_2A_3K_{0001}K_{2000}L_{0001}\omega_d{}^2(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1) + 2A_1A_2A_4K_{0001}L_{0001}L_{2000}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(\varphi_1-\varphi_2) + 2A_4(A_1{}^2K_{0001}{}^2 + A_2{}^2L_{0001}{}^2)L_{2000}\omega_d{}^2(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_2) + 2A_1A_2A_4K_{0001}L_{0001}L_{2000}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(\varphi_1+\varphi_2) + 4A_3e^{\sigma t}K_{2000}\omega_d{}^2(\sigma^2+\omega_d{}^2)(-A_1{}^2K_{0001}{}^2(\sigma^2$$

$$+ 9\omega_d{}^2) + A_2{}^2L_{0001}{}^2(\sigma^3 t - 12\omega_d{}^2 + 9\sigma t\omega_d{}^2))\cos(t\omega_d)$$

$$- 2A_2{}^2A_3K_{2000}L_{0001}{}^2\omega_d{}^2(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(2t\omega_d)$$

$$+ A_2{}^2A_4L_{0001}{}^2L_{2000}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_2-2t\omega_d)$$

$$- 2A_1A_2A_3e^{\sigma t}K_{0001}K_{2000}L_{0001}(\sigma^6+7\sigma^4\omega_d{}^2+27\sigma^2\omega_d{}^4+21\omega_d{}^6)\cos(\varphi_1$$

$$- t\omega_d) - A_1{}^2A_3e^{\sigma t}K_{0001}{}^2K_{2000}(\sigma^2+\omega_d{}^2)^2(\sigma^2+3\omega_d{}^2)\cos(2\varphi_1-t\omega_d)$$

$$- A_1A_2A_4e^{\sigma t}K_{0001}L_{0001}L_{2000}(\sigma^6+3\sigma^4\omega_d{}^2+35\sigma^2\omega_d{}^4+33\omega_d{}^6)\cos(\varphi_1$$

$$- \varphi_2 - t\omega_d) - A_1{}^2A_4e^{\sigma t}K_{0001}{}^2L_{2000}(\sigma^2+\omega_d{}^2)^2(\sigma^2+3\omega_d{}^2)\cos(2\varphi_1-\varphi_2$$

$$- t\omega_d) - A_4e^{\sigma t}L_{2000}(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)(2A_2{}^2L_{0001}{}^2\omega_d{}^2$$

$$+ A_1{}^2K_{0001}{}^2(\sigma^2+\omega_d{}^2))\cos(\varphi_2-t\omega_d)$$

$$- A_1A_2A_4e^{\sigma t}K_{0001}L_{0001}L_{2000}(\sigma^2+\omega_d{}^2)^2(\sigma^2+9\omega_d{}^2)\cos(\varphi_1+\varphi_2-t\omega_d)$$

$$+ 2A_1A_2A_3e^{\sigma t}K_{0001}K_{2000}L_{0001}(\sigma^2-3\omega_d{}^2+4\sigma t\omega_d{}^2)(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(\varphi_1+t\omega_d) - 2A_1{}^2A_3K_{0001}{}^2K_{2000}\omega_d{}^2(\sigma^4-12\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(2(\varphi_1+t\omega_d)) + A_1{}^2A_3e^{\sigma t}K_{0001}{}^2K_{2000}(\sigma^2-\omega_d{}^2+4\sigma t\omega_d{}^2)(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(2\varphi_1+t\omega_d) + A_1A_2A_4e^{\sigma t}K_{0001}L_{0001}L_{2000}(\sigma^6$$

$$+ 7\sigma^4\omega_d{}^2 - 21\sigma^2\omega_d{}^4 - 27\omega_d{}^6)\cos(\varphi_1-\varphi_2+t\omega_d)$$

$$+ A_1{}^2A_4e^{\sigma t}K_{0001}{}^2L_{2000}(\sigma^6+9\sigma^4\omega_d{}^2-\sigma^2\omega_d{}^4-9\omega_d{}^6)\cos(2\varphi_1-\varphi_2+t\omega_d)$$

$$+ A_4 e^{\sigma t} L_{2000}(\sigma^2 + \omega_d{}^2)(2A_2{}^2 L_{0001}{}^2 \omega_d{}^2(\sigma^2 + 2\sigma^3 t - 15\omega_d{}^2 + 18\sigma t\omega_d{}^2)$$

$$+ A_1{}^2 K_{0001}{}^2(\sigma^4 + 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4))\cos(\varphi_2 + t\omega_d)$$

$$+ A_1 A_2 A_4 e^{\sigma t} K_{0001} L_{0001} L_{2000}(\sigma^2 - 3\omega_d{}^2 + 8\sigma t\omega_d{}^2)(\sigma^4 + 10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(\varphi_1 + \varphi_2 + t\omega_d) + 4A_1{}^2 A_4 e^{\sigma t} K_{0001}{}^2 L_{2000}\sigma t\omega_d{}^2(\sigma^4 + 10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(2\varphi_1 + \varphi_2 + t\omega_d) - 4A_1 A_2 A_3 K_{0001} K_{2000} L_{0001}\omega_d{}^2(\sigma^4 - 12\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(\varphi_1 + 2t\omega_d) + 2A_1 A_2 A_4 K_{0001} L_{0001} L_{2000}\omega_d{}^2(-5\sigma^4 - 2\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(\varphi_1 - \varphi_2 + 2t\omega_d) + A_1{}^2 A_4 K_{0001}{}^2 L_{2000}\omega_d{}^2(-5\sigma^4 - 2\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(2\varphi_1 - \varphi_2 + 2t\omega_d) + A_2{}^2 A_4 L_{0001}{}^2 L_{2000}\omega_d{}^2(3\sigma^4 + 26\sigma^2\omega_d{}^2$$

$$- 9\omega_d{}^4)\cos(\varphi_2 + 2t\omega_d) + 2A_1 A_2 A_4 K_{0001} L_{0001} L_{2000}\omega_d{}^2(3\sigma^4 + 26\sigma^2\omega_d{}^2$$

$$- 9\omega_d{}^4)\cos(\varphi_1 + \varphi_2 + 2t\omega_d) + A_1{}^2 A_4 K_{0001}{}^2 L_{2000}\omega_d{}^2(3\sigma^4 + 26\sigma^2\omega_d{}^2$$

$$- 9\omega_d{}^4)\cos(2\varphi_1 + \varphi_2 + 2t\omega_d) + 2A_1 A_2 A_4 K_{0001} L_{0001} L_{2000}\sigma\omega_d(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 - \varphi_2) - 2A_4(A_1{}^2 K_{0001}{}^2$$

$$+ A_2{}^2 L_{0001}{}^2)L_{2000}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_2) + 4e^{2\sigma t}\omega_d(\sigma^2$$

$$+ 9\omega_d{}^2)(A_2 L_{0001}\omega_d + A_1 K_{0001}\omega_d\cos(\varphi_1) + A_1 K_{0001}\sigma\sin(\varphi_1))^2(A_3 K_{2000}\omega_d$$

$$+ A_4 L_{2000}\omega_d\cos(\varphi_2) + A_4 L_{2000}\sigma\sin(\varphi_2)) - 2A_1 A_2 A_4 K_{0001} L_{0001} L_{2000}\sigma\omega_d(\sigma^4$$

$$+ 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 + \varphi_2) - 2A_3 e^{\sigma t} K_{2000}\omega_d(\sigma^2$$

$$+ \omega_d{}^2)(A_1{}^2 K_{0001}{}^2 t(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4) + 2A_2{}^2 L_{0001}{}^2(\sigma^3 + 5\sigma\omega_d{}^2$$

$$+ \sigma^2 t\omega_d{}^2 + 9t\omega_d{}^4))\sin(t\omega_d) + 4A_2{}^2 A_3 K_{2000} L_{0001}{}^2\sigma\omega_d{}^3(3\sigma^2$$

$$- 5\omega_d{}^2)\sin(2t\omega_d) + A_2{}^2 A_4 L_{0001}{}^2 L_{2000}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_2$$

$$- 2t\omega_d) + 2A_1 A_2 A_3 e^{\sigma t} K_{0001} K_{2000} L_{0001}\omega_d(\sigma^2 + \omega_d{}^2)^2(2\sigma + \sigma^2 t$$

$$+ 9t\omega_d{}^2)\sin(\varphi_1 - t\omega_d) + 2A_1{}^2 A_3 e^{\sigma t} K_{0001}{}^2 K_{2000}\sigma\omega_d(\sigma^2 + \omega_d{}^2)^2\sin(2\varphi_1$$

$$- t\omega_d) + 2A_1 A_2 A_4 e^{\sigma t} K_{0001} L_{0001} L_{2000}\omega_d(\sigma^2 + \omega_d{}^2)^2(2\sigma + \sigma^2 t$$

$$+ 9t\omega_d{}^2)\sin(\varphi_1 - \varphi_2 - t\omega_d)$$

$$
\begin{aligned}
&+2A_1{}^2A_4e^{\sigma t}K_{0001}{}^2L_{2000}\sigma\omega_d(\sigma^2+\omega_d{}^2)^2\sin(2\varphi_1-\varphi_2-t\omega_d)\\
&+2A_2{}^2A_4e^{\sigma t}L_{0001}{}^2L_{2000}\sigma\omega_d(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\sin(\varphi_2-t\omega_d)\\
&+2A_1A_2A_3e^{\sigma t}K_{0001}K_{2000}L_{0001}\omega_d(-2\sigma+\sigma^2t-3t\omega_d{}^2)(\sigma^4+10\sigma^2\omega_d{}^2\\
&+9\omega_d{}^4)\sin(\varphi_1+t\omega_d)+4A_1{}^2A_3K_{0001}{}^2K_{2000}\sigma\omega_d{}^3(3\sigma^2-5\omega_d{}^2)\sin(2(\varphi_1\\
&+t\omega_d))-2A_1{}^2A_3e^{\sigma t}K_{0001}{}^2K_{2000}\omega_d(\sigma-\sigma^2t+t\omega_d{}^2)(\sigma^4+10\sigma^2\omega_d{}^2\\
&+9\omega_d{}^4)\sin(2\varphi_1+t\omega_d)-4A_1A_2A_4e^{\sigma t}K_{0001}L_{0001}L_{2000}\sigma\omega_d(\sigma^4+10\sigma^2\omega_d{}^2\\
&+9\omega_d{}^4)\sin(\varphi_1-\varphi_2+t\omega_d)-2A_1{}^2A_4e^{\sigma t}K_{0001}{}^2L_{2000}\sigma\omega_d(\sigma^4+10\sigma^2\omega_d{}^2\\
&+9\omega_d{}^4)\sin(2\varphi_1-\varphi_2+t\omega_d)-2A_4e^{\sigma t}L_{2000}\omega_d(\sigma^2+\omega_d{}^2)(A_1{}^2K_{0001}{}^2t(\sigma^4\\
&+10\sigma^2\omega_d{}^2+9\omega_d{}^4)+A_2{}^2L_{0001}{}^2(\sigma^3+\sigma\omega_d{}^2+2\sigma^2t\omega_d{}^2+18t\omega_d{}^4))\sin(\varphi_2\\
&+t\omega_d)+2A_1A_2A_4e^{\sigma t}K_{0001}L_{0001}L_{2000}t\omega_d(\sigma^6+7\sigma^4\omega_d{}^2-21\sigma^2\omega_d{}^4\\
&-27\omega_d{}^6)\sin(\varphi_1+\varphi_2+t\omega_d)+2A_1{}^2A_4e^{\sigma t}K_{0001}{}^2L_{2000}t\omega_d(\sigma^6+9\sigma^4\omega_d{}^2\\
&-\sigma^2\omega_d{}^4-9\omega_d{}^6)\sin(2\varphi_1+\varphi_2+t\omega_d)+8A_1A_2A_3K_{0001}K_{2000}L_{0001}\sigma\omega_d{}^3(3\sigma^2\\
&-5\omega_d{}^2)\sin(\varphi_1+2t\omega_d)-2A_1A_2A_4K_{0001}L_{0001}L_{2000}\sigma\omega_d(\sigma^4-6\sigma^2\omega_d{}^2\\
&-7\omega_d{}^4)\sin(\varphi_1-\varphi_2+2t\omega_d)+A_1{}^2A_4K_{0001}{}^2L_{2000}\sigma\omega_d(-\sigma^4+6\sigma^2\omega_d{}^2\\
&+7\omega_d{}^4)\sin(2\varphi_1-\varphi_2+2t\omega_d)+A_2{}^2A_4L_{0001}{}^2L_{2000}\sigma\omega_d(\sigma^4+6\sigma^2\omega_d{}^2\\
&-27\omega_d{}^4)\sin(\varphi_2+2t\omega_d)+2A_1A_2A_4K_{0001}L_{0001}L_{2000}\sigma\omega_d(\sigma^4+6\sigma^2\omega_d{}^2\\
&-27\omega_d{}^4)\sin(\varphi_1+\varphi_2+2t\omega_d)+A_1{}^2A_4K_{0001}{}^2L_{2000}\sigma\omega_d(\sigma^4+6\sigma^2\omega_d{}^2\\
&-27\omega_d{}^4)\sin(2\varphi_1+\varphi_2+2t\omega_d)\}
\end{aligned}
\tag{C.40}
$$

$$x_{2,2}^{bs1s2}(t) = x_{2,2}^{bs1s2,u_1,u_1}(t) \times u_1u_1 + x_{2,2}^{bs1s2,u_1,u_2}(t) \times u_1u_2 + x_{2,2}^{bs1s2,u_2,u_1}(t) \times u_2u_1 + x_{2,2}^{bs1s2,u_2,u_2}(t) \times u_2u_2 \tag{C.41}$$

$$x_{2,2}^{bs1s2,u_1,u_1}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{A_1K_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_4{}^2(1$$

$$+2e^{2\sigma t})L_{0010}L_{1100}(\sigma^2+\omega_d{}^2)+2A_3{}^2K_{0010}K_{1100}(\sigma^2+(1$$

$$+2e^{2\sigma t})\omega_d{}^2))\cos(\varphi_1)+A_1A_4{}^2(1+e^{2\sigma t})K_{0010}L_{0010}L_{1100}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+9\omega_d{}^4)\cos(\varphi_1-2\varphi_2)+\frac{1}{2}[2\omega_d{}^2(A_2L_{0010}(\sigma^2+9\omega_d{}^2)(A_4{}^2(1$$

$$+2e^{2\sigma t})L_{0010}L_{1100}(\sigma^2+\omega_d{}^2)+2A_3{}^2K_{0010}K_{1100}(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2))$$

$$+A_1A_3A_4K_{0010}(2(1+e^{2\sigma t})K_{1100}L_{0010}+(1+2e^{2\sigma t})K_{0010}L_{1100})(\sigma^4$$

$$+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1-\varphi_2))+4A_2A_3A_4L_{0010}(K_{1100}L_{0010}$$

$$+K_{0010}L_{1100})\omega_d{}^2(\sigma^2+9\omega_d{}^2)(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_2)$$

$$+2A_2A_4{}^2L_{0010}{}^2L_{1100}\omega_d{}^2(\sigma^2+9\omega_d{}^2)((1-2e^{2\sigma t})\sigma^2+(1$$

$$+2e^{2\sigma t})\omega_d{}^2)\cos(2\varphi_2)+2A_1A_3A_4K_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(-2e^{2\sigma t}(K_{1100}L_{0010}$$

$$+K_{0010}L_{1100})(\sigma^2-\omega_d{}^2)+K_{0010}L_{1100}(\sigma^2+\omega_d{}^2))\cos(\varphi_1+\varphi_2)$$

$$+2A_1A_4{}^2e^{2\sigma t}K_{0010}L_{0010}L_{1100}\omega_d{}^2(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1+2\varphi_2)$$

$$+4A_2e^{\sigma t}L_{0010}\omega_d{}^2(\sigma^2+\omega_d{}^2)(A_4{}^2L_{0010}L_{1100}(\sigma^2-15\omega_d{}^2)$$

$$+2A_3{}^2K_{0010}K_{1100}(\sigma^3t-12\omega_d{}^2+9\sigma t\omega_d{}^2))\cos(t\omega_d)$$

$$-2A_2L_{0010}\omega_d{}^2(A_4{}^2L_{0010}L_{1100}(5\sigma^4+2\sigma^2\omega_d{}^2-3\omega_d{}^4)+2A_3{}^2K_{0010}K_{1100}(\sigma^4$$

$$-12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(2t\omega_d)+2A_2A_3A_4K_{0010}L_{0010}L_{1100}\omega_d{}^2(-5\sigma^4$$

$$-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_2-2t\omega_d)-A_1e^{\sigma t}K_{0010}(\sigma^2$$

$$+\omega_d{}^2)(3A_4{}^2L_{0010}L_{1100}(\sigma^4+6\sigma^2\omega_d{}^2+5\omega_d{}^4)+2A_3{}^2K_{0010}K_{1100}(\sigma^4$$

$$+6\sigma^2\omega_d{}^2+21\omega_d{}^4))\cos(\varphi_1-t\omega_d)+A_1A_4{}^2e^{\sigma t}K_{0010}L_{0010}L_{1100}(\sigma^6$$

$$+7\sigma^4\omega_d{}^2-21\sigma^2\omega_d{}^4-27\omega_d{}^6)\cos(\varphi_1-2\varphi_2-t\omega_d)-A_1A_3A_4e^{\sigma t}K_{0010}(\sigma^2$$

$$+\omega_d{}^2)(4K_{1100}L_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2)+K_{0010}L_{1100}(\sigma^4+2\sigma^2\omega_d{}^2$$

$$+33\omega_d{}^4))\cos(\varphi_1-\varphi_2-t\omega_d)-2A_2A_3A_4e^{\sigma t}L_{0010}(\sigma^2$$

$$+ \omega_d{}^2)(2K_{0010}L_{1100}\omega_d{}^2(\sigma^2 + 9\omega_d{}^2) + K_{1100}L_{0010}(\sigma^4 + 6\sigma^2\omega_d{}^2$$

$$+ 21\omega_d{}^4))\cos(\varphi_2 - t\omega_d) - A_1A_3A_4e^{\sigma t}K_{0010}(\sigma^2 + \omega_d{}^2)^2(2K_{1100}L_{0010}(\sigma^2$$

$$+ 3\omega_d{}^2) + K_{0010}L_{1100}(\sigma^2 + 9\omega_d{}^2))\cos(\varphi_1 + \varphi_2 - t\omega_d)$$

$$- A_2A_4{}^2e^{\sigma t}L_{0010}{}^2L_{1100}(\sigma^2 + \omega_d{}^2)^2(\sigma^2 + 9\omega_d{}^2)\cos(2\varphi_2 - t\omega_d)$$

$$+ A_1e^{\sigma t}K_{0010}(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(A_4{}^2L_{0010}L_{1100}(3\sigma^2 - 5\omega_d{}^2)$$

$$+ 2A_3{}^2K_{0010}K_{1100}(\sigma^2 - 3\omega_d{}^2 + 4\sigma t\omega_d{}^2))\cos(\varphi_1 + t\omega_d)$$

$$- A_1A_4{}^2e^{\sigma t}K_{0010}L_{0010}L_{1100}(\sigma^2 + \omega_d{}^2)^2(\sigma^2 + 9\omega_d{}^2)\cos(\varphi_1 - 2\varphi_2 + t\omega_d)$$

$$+ A_1A_3A_4e^{\sigma t}K_{0010}(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(-4K_{1100}L_{0010}\omega_d{}^2$$

$$+ K_{0010}L_{1100}(\sigma^2 - 3\omega_d{}^2))\cos(\varphi_1 - \varphi_2 + t\omega_d) + 2A_2A_3A_4e^{\sigma t}L_{0010}(\sigma^2$$

$$+ \omega_d{}^2)(K_{1100}L_{0010}(\sigma^2 + 9\omega_d{}^2)(\sigma^2 - 3\omega_d{}^2 + 4\sigma t\omega_d{}^2) + 2K_{0010}L_{1100}\omega_d{}^2(\sigma^2$$

$$+ 2\sigma^3t - 15\omega_d{}^2 + 18\sigma t\omega_d{}^2))\cos(\varphi_2 + t\omega_d) + 2A_2A_4{}^2L_{0010}{}^2L_{1100}\omega_d{}^2(3\sigma^4$$

$$+ 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4)\cos(2(\varphi_2 + t\omega_d)) + A_1A_3A_4e^{\sigma t}K_{0010}(\sigma^4 + 10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)(2K_{1100}L_{0010}(\sigma^2 - \omega_d{}^2 + 4\sigma t\omega_d{}^2) + K_{0010}L_{1100}(\sigma^2 - 3\omega_d{}^2$$

$$+ 8\sigma t\omega_d{}^2))\cos(\varphi_1 + \varphi_2 + t\omega_d) + A_2A_4{}^2e^{\sigma t}L_{0010}{}^2L_{1100}(\sigma^2 - 3\omega_d{}^2$$

$$+ 8\sigma t\omega_d{}^2)(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\cos(2\varphi_2 + t\omega_d)$$

$$+ 8A_1A_4{}^2e^{\sigma t}K_{0010}L_{0010}L_{1100}\sigma t\omega_d{}^2(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\cos(\varphi_1 + 2\varphi_2$$

$$+ t\omega_d) - 2A_1K_{0010}\omega_d{}^2(A_4{}^2L_{0010}L_{1100}(5\sigma^4 + 2\sigma^2\omega_d{}^2 - 3\omega_d{}^4)$$

$$+ 2A_3{}^2K_{0010}K_{1100}(\sigma^4 - 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))\cos(\varphi_1 + 2t\omega_d)$$

$$+ 2A_1A_3A_4K_{0010}{}^2L_{1100}\omega_d{}^2(-5\sigma^4 - 2\sigma^2\omega_d{}^2 + 3\omega_d{}^4)\cos(\varphi_1 - \varphi_2 + 2t\omega_d)$$

$$+ 2A_2A_3A_4L_{0010}\omega_d{}^2(K_{0010}L_{1100}(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4) - 2K_{1100}L_{0010}(\sigma^4$$

$$- 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))\cos(\varphi_2 + 2t\omega_d) + 2A_1A_3A_4K_{0010}\omega_d{}^2(K_{0010}L_{1100}(3\sigma^4$$

$$+ 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4) - 2K_{1100}L_{0010}(\sigma^4 - 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))\cos(\varphi_1 + \varphi_2$$

$$+ 2t\omega_d) + 2A_1A_4{}^2K_{0010}L_{0010}L_{1100}\omega_d{}^2(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4)\cos(\varphi_1$$

$$+ 2(\varphi_2 + t\omega_d)) + 2A_1K_{0010}\sigma\omega_d(\sigma^2 + 9{\omega_d}^2)(4{A_3}^2e^{2\sigma t}K_{0010}K_{1100}{\omega_d}^2$$

$$+ {A_4}^2(-1 + 2e^{2\sigma t})L_{0010}L_{1100}(\sigma^2 + {\omega_d}^2))\sin(\varphi_1) - 2A_1{A_4}^2(-1$$

$$+ e^{2\sigma t})K_{0010}L_{0010}L_{1100}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_1 - 2\varphi_2)$$

$$+ 2A_1A_3A_4{K_{0010}}^2L_{1100}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_1 - \varphi_2)$$

$$- 4A_2A_3A_4L_{0010}\omega_d(\sigma^3 + 9\sigma{\omega_d}^2)(-2e^{2\sigma t}K_{1100}L_{0010}{\omega_d}^2 + K_{0010}L_{1100}(\sigma^2$$

$$+ (1 - 2e^{2\sigma t}){\omega_d}^2))\sin(\varphi_2) - 2A_2{A_4}^2{L_{0010}}^2L_{1100}\omega_d(\sigma^2 + (1$$

$$- 4e^{2\sigma t}){\omega_d}^2)(\sigma^3 + 9\sigma{\omega_d}^2)\sin(2\varphi_2) - 2A_1A_3A_4K_{0010}\omega_d(\sigma^3$$

$$+ 9\sigma{\omega_d}^2)(-4e^{2\sigma t}K_{1100}L_{0010}{\omega_d}^2 + K_{0010}L_{1100}(\sigma^2 + (1 - 4e^{2\sigma t}){\omega_d}^2))\sin(\varphi_1$$

$$+ \varphi_2) - 2A_1{A_4}^2e^{2\sigma t}K_{0010}L_{0010}L_{1100}\omega_d(\sigma^5 + 6\sigma^3{\omega_d}^2 - 27\sigma{\omega_d}^4)\sin(\varphi_1$$

$$+ 2\varphi_2) - 2A_2e^{\sigma t}L_{0010}\omega_d(\sigma^2 + {\omega_d}^2)(4{A_3}^2K_{0010}K_{1100}(\sigma^3 + 5\sigma{\omega_d}^2 + \sigma^2t{\omega_d}^2$$

$$+ 9t{\omega_d}^4) + {A_4}^2L_{0010}L_{1100}(4\sigma^3 + \sigma^4t + 20\sigma{\omega_d}^2 + 10\sigma^2t{\omega_d}^2$$

$$+ 9t{\omega_d}^4))\sin(t\omega_d) + 2A_2L_{0010}\sigma\omega_d(4{A_3}^2K_{0010}K_{1100}{\omega_d}^2(3\sigma^2 - 5{\omega_d}^2)$$

$$+ {A_4}^2L_{0010}L_{1100}(-\sigma^4 + 6\sigma^2{\omega_d}^2 + 7{\omega_d}^4))\sin(2t\omega_d)$$

$$+ 2A_2A_3A_4K_{0010}L_{0010}L_{1100}\sigma\omega_d(\sigma^4 - 6\sigma^2{\omega_d}^2 - 7{\omega_d}^4)\sin(\varphi_2 - 2t\omega_d)$$

$$+ 2A_1e^{\sigma t}K_{0010}\omega_d(\sigma^2 + {\omega_d}^2)^2(2{A_4}^2L_{0010}L_{1100}\sigma + {A_3}^2K_{0010}K_{1100}(2\sigma + \sigma^2t$$

$$+ 9t{\omega_d}^2))\sin(\varphi_1 - t\omega_d) + 2A_1{A_4}^2e^{\sigma t}K_{0010}L_{0010}L_{1100}t\omega_d(\sigma^2 + {\omega_d}^2)^2(\sigma^2$$

$$+ 9{\omega_d}^2)\sin(\varphi_1 - 2\varphi_2 - t\omega_d)$$

$$+ 2A_1A_3A_4e^{\sigma t}K_{0010}\omega_d(\sigma^2 + {\omega_d}^2)^2(K_{1100}L_{0010}t(\sigma^2 + 9{\omega_d}^2) + K_{0010}L_{1100}(2\sigma$$

$$+ \sigma^2t + 9t{\omega_d}^2))\sin(\varphi_1 - \varphi_2 - t\omega_d) + 2A_2A_3A_4e^{\sigma t}L_{0010}\omega_d(\sigma^2$$

$$+ {\omega_d}^2)(2K_{0010}L_{1100}\sigma(\sigma^2 + 9{\omega_d}^2) + K_{1100}L_{0010}(\sigma^2 + {\omega_d}^2)(2\sigma + \sigma^2t$$

$$+ 9t{\omega_d}^2))\sin(\varphi_2 - t\omega_d)$$

$$+ 4A_1A_3A_4e^{\sigma t}K_{0010}K_{1100}L_{0010}\sigma\omega_d(\sigma^2 + {\omega_d}^2)^2\sin(\varphi_1 + \varphi_2 - t\omega_d)$$

$$- 2A_1e^{\sigma t}K_{0010}\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)({A_4}^2L_{0010}L_{1100}(2\sigma + \sigma^2t + t{\omega_d}^2)$$

$$+ A_3{}^2K_{0010}K_{1100}(2\sigma - \sigma^2t + 3t\omega_d{}^2))\sin(\varphi_1 + t\omega_d)$$

$$- 2A_1A_3A_4e^{\sigma t}K_{0010}\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(2K_{0010}L_{1100}\sigma$$

$$+ K_{1100}L_{0010}t(\sigma^2 + \omega_d{}^2))\sin(\varphi_1 - \varphi_2 + t\omega_d) - 2A_2A_3A_4e^{\sigma t}L_{0010}\omega_d(\sigma^2$$

$$+ \omega_d{}^2)(-K_{1100}L_{0010}(\sigma^2 + 9\omega_d{}^2)(-2\sigma + \sigma^2t - 3t\omega_d{}^2) + 2K_{0010}L_{1100}(\sigma^3$$

$$+ \sigma\omega_d{}^2 + 2\sigma^2t\omega_d{}^2 + 18t\omega_d{}^4))\sin(\varphi_2 + t\omega_d) + 2A_2A_4{}^2L_{0010}{}^2L_{1100}\sigma\omega_d(\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4)\sin(2(\varphi_2 + t\omega_d)) - 2A_1A_3A_4e^{\sigma t}K_{0010}\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)(-K_{0010}L_{1100}t(\sigma^2 - 3\omega_d{}^2) + 2K_{1100}L_{0010}(\sigma - \sigma^2t + t\omega_d{}^2))\sin(\varphi_1$$

$$+ \varphi_2 + t\omega_d) + 2A_2A_4{}^2e^{\sigma t}L_{0010}{}^2L_{1100}t\omega_d(\sigma^6 + 7\sigma^4\omega_d{}^2 - 21\sigma^2\omega_d{}^4$$

$$- 27\omega_d{}^6)\sin(2\varphi_2 + t\omega_d) + 4A_1A_4{}^2e^{\sigma t}K_{0010}L_{0010}L_{1100}t\omega_d(\sigma^6 + 9\sigma^4\omega_d{}^2$$

$$- \sigma^2\omega_d{}^4 - 9\omega_d{}^6)\sin(\varphi_1 + 2\varphi_2 + t\omega_d)$$

$$+ 2A_1K_{0010}\sigma\omega_d(4A_3{}^2K_{0010}K_{1100}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + A_4{}^2L_{0010}L_{1100}(-\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2 + 7\omega_d{}^4))\sin(\varphi_1 + 2t\omega_d) - 2A_1A_3A_4K_{0010}{}^2L_{1100}\sigma\omega_d(\sigma^4$$

$$- 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_1 - \varphi_2 + 2t\omega_d)$$

$$+ 2A_2A_3A_4L_{0010}\sigma\omega_d(4K_{1100}L_{0010}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + K_{0010}L_{1100}(\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4))\sin(\varphi_2 + 2t\omega_d)$$

$$+ 2A_1A_3A_4K_{0010}\sigma\omega_d(4K_{1100}L_{0010}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + K_{0010}L_{1100}(\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4))\sin(\varphi_1 + \varphi_2 + 2t\omega_d) + 2A_1A_4{}^2K_{0010}L_{0010}L_{1100}\sigma\omega_d(\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4)\sin(\varphi_1 + 2(\varphi_2 + t\omega_d))]\}$$

(C. 42)

$$x_{2,2}^{bs1s2,u_1,u_2}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)}e^{-2\sigma t}\{A_1K_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_4{}^2(1$$

$$+2e^{2\sigma t})L_{0001}L_{1100}(\sigma^2+\omega_d{}^2)+2A_3{}^2K_{0001}K_{1100}(\sigma^2+(1$$

$$+2e^{2\sigma t})\omega_d{}^2))\cos(\varphi_1)+A_1A_4{}^2(1+e^{2\sigma t})K_{0010}L_{0001}L_{1100}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+9\omega_d{}^4)\cos(\varphi_1-2\varphi_2)+\frac{1}{2}[2\omega_d{}^2(A_2L_{0010}(\sigma^2+9\omega_d{}^2)(A_4{}^2(1$$

$$+2e^{2\sigma t})L_{0001}L_{1100}(\sigma^2+\omega_d{}^2)+2A_3{}^2K_{0001}K_{1100}(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2))$$

$$+A_1A_3A_4K_{0010}(2(1+e^{2\sigma t})K_{1100}L_{0001}+(1+2e^{2\sigma t})K_{0001}L_{1100})(\sigma^4$$

$$+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1-\varphi_2))+4A_2A_3A_4L_{0010}(K_{1100}L_{0001}$$

$$+K_{0001}L_{1100})\omega_d{}^2(\sigma^2+9\omega_d{}^2)(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_2)$$

$$+2A_2A_4{}^2L_{0001}L_{0010}L_{1100}\omega_d{}^2(\sigma^2+9\omega_d{}^2)((1-2e^{2\sigma t})\sigma^2+(1$$

$$+2e^{2\sigma t})\omega_d{}^2)\cos(2\varphi_2)+2A_1A_3A_4K_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(-2e^{2\sigma t}(K_{1100}L_{0001}$$

$$+K_{0001}L_{1100})(\sigma^2-\omega_d{}^2)+K_{0001}L_{1100}(\sigma^2+\omega_d{}^2))\cos(\varphi_1+\varphi_2)$$

$$+2A_1A_4{}^2e^{2\sigma t}K_{0010}L_{0001}L_{1100}\omega_d{}^2(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1+2\varphi_2)$$

$$+4A_2e^{\sigma t}L_{0010}\omega_d{}^2(\sigma^2+\omega_d{}^2)(A_4{}^2L_{0001}L_{1100}(\sigma^2-15\omega_d{}^2)$$

$$+2A_3{}^2K_{0001}K_{1100}(\sigma^3t-12\omega_d{}^2+9\sigma t\omega_d{}^2))\cos(t\omega_d)$$

$$-2A_2L_{0010}\omega_d{}^2(A_4{}^2L_{0001}L_{1100}(5\sigma^4+2\sigma^2\omega_d{}^2-3\omega_d{}^4)+2A_3{}^2K_{0001}K_{1100}(\sigma^4$$

$$-12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(2t\omega_d)+2A_2A_3A_4K_{0001}L_{0010}L_{1100}\omega_d{}^2(-5\sigma^4$$

$$-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_2-2t\omega_d)-A_1e^{\sigma t}K_{0010}(\sigma^2$$

$$+\omega_d{}^2)(3A_4{}^2L_{0001}L_{1100}(\sigma^4+6\sigma^2\omega_d{}^2+5\omega_d{}^4)+2A_3{}^2K_{0001}K_{1100}(\sigma^4$$

$$+6\sigma^2\omega_d{}^2+21\omega_d{}^4))\cos(\varphi_1-t\omega_d)+A_1A_4{}^2e^{\sigma t}K_{0010}L_{0001}L_{1100}(\sigma^6$$

$$+7\sigma^4\omega_d{}^2-21\sigma^2\omega_d{}^4-27\omega_d{}^6)\cos(\varphi_1-2\varphi_2-t\omega_d)-A_1A_3A_4e^{\sigma t}K_{0010}(\sigma^2$$

$$+\omega_d{}^2)(4K_{1100}L_{0001}\omega_d{}^2(\sigma^2+9\omega_d{}^2)+K_{0001}L_{1100}(\sigma^4+2\sigma^2\omega_d{}^2$$

$$+33\omega_d{}^4))\cos(\varphi_1-\varphi_2-t\omega_d)-2A_2A_3A_4e^{\sigma t}L_{0010}(\sigma^2$$

$$+ \omega_d{}^2)(2K_{0001}L_{1100}\omega_d{}^2(\sigma^2 + 9\omega_d{}^2) + K_{1100}L_{0001}(\sigma^4 + 6\sigma^2\omega_d{}^2$$

$$+ 21\omega_d{}^4))\cos(\varphi_2 - t\omega_d) - A_1A_3A_4e^{\sigma t}K_{0010}(\sigma^2 + \omega_d{}^2)^2(2K_{1100}L_{0001}(\sigma^2$$

$$+ 3\omega_d{}^2) + K_{0001}L_{1100}(\sigma^2 + 9\omega_d{}^2))\cos(\varphi_1 + \varphi_2 - t\omega_d)$$

$$- A_2A_4{}^2e^{\sigma t}L_{0001}L_{0010}L_{1100}(\sigma^2 + \omega_d{}^2)^2(\sigma^2 + 9\omega_d{}^2)\cos(2\varphi_2 - t\omega_d)$$

$$+ A_1e^{\sigma t}K_{0010}(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(A_4{}^2L_{0001}L_{1100}(3\sigma^2 - 5\omega_d{}^2)$$

$$+ 2A_3{}^2K_{0001}K_{1100}(\sigma^2 - 3\omega_d{}^2 + 4\sigma t\omega_d{}^2))\cos(\varphi_1 + t\omega_d)$$

$$- A_1A_4{}^2e^{\sigma t}K_{0010}L_{0001}L_{1100}(\sigma^2 + \omega_d{}^2)^2(\sigma^2 + 9\omega_d{}^2)\cos(\varphi_1 - 2\varphi_2 + t\omega_d)$$

$$+ A_1A_3A_4e^{\sigma t}K_{0010}(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(-4K_{1100}L_{0001}\omega_d{}^2$$

$$+ K_{0001}L_{1100}(\sigma^2 - 3\omega_d{}^2))\cos(\varphi_1 - \varphi_2 + t\omega_d) + 2A_2A_3A_4e^{\sigma t}L_{0010}(\sigma^2$$

$$+ \omega_d{}^2)(K_{1100}L_{0001}(\sigma^2 + 9\omega_d{}^2)(\sigma^2 - 3\omega_d{}^2 + 4\sigma t\omega_d{}^2) + 2K_{0001}L_{1100}\omega_d{}^2(\sigma^2$$

$$+ 2\sigma^3 t - 15\omega_d{}^2 + 18\sigma t\omega_d{}^2))\cos(\varphi_2 + t\omega_d)$$

$$+ 2A_2A_4{}^2L_{0001}L_{0010}L_{1100}\omega_d{}^2(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4)\cos(2(\varphi_2 + t\omega_d))$$

$$+ A_1A_3A_4e^{\sigma t}K_{0010}(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(2K_{1100}L_{0001}(\sigma^2 - \omega_d{}^2$$

$$+ 4\sigma t\omega_d{}^2) + K_{0001}L_{1100}(\sigma^2 - 3\omega_d{}^2 + 8\sigma t\omega_d{}^2))\cos(\varphi_1 + \varphi_2 + t\omega_d)$$

$$+ A_2A_4{}^2e^{\sigma t}L_{0001}L_{0010}L_{1100}(\sigma^2 - 3\omega_d{}^2 + 8\sigma t\omega_d{}^2)(\sigma^4 + 10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(2\varphi_2 + t\omega_d) + 8A_1A_4{}^2e^{\sigma t}K_{0010}L_{0001}L_{1100}\sigma t\omega_d{}^2(\sigma^4 + 10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)\cos(\varphi_1 + 2\varphi_2 + t\omega_d) - 2A_1K_{0010}\omega_d{}^2(A_4{}^2L_{0001}L_{1100}(5\sigma^4 + 2\sigma^2\omega_d{}^2$$

$$- 3\omega_d{}^4) + 2A_3{}^2K_{0001}K_{1100}(\sigma^4 - 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))\cos(\varphi_1 + 2t\omega_d)$$

$$+ 2A_1A_3A_4K_{0001}K_{0010}L_{1100}\omega_d{}^2(-5\sigma^4 - 2\sigma^2\omega_d{}^2 + 3\omega_d{}^4)\cos(\varphi_1 - \varphi_2$$

$$+ 2t\omega_d) + 2A_2A_3A_4L_{0010}\omega_d{}^2(K_{0001}L_{1100}(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4)$$

$$- 2K_{1100}L_{0001}(\sigma^4 - 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))\cos(\varphi_2 + 2t\omega_d)$$

$$+ 2A_1A_3A_4K_{0010}\omega_d{}^2(K_{0001}L_{1100}(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4) - 2K_{1100}L_{0001}(\sigma^4$$

$$- 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))\cos(\varphi_1 + \varphi_2 + 2t\omega_d)$$

$+ 2A_1A_4{}^2K_{0010}L_{0001}L_{1100}\omega_d{}^2(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4)\cos(\varphi_1 + 2(\varphi_2$

$+ t\omega_d)) + 2A_1K_{0010}\sigma\omega_d(\sigma^2 + 9\omega_d{}^2)(4A_3{}^2e^{2\sigma t}K_{0001}K_{1100}\omega_d{}^2 + A_4{}^2(-1$

$+ 2e^{2\sigma t})L_{0001}L_{1100}(\sigma^2 + \omega_d{}^2))\sin(\varphi_1) - 2A_1A_4{}^2(-1$

$+ e^{2\sigma t})K_{0010}L_{0001}L_{1100}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 - 2\varphi_2)$

$+ 2A_1A_3A_4K_{0001}K_{0010}L_{1100}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 - \varphi_2)$

$- 4A_2A_3A_4L_{0010}\omega_d(\sigma^3 + 9\sigma\omega_d{}^2)(-2e^{2\sigma t}K_{1100}L_{0001}\omega_d{}^2 + K_{0001}L_{1100}(\sigma^2$

$+ (1 - 2e^{2\sigma t})\omega_d{}^2))\sin(\varphi_2) - 2A_2A_4{}^2L_{0001}L_{0010}L_{1100}\omega_d(\sigma^2 + (1$

$- 4e^{2\sigma t})\omega_d{}^2)(\sigma^3 + 9\sigma\omega_d{}^2)\sin(2\varphi_2) - 2A_1A_3A_4K_{0010}\omega_d(\sigma^3$

$+ 9\sigma\omega_d{}^2)(-4e^{2\sigma t}K_{1100}L_{0001}\omega_d{}^2 + K_{0001}L_{1100}(\sigma^2 + (1 - 4e^{2\sigma t})\omega_d{}^2))\sin(\varphi_1$

$+ \varphi_2) - 2A_1A_4{}^2e^{2\sigma t}K_{0010}L_{0001}L_{1100}\omega_d(\sigma^5 + 6\sigma^3\omega_d{}^2 - 27\sigma\omega_d{}^4)\sin(\varphi_1$

$+ 2\varphi_2) - 2A_2e^{\sigma t}L_{0010}\omega_d(\sigma^2 + \omega_d{}^2)(4A_3{}^2K_{0001}K_{1100}(\sigma^3 + 5\sigma\omega_d{}^2 + \sigma^2t\omega_d{}^2$

$+ 9t\omega_d{}^4) + A_4{}^2L_{0001}L_{1100}(4\sigma^3 + \sigma^4t + 20\sigma\omega_d{}^2 + 10\sigma^2t\omega_d{}^2$

$+ 9t\omega_d{}^4))\sin(t\omega_d) + 2A_2L_{0010}\sigma\omega_d(4A_3{}^2K_{0001}K_{1100}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2)$

$+ A_4{}^2L_{0001}L_{1100}(-\sigma^4 + 6\sigma^2\omega_d{}^2 + 7\omega_d{}^4))\sin(2t\omega_d)$

$+ 2A_2A_3A_4K_{0001}L_{0010}L_{1100}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_2 - 2t\omega_d)$

$+ 2A_1e^{\sigma t}K_{0010}\omega_d(\sigma^2 + \omega_d{}^2)^2(2A_4{}^2L_{0001}L_{1100}\sigma + A_3{}^2K_{0001}K_{1100}(2\sigma + \sigma^2t$

$+ 9t\omega_d{}^2))\sin(\varphi_1 - t\omega_d) + 2A_1A_4{}^2e^{\sigma t}K_{0010}L_{0001}L_{1100}t\omega_d(\sigma^2 + \omega_d{}^2)^2(\sigma^2$

$+ 9\omega_d{}^2)\sin(\varphi_1 - 2\varphi_2 - t\omega_d)$

$+ 2A_1A_3A_4e^{\sigma t}K_{0010}\omega_d(\sigma^2 + \omega_d{}^2)^2(K_{1100}L_{0001}t(\sigma^2 + 9\omega_d{}^2) + K_{0001}L_{1100}(2\sigma$

$+ \sigma^2t + 9t\omega_d{}^2))\sin(\varphi_1 - \varphi_2 - t\omega_d) + 2A_2A_3A_4e^{\sigma t}L_{0010}\omega_d(\sigma^2$

$+ \omega_d{}^2)(2K_{0001}L_{1100}\sigma(\sigma^2 + 9\omega_d{}^2) + K_{1100}L_{0001}(\sigma^2 + \omega_d{}^2)(2\sigma + \sigma^2t$

$+ 9t\omega_d{}^2))\sin(\varphi_2 - t\omega_d)$

$+ 4A_1A_3A_4e^{\sigma t}K_{0010}K_{1100}L_{0001}\sigma\omega_d(\sigma^2 + \omega_d{}^2)^2\sin(\varphi_1 + \varphi_2 - t\omega_d)$

$$- 2A_1e^{\sigma t}K_{0010}\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)({A_4}^2L_{0001}L_{1100}(2\sigma + \sigma^2 t + t{\omega_d}^2)$$

$$+ {A_3}^2K_{0001}K_{1100}(2\sigma - \sigma^2 t + 3t{\omega_d}^2))\sin(\varphi_1 + t\omega_d)$$

$$- 2A_1A_3A_4e^{\sigma t}K_{0010}\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)(2K_{0001}L_{1100}\sigma$$

$$+ K_{1100}L_{0001}t(\sigma^2 + {\omega_d}^2))\sin(\varphi_1 - \varphi_2 + t\omega_d) - 2A_2A_3A_4e^{\sigma t}L_{0010}\omega_d(\sigma^2$$

$$+ {\omega_d}^2)(-K_{1100}L_{0001}(\sigma^2 + 9{\omega_d}^2)(-2\sigma + \sigma^2 t - 3t{\omega_d}^2) + 2K_{0001}L_{1100}(\sigma^3$$

$$+ \sigma{\omega_d}^2 + 2\sigma^2 t{\omega_d}^2 + 18t{\omega_d}^4))\sin(\varphi_2 + t\omega_d)$$

$$+ 2A_2{A_4}^2L_{0001}L_{0010}L_{1100}\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4)\sin(2(\varphi_2 + t\omega_d))$$

$$- 2A_1A_3A_4e^{\sigma t}K_{0010}\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)(-K_{0001}L_{1100}t(\sigma^2 - 3{\omega_d}^2)$$

$$+ 2K_{1100}L_{0001}(\sigma - \sigma^2 t + t{\omega_d}^2))\sin(\varphi_1 + \varphi_2 + t\omega_d)$$

$$+ 2A_2{A_4}^2e^{\sigma t}L_{0001}L_{0010}L_{1100}t\omega_d(\sigma^6 + 7\sigma^4{\omega_d}^2 - 21\sigma^2{\omega_d}^4 - 27{\omega_d}^6)\sin(2\varphi_2$$

$$+ t\omega_d) + 4A_1{A_4}^2e^{\sigma t}K_{0010}L_{0001}L_{1100}t\omega_d(\sigma^6 + 9\sigma^4{\omega_d}^2 - \sigma^2{\omega_d}^4$$

$$- 9{\omega_d}^6)\sin(\varphi_1 + 2\varphi_2 + t\omega_d) + 2A_1K_{0010}\sigma\omega_d(4{A_3}^2K_{0001}K_{1100}{\omega_d}^2(3\sigma^2$$

$$- 5{\omega_d}^2) + {A_4}^2L_{0001}L_{1100}(-\sigma^4 + 6\sigma^2{\omega_d}^2 + 7{\omega_d}^4))\sin(\varphi_1 + 2t\omega_d)$$

$$- 2A_1A_3A_4K_{0001}K_{0010}L_{1100}\sigma\omega_d(\sigma^4 - 6\sigma^2{\omega_d}^2 - 7{\omega_d}^4)\sin(\varphi_1 - \varphi_2 + 2t\omega_d)$$

$$+ 2A_2A_3A_4L_{0010}\sigma\omega_d(4K_{1100}L_{0001}{\omega_d}^2(3\sigma^2 - 5{\omega_d}^2) + K_{0001}L_{1100}(\sigma^4$$

$$+ 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4))\sin(\varphi_2 + 2t\omega_d)$$

$$+ 2A_1A_3A_4K_{0010}\sigma\omega_d(4K_{1100}L_{0001}{\omega_d}^2(3\sigma^2 - 5{\omega_d}^2) + K_{0001}L_{1100}(\sigma^4$$

$$+ 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4))\sin(\varphi_1 + \varphi_2 + 2t\omega_d) + 2A_1{A_4}^2K_{0010}L_{0001}L_{1100}\sigma\omega_d(\sigma^4$$

$$+ 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4)\sin(\varphi_1 + 2(\varphi_2 + t\omega_d))]\}$$

(C. 43)

$$x_{2,2}^{bs1s2,u_2,u_1}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{A_1K_{0001}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(A_4{}^2(1$$

$$+2e^{2\sigma t})L_{0010}L_{1100}(\sigma^2+\omega_d{}^2)+2A_3{}^2K_{0010}K_{1100}(\sigma^2+(1$$

$$+2e^{2\sigma t})\omega_d{}^2))\cos(\varphi_1)+A_1A_4{}^2(1+e^{2\sigma t})K_{0001}L_{0010}L_{1100}\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2$$

$$+9\omega_d{}^4)\cos(\varphi_1-2\varphi_2)+\frac{1}{2}[2\omega_d{}^2(A_2L_{0001}(\sigma^2+9\omega_d{}^2)(A_4{}^2(1$$

$$+2e^{2\sigma t})L_{0010}L_{1100}(\sigma^2+\omega_d{}^2)+2A_3{}^2K_{0010}K_{1100}(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2))$$

$$+A_1A_3A_4K_{0001}(2(1+e^{2\sigma t})K_{1100}L_{0010}+(1+2e^{2\sigma t})K_{0010}L_{1100})(\sigma^4$$

$$+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1-\varphi_2))+4A_2A_3A_4L_{0001}(K_{1100}L_{0010}$$

$$+K_{0010}L_{1100})\omega_d{}^2(\sigma^2+9\omega_d{}^2)(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_2)$$

$$+2A_2A_4{}^2L_{0001}L_{0010}L_{1100}\omega_d{}^2(\sigma^2+9\omega_d{}^2)((1-2e^{2\sigma t})\sigma^2+(1$$

$$+2e^{2\sigma t})\omega_d{}^2)\cos(2\varphi_2)+2A_1A_3A_4K_{0001}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(-2e^{2\sigma t}(K_{1100}L_{0010}$$

$$+K_{0010}L_{1100})(\sigma^2-\omega_d{}^2)+K_{0010}L_{1100}(\sigma^2+\omega_d{}^2))\cos(\varphi_1+\varphi_2)$$

$$+2A_1A_4{}^2e^{2\sigma t}K_{0001}L_{0010}L_{1100}\omega_d{}^2(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(\varphi_1+2\varphi_2)$$

$$+4A_2e^{\sigma t}L_{0001}\omega_d{}^2(\sigma^2+\omega_d{}^2)(A_4{}^2L_{0010}L_{1100}(\sigma^2-15\omega_d{}^2)$$

$$+2A_3{}^2K_{0010}K_{1100}(\sigma^3t-12\omega_d{}^2+9\sigma t\omega_d{}^2))\cos(t\omega_d)$$

$$-2A_2L_{0001}\omega_d{}^2(A_4{}^2L_{0010}L_{1100}(5\sigma^4+2\sigma^2\omega_d{}^2-3\omega_d{}^4)+2A_3{}^2K_{0010}K_{1100}(\sigma^4$$

$$-12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(2t\omega_d)+2A_2A_3A_4K_{0010}L_{0001}L_{1100}\omega_d{}^2(-5\sigma^4$$

$$-2\sigma^2\omega_d{}^2+3\omega_d{}^4)\cos(\varphi_2-2t\omega_d)-A_1e^{\sigma t}K_{0001}(\sigma^2$$

$$+\omega_d{}^2)(3A_4{}^2L_{0010}L_{1100}(\sigma^4+6\sigma^2\omega_d{}^2+5\omega_d{}^4)+2A_3{}^2K_{0010}K_{1100}(\sigma^4$$

$$+6\sigma^2\omega_d{}^2+21\omega_d{}^4))\cos(\varphi_1-t\omega_d)+A_1A_4{}^2e^{\sigma t}K_{0001}L_{0010}L_{1100}(\sigma^6$$

$$+7\sigma^4\omega_d{}^2-21\sigma^2\omega_d{}^4-27\omega_d{}^6)\cos(\varphi_1-2\varphi_2-t\omega_d)-A_1A_3A_4e^{\sigma t}K_{0001}(\sigma^2$$

$$+\omega_d{}^2)(4K_{1100}L_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2)+K_{0010}L_{1100}(\sigma^4+2\sigma^2\omega_d{}^2$$

$$+33\omega_d{}^4))\cos(\varphi_1-\varphi_2-t\omega_d)-2A_2A_3A_4e^{\sigma t}L_{0001}(\sigma^2$$

$+ \omega_d{}^2)(2K_{0010}L_{1100}\omega_d{}^2(\sigma^2 + 9\omega_d{}^2) + K_{1100}L_{0010}(\sigma^4 + 6\sigma^2\omega_d{}^2$

$+ 21\omega_d{}^4))\cos(\varphi_2 - t\omega_d) - A_1A_3A_4e^{\sigma t}K_{0001}(\sigma^2 + \omega_d{}^2)^2(2K_{1100}L_{0010}(\sigma^2$

$+ 3\omega_d{}^2) + K_{0010}L_{1100}(\sigma^2 + 9\omega_d{}^2))\cos(\varphi_1 + \varphi_2 - t\omega_d)$

$- A_2A_4{}^2e^{\sigma t}L_{0001}L_{0010}L_{1100}(\sigma^2 + \omega_d{}^2)^2(\sigma^2 + 9\omega_d{}^2)\cos(2\varphi_2 - t\omega_d)$

$+ A_1e^{\sigma t}K_{0001}(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(A_4{}^2L_{0010}L_{1100}(3\sigma^2 - 5\omega_d{}^2)$

$+ 2A_3{}^2K_{0010}K_{1100}(\sigma^2 - 3\omega_d{}^2 + 4\sigma t\omega_d{}^2))\cos(\varphi_1 + t\omega_d)$

$- A_1A_4{}^2e^{\sigma t}K_{0001}L_{0010}L_{1100}(\sigma^2 + \omega_d{}^2)^2(\sigma^2 + 9\omega_d{}^2)\cos(\varphi_1 - 2\varphi_2 + t\omega_d)$

$+ A_1A_3A_4e^{\sigma t}K_{0001}(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(-4K_{1100}L_{0010}\omega_d{}^2$

$+ K_{0010}L_{1100}(\sigma^2 - 3\omega_d{}^2))\cos(\varphi_1 - \varphi_2 + t\omega_d) + 2A_2A_3A_4e^{\sigma t}L_{0001}(\sigma^2$

$+ \omega_d{}^2)(K_{1100}L_{0010}(\sigma^2 + 9\omega_d{}^2)(\sigma^2 - 3\omega_d{}^2 + 4\sigma t\omega_d{}^2) + 2K_{0010}L_{1100}\omega_d{}^2(\sigma^2$

$+ 2\sigma^3 t - 15\omega_d{}^2 + 18\sigma t\omega_d{}^2))\cos(\varphi_2 + t\omega_d)$

$+ 2A_2A_4{}^2L_{0001}L_{0010}L_{1100}\omega_d{}^2(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4)\cos(2(\varphi_2 + t\omega_d))$

$+ A_1A_3A_4e^{\sigma t}K_{0001}(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)(2K_{1100}L_{0010}(\sigma^2 - \omega_d{}^2$

$+ 4\sigma t\omega_d{}^2) + K_{0010}L_{1100}(\sigma^2 - 3\omega_d{}^2 + 8\sigma t\omega_d{}^2))\cos(\varphi_1 + \varphi_2 + t\omega_d)$

$+ A_2A_4{}^2e^{\sigma t}L_{0001}L_{0010}L_{1100}(\sigma^2 - 3\omega_d{}^2 + 8\sigma t\omega_d{}^2)(\sigma^4 + 10\sigma^2\omega_d{}^2$

$+ 9\omega_d{}^4)\cos(2\varphi_2 + t\omega_d) + 8A_1A_4{}^2e^{\sigma t}K_{0001}L_{0010}L_{1100}\sigma t\omega_d{}^2(\sigma^4 + 10\sigma^2\omega_d{}^2$

$+ 9\omega_d{}^4)\cos(\varphi_1 + 2\varphi_2 + t\omega_d) - 2A_1K_{0001}\omega_d{}^2(A_4{}^2L_{0010}L_{1100}(5\sigma^4 + 2\sigma^2\omega_d{}^2$

$- 3\omega_d{}^4) + 2A_3{}^2K_{0010}K_{1100}(\sigma^4 - 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))\cos(\varphi_1 + 2t\omega_d)$

$+ 2A_1A_3A_4K_{0001}K_{0010}L_{1100}\omega_d{}^2(-5\sigma^4 - 2\sigma^2\omega_d{}^2 + 3\omega_d{}^4)\cos(\varphi_1 - \varphi_2$

$+ 2t\omega_d) + 2A_2A_3A_4L_{0001}\omega_d{}^2(K_{0010}L_{1100}(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4)$

$- 2K_{1100}L_{0010}(\sigma^4 - 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))\cos(\varphi_2 + 2t\omega_d)$

$+ 2A_1A_3A_4K_{0001}\omega_d{}^2(K_{0010}L_{1100}(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4) - 2K_{1100}L_{0010}(\sigma^4$

$- 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))\cos(\varphi_1 + \varphi_2 + 2t\omega_d)$

$$+ 2A_1A_4{}^2K_{0001}L_{0010}L_{1100}\omega_d{}^2(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4)\cos(\varphi_1 + 2(\varphi_2$$

$$+ t\omega_d)) + 2A_1K_{0001}\sigma\omega_d(\sigma^2 + 9\omega_d{}^2)(4A_3{}^2e^{2\sigma t}K_{0010}K_{1100}\omega_d{}^2 + A_4{}^2(-1$$

$$+ 2e^{2\sigma t})L_{0010}L_{1100}(\sigma^2 + \omega_d{}^2))\sin(\varphi_1) - 2A_1A_4{}^2(-1$$

$$+ e^{2\sigma t})K_{0001}L_{0010}L_{1100}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 - 2\varphi_2)$$

$$+ 2A_1A_3A_4K_{0001}K_{0010}L_{1100}\sigma\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)\sin(\varphi_1 - \varphi_2)$$

$$- 4A_2A_3A_4L_{0001}\omega_d(\sigma^3 + 9\sigma\omega_d{}^2)(-2e^{2\sigma t}K_{1100}L_{0010}\omega_d{}^2 + K_{0010}L_{1100}(\sigma^2$$

$$+ (1 - 2e^{2\sigma t})\omega_d{}^2))\sin(\varphi_2) - 2A_2A_4{}^2L_{0001}L_{0010}L_{1100}\omega_d(\sigma^2 + (1$$

$$- 4e^{2\sigma t})\omega_d{}^2)(\sigma^3 + 9\sigma\omega_d{}^2)\sin(2\varphi_2) - 2A_1A_3A_4K_{0001}\omega_d(\sigma^3$$

$$+ 9\sigma\omega_d{}^2)(-4e^{2\sigma t}K_{1100}L_{0010}\omega_d{}^2 + K_{0010}L_{1100}(\sigma^2 + (1 - 4e^{2\sigma t})\omega_d{}^2))\sin(\varphi_1$$

$$+ \varphi_2) - 2A_1A_4{}^2e^{2\sigma t}K_{0001}L_{0010}L_{1100}\omega_d(\sigma^5 + 6\sigma^3\omega_d{}^2 - 27\sigma\omega_d{}^4)\sin(\varphi_1$$

$$+ 2\varphi_2) - 2A_2e^{\sigma t}L_{0001}\omega_d(\sigma^2 + \omega_d{}^2)(4A_3{}^2K_{0010}K_{1100}(\sigma^3 + 5\sigma\omega_d{}^2 + \sigma^2t\omega_d{}^2$$

$$+ 9t\omega_d{}^4) + A_4{}^2L_{0010}L_{1100}(4\sigma^3 + \sigma^4t + 20\sigma\omega_d{}^2 + 10\sigma^2t\omega_d{}^2$$

$$+ 9t\omega_d{}^4))\sin(t\omega_d) + 2A_2L_{0001}\sigma\omega_d(4A_3{}^2K_{0010}K_{1100}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2)$$

$$+ A_4{}^2L_{0010}L_{1100}(-\sigma^4 + 6\sigma^2\omega_d{}^2 + 7\omega_d{}^4))\sin(2t\omega_d)$$

$$+ 2A_2A_3A_4K_{0010}L_{0001}L_{1100}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2 - 7\omega_d{}^4)\sin(\varphi_2 - 2t\omega_d)$$

$$+ 2A_1e^{\sigma t}K_{0001}\omega_d(\sigma^2 + \omega_d{}^2)^2(2A_4{}^2L_{0010}L_{1100}\sigma + A_3{}^2K_{0010}K_{1100}(2\sigma + \sigma^2t$$

$$+ 9t\omega_d{}^2))\sin(\varphi_1 - t\omega_d) + 2A_1A_4{}^2e^{\sigma t}K_{0001}L_{0010}L_{1100}t\omega_d(\sigma^2 + \omega_d{}^2)^2(\sigma^2$$

$$+ 9\omega_d{}^2)\sin(\varphi_1 - 2\varphi_2 - t\omega_d)$$

$$+ 2A_1A_3A_4e^{\sigma t}K_{0001}\omega_d(\sigma^2 + \omega_d{}^2)^2(K_{1100}L_{0010}t(\sigma^2 + 9\omega_d{}^2) + K_{0010}L_{1100}(2\sigma$$

$$+ \sigma^2t + 9t\omega_d{}^2))\sin(\varphi_1 - \varphi_2 - t\omega_d) + 2A_2A_3A_4e^{\sigma t}L_{0001}\omega_d(\sigma^2$$

$$+ \omega_d{}^2)(2K_{0010}L_{1100}\sigma(\sigma^2 + 9\omega_d{}^2) + K_{1100}L_{0010}(\sigma^2 + \omega_d{}^2)(2\sigma + \sigma^2t$$

$$+ 9t\omega_d{}^2))\sin(\varphi_2 - t\omega_d)$$

$$+ 4A_1A_3A_4e^{\sigma t}K_{0001}K_{1100}L_{0010}\sigma\omega_d(\sigma^2 + \omega_d{}^2)^2\sin(\varphi_1 + \varphi_2 - t\omega_d)$$

$$- 2A_1e^{\sigma t}K_{0001}\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)({A_4}^2L_{0010}L_{1100}(2\sigma + \sigma^2 t + t{\omega_d}^2)$$

$$+ {A_3}^2K_{0010}K_{1100}(2\sigma - \sigma^2 t + 3t{\omega_d}^2))\sin(\varphi_1 + t\omega_d)$$

$$- 2A_1A_3A_4e^{\sigma t}K_{0001}\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)(2K_{0010}L_{1100}\sigma$$

$$+ K_{1100}L_{0010}t(\sigma^2 + {\omega_d}^2))\sin(\varphi_1 - \varphi_2 + t\omega_d) - 2A_2A_3A_4e^{\sigma t}L_{0001}\omega_d(\sigma^2$$

$$+ {\omega_d}^2)(-K_{1100}L_{0010}(\sigma^2 + 9{\omega_d}^2)(-2\sigma + \sigma^2 t - 3t{\omega_d}^2) + 2K_{0010}L_{1100}(\sigma^3$$

$$+ \sigma{\omega_d}^2 + 2\sigma^2 t{\omega_d}^2 + 18t{\omega_d}^4))\sin(\varphi_2 + t\omega_d)$$

$$+ 2A_2{A_4}^2L_{0001}L_{0010}L_{1100}\sigma\omega_d(\sigma^4 + 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4)\sin(2(\varphi_2 + t\omega_d))$$

$$- 2A_1A_3A_4e^{\sigma t}K_{0001}\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)(-K_{0010}L_{1100}t(\sigma^2 - 3{\omega_d}^2)$$

$$+ 2K_{1100}L_{0010}(\sigma - \sigma^2 t + t{\omega_d}^2))\sin(\varphi_1 + \varphi_2 + t\omega_d)$$

$$+ 2A_2{A_4}^2e^{\sigma t}L_{0001}L_{0010}L_{1100}t\omega_d(\sigma^6 + 7\sigma^4{\omega_d}^2 - 21\sigma^2{\omega_d}^4 - 27{\omega_d}^6)\sin(2\varphi_2$$

$$+ t\omega_d) + 4A_1{A_4}^2e^{\sigma t}K_{0001}L_{0010}L_{1100}t\omega_d(\sigma^6 + 9\sigma^4{\omega_d}^2 - \sigma^2{\omega_d}^4$$

$$- 9{\omega_d}^6)\sin(\varphi_1 + 2\varphi_2 + t\omega_d) + 2A_1K_{0001}\sigma\omega_d(4{A_3}^2K_{0010}K_{1100}{\omega_d}^2(3\sigma^2$$

$$- 5{\omega_d}^2) + {A_4}^2L_{0010}L_{1100}(-\sigma^4 + 6\sigma^2{\omega_d}^2 + 7{\omega_d}^4))\sin(\varphi_1 + 2t\omega_d)$$

$$- 2A_1A_3A_4K_{0001}K_{0010}L_{1100}\sigma\omega_d(\sigma^4 - 6\sigma^2{\omega_d}^2 - 7{\omega_d}^4)\sin(\varphi_1 - \varphi_2 + 2t\omega_d)$$

$$+ 2A_2A_3A_4L_{0001}\sigma\omega_d(4K_{1100}L_{0010}{\omega_d}^2(3\sigma^2 - 5{\omega_d}^2) + K_{0010}L_{1100}(\sigma^4$$

$$+ 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4))\sin(\varphi_2 + 2t\omega_d)$$

$$+ 2A_1A_3A_4K_{0001}\sigma\omega_d(4K_{1100}L_{0010}{\omega_d}^2(3\sigma^2 - 5{\omega_d}^2) + K_{0010}L_{1100}(\sigma^4$$

$$+ 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4))\sin(\varphi_1 + \varphi_2 + 2t\omega_d) + 2A_1{A_4}^2K_{0001}L_{0010}L_{1100}\sigma\omega_d(\sigma^4$$

$$+ 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4)\sin(\varphi_1 + 2(\varphi_2 + t\omega_d))]\}$$

(C. 44)

$$\boldsymbol{x_{2,2}^{bs1s2,u_2,u_2}(t)} = \frac{1}{4\omega_d(\sigma^2+{\omega_d}^2)^3(\sigma^2+9{\omega_d}^2)} e^{-2\sigma t}\{A_1K_{0001}{\omega_d}^2(\sigma^2+9{\omega_d}^2)({A_4}^2(1$$

$$+2e^{2\sigma t})L_{0001}L_{1100}(\sigma^2+{\omega_d}^2)+2{A_3}^2K_{0001}K_{1100}(\sigma^2+(1$$

$$+2e^{2\sigma t}){\omega_d}^2))\cos(\varphi_1)+A_1{A_4}^2(1+e^{2\sigma t})K_{0001}L_{0001}L_{1100}{\omega_d}^2(\sigma^4+10\sigma^2{\omega_d}^2$$

$$+9{\omega_d}^4)\cos(\varphi_1-2\varphi_2)+\frac{1}{2}[2{\omega_d}^2(A_2L_{0001}(\sigma^2+9{\omega_d}^2)({A_4}^2(1$$

$$+2e^{2\sigma t})L_{0001}L_{1100}(\sigma^2+{\omega_d}^2)+2{A_3}^2K_{0001}K_{1100}(\sigma^2+(1+2e^{2\sigma t}){\omega_d}^2))$$

$$+A_1A_3A_4K_{0001}(2(1+e^{2\sigma t})K_{1100}L_{0001}+(1+2e^{2\sigma t})K_{0001}L_{1100})(\sigma^4$$

$$+10\sigma^2{\omega_d}^2+9{\omega_d}^4)\cos(\varphi_1-\varphi_2))+4A_2A_3A_4L_{0001}(K_{1100}L_{0001}$$

$$+K_{0001}L_{1100}){\omega_d}^2(\sigma^2+9{\omega_d}^2)(\sigma^2+(1+2e^{2\sigma t}){\omega_d}^2)\cos(\varphi_2)$$

$$+2A_2{A_4}^2{L_{0001}}^2L_{1100}{\omega_d}^2(\sigma^2+9{\omega_d}^2)((1-2e^{2\sigma t})\sigma^2+(1$$

$$+2e^{2\sigma t}){\omega_d}^2)\cos(2\varphi_2)+2A_1A_3A_4K_{0001}{\omega_d}^2(\sigma^2+9{\omega_d}^2)(-2e^{2\sigma t}(K_{1100}L_{0001}$$

$$+K_{0001}L_{1100})(\sigma^2-{\omega_d}^2)+K_{0001}L_{1100}(\sigma^2+{\omega_d}^2))\cos(\varphi_1+\varphi_2)$$

$$+2A_1{A_4}^2e^{2\sigma t}K_{0001}L_{0001}L_{1100}{\omega_d}^2(-3\sigma^4-26\sigma^2{\omega_d}^2+9{\omega_d}^4)\cos(\varphi_1+2\varphi_2)$$

$$+4A_2e^{\sigma t}L_{0001}{\omega_d}^2(\sigma^2+{\omega_d}^2)({A_4}^2L_{0001}L_{1100}(\sigma^2-15{\omega_d}^2)$$

$$+2{A_3}^2K_{0001}K_{1100}(\sigma^3t-12{\omega_d}^2+9\sigma t{\omega_d}^2))\cos(t\omega_d)$$

$$-2A_2L_{0001}{\omega_d}^2({A_4}^2L_{0001}L_{1100}(5\sigma^4+2\sigma^2{\omega_d}^2-3{\omega_d}^4)+2{A_3}^2K_{0001}K_{1100}(\sigma^4$$

$$-12\sigma^2{\omega_d}^2+3{\omega_d}^4))\cos(2t\omega_d)+2A_2A_3A_4K_{0001}L_{0001}L_{1100}{\omega_d}^2(-5\sigma^4$$

$$-2\sigma^2{\omega_d}^2+3{\omega_d}^4)\cos(\varphi_2-2t\omega_d)-A_1e^{\sigma t}K_{0001}(\sigma^2$$

$$+{\omega_d}^2)(3{A_4}^2L_{0001}L_{1100}(\sigma^4+6\sigma^2{\omega_d}^2+5{\omega_d}^4)+2{A_3}^2K_{0001}K_{1100}(\sigma^4$$

$$+6\sigma^2{\omega_d}^2+21{\omega_d}^4))\cos(\varphi_1-t\omega_d)+A_1{A_4}^2e^{\sigma t}K_{0001}L_{0001}L_{1100}(\sigma^6$$

$$+7\sigma^4{\omega_d}^2-21\sigma^2{\omega_d}^4-27{\omega_d}^6)\cos(\varphi_1-2\varphi_2-t\omega_d)-A_1A_3A_4e^{\sigma t}K_{0001}(\sigma^2$$

$$+{\omega_d}^2)(4K_{1100}L_{0001}{\omega_d}^2(\sigma^2+9{\omega_d}^2)+K_{0001}L_{1100}(\sigma^4+2\sigma^2{\omega_d}^2$$

$$+33{\omega_d}^4))\cos(\varphi_1-\varphi_2-t\omega_d)-2A_2A_3A_4e^{\sigma t}L_{0001}(\sigma^2$$

$$+ {\omega_d}^2)(2K_{0001}L_{1100}{\omega_d}^2(\sigma^2 + 9{\omega_d}^2) + K_{1100}L_{0001}(\sigma^4 + 6\sigma^2{\omega_d}^2$$

$$+ 21{\omega_d}^4))\cos(\varphi_2 - t\omega_d) - A_1A_3A_4e^{\sigma t}K_{0001}(\sigma^2 + {\omega_d}^2)^2(2K_{1100}L_{0001}(\sigma^2$$

$$+ 3{\omega_d}^2) + K_{0001}L_{1100}(\sigma^2 + 9{\omega_d}^2))\cos(\varphi_1 + \varphi_2 - t\omega_d)$$

$$- A_2{A_4}^2e^{\sigma t}{L_{0001}}^2L_{1100}(\sigma^2 + {\omega_d}^2)^2(\sigma^2 + 9{\omega_d}^2)\cos(2\varphi_2 - t\omega_d)$$

$$+ A_1e^{\sigma t}K_{0001}(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)({A_4}^2L_{0001}L_{1100}(3\sigma^2 - 5{\omega_d}^2)$$

$$+ 2{A_3}^2K_{0001}K_{1100}(\sigma^2 - 3{\omega_d}^2 + 4\sigma t{\omega_d}^2))\cos(\varphi_1 + t\omega_d)$$

$$- A_1{A_4}^2e^{\sigma t}K_{0001}L_{0001}L_{1100}(\sigma^2 + {\omega_d}^2)^2(\sigma^2 + 9{\omega_d}^2)\cos(\varphi_1 - 2\varphi_2 + t\omega_d)$$

$$+ A_1A_3A_4e^{\sigma t}K_{0001}(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)(-4K_{1100}L_{0001}{\omega_d}^2$$

$$+ K_{0001}L_{1100}(\sigma^2 - 3{\omega_d}^2))\cos(\varphi_1 - \varphi_2 + t\omega_d) + 2A_2A_3A_4e^{\sigma t}L_{0001}(\sigma^2$$

$$+ {\omega_d}^2)(K_{1100}L_{0001}(\sigma^2 + 9{\omega_d}^2)(\sigma^2 - 3{\omega_d}^2 + 4\sigma t{\omega_d}^2) + 2K_{0001}L_{1100}{\omega_d}^2(\sigma^2$$

$$+ 2\sigma^3t - 15{\omega_d}^2 + 18\sigma t{\omega_d}^2))\cos(\varphi_2 + t\omega_d) + 2A_2{A_4}^2{L_{0001}}^2L_{1100}{\omega_d}^2(3\sigma^4$$

$$+ 26\sigma^2{\omega_d}^2 - 9{\omega_d}^4)\cos(2(\varphi_2 + t\omega_d)) + A_1A_3A_4e^{\sigma t}K_{0001}(\sigma^4 + 10\sigma^2{\omega_d}^2$$

$$+ 9{\omega_d}^4)(2K_{1100}L_{0001}(\sigma^2 - {\omega_d}^2 + 4\sigma t{\omega_d}^2) + K_{0001}L_{1100}(\sigma^2 - 3{\omega_d}^2$$

$$+ 8\sigma t{\omega_d}^2))\cos(\varphi_1 + \varphi_2 + t\omega_d) + A_2{A_4}^2e^{\sigma t}{L_{0001}}^2L_{1100}(\sigma^2 - 3{\omega_d}^2$$

$$+ 8\sigma t{\omega_d}^2)(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\cos(2\varphi_2 + t\omega_d)$$

$$+ 8A_1{A_4}^2e^{\sigma t}K_{0001}L_{0001}L_{1100}\sigma t{\omega_d}^2(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\cos(\varphi_1 + 2\varphi_2$$

$$+ t\omega_d) - 2A_1K_{0001}{\omega_d}^2({A_4}^2L_{0001}L_{1100}(5\sigma^4 + 2\sigma^2{\omega_d}^2 - 3{\omega_d}^4)$$

$$+ 2{A_3}^2K_{0001}K_{1100}(\sigma^4 - 12\sigma^2{\omega_d}^2 + 3{\omega_d}^4))\cos(\varphi_1 + 2t\omega_d)$$

$$+ 2A_1A_3A_4{K_{0001}}^2L_{1100}{\omega_d}^2(-5\sigma^4 - 2\sigma^2{\omega_d}^2 + 3{\omega_d}^4)\cos(\varphi_1 - \varphi_2 + 2t\omega_d)$$

$$+ 2A_2A_3A_4L_{0001}{\omega_d}^2(K_{0001}L_{1100}(3\sigma^4 + 26\sigma^2{\omega_d}^2 - 9{\omega_d}^4) - 2K_{1100}L_{0001}(\sigma^4$$

$$- 12\sigma^2{\omega_d}^2 + 3{\omega_d}^4))\cos(\varphi_2 + 2t\omega_d) + 2A_1A_3A_4K_{0001}{\omega_d}^2(K_{0001}L_{1100}(3\sigma^4$$

$$+ 26\sigma^2{\omega_d}^2 - 9{\omega_d}^4) - 2K_{1100}L_{0001}(\sigma^4 - 12\sigma^2{\omega_d}^2 + 3{\omega_d}^4))\cos(\varphi_1 + \varphi_2$$

$$+ 2t\omega_d) + 2A_1{A_4}^2K_{0001}L_{0001}L_{1100}{\omega_d}^2(3\sigma^4 + 26\sigma^2{\omega_d}^2 - 9{\omega_d}^4)\cos(\varphi_1$$

$$+ 2(\varphi_2 + t\omega_d)) + 2A_1K_{0001}\sigma\omega_d(\sigma^2 + 9{\omega_d}^2)(4{A_3}^2e^{2\sigma t}K_{0001}K_{1100}{\omega_d}^2$$

$$+ {A_4}^2(-1 + 2e^{2\sigma t})L_{0001}L_{1100}(\sigma^2 + {\omega_d}^2))\sin(\varphi_1) - 2A_1{A_4}^2(-1$$

$$+ e^{2\sigma t})K_{0001}L_{0001}L_{1100}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_1 - 2\varphi_2)$$

$$+ 2A_1A_3A_4{K_{0001}}^2L_{1100}\sigma\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)\sin(\varphi_1 - \varphi_2)$$

$$- 4A_2A_3A_4L_{0001}\omega_d(\sigma^3 + 9\sigma{\omega_d}^2)(-2e^{2\sigma t}K_{1100}L_{0001}{\omega_d}^2 + K_{0001}L_{1100}(\sigma^2$$

$$+ (1 - 2e^{2\sigma t}){\omega_d}^2))\sin(\varphi_2) - 2A_2{A_4}^2{L_{0001}}^2L_{1100}\omega_d(\sigma^2 + (1$$

$$- 4e^{2\sigma t}){\omega_d}^2)(\sigma^3 + 9\sigma{\omega_d}^2)\sin(2\varphi_2) - 2A_1A_3A_4K_{0001}\omega_d(\sigma^3$$

$$+ 9\sigma{\omega_d}^2)(-4e^{2\sigma t}K_{1100}L_{0001}{\omega_d}^2 + K_{0001}L_{1100}(\sigma^2 + (1 - 4e^{2\sigma t}){\omega_d}^2))\sin(\varphi_1$$

$$+ \varphi_2) - 2A_1{A_4}^2e^{2\sigma t}K_{0001}L_{0001}L_{1100}\omega_d(\sigma^5 + 6\sigma^3{\omega_d}^2 - 27\sigma{\omega_d}^4)\sin(\varphi_1$$

$$+ 2\varphi_2) - 2A_2e^{\sigma t}L_{0001}\omega_d(\sigma^2 + {\omega_d}^2)(4{A_3}^2K_{0001}K_{1100}(\sigma^3 + 5\sigma{\omega_d}^2 + \sigma^2t{\omega_d}^2$$

$$+ 9t{\omega_d}^4) + {A_4}^2L_{0001}L_{1100}(4\sigma^3 + \sigma^4t + 20\sigma{\omega_d}^2 + 10\sigma^2t{\omega_d}^2$$

$$+ 9t{\omega_d}^4))\sin(t\omega_d) + 2A_2L_{0001}\sigma\omega_d(4{A_3}^2K_{0001}K_{1100}{\omega_d}^2(3\sigma^2 - 5{\omega_d}^2)$$

$$+ {A_4}^2L_{0001}L_{1100}(-\sigma^4 + 6\sigma^2{\omega_d}^2 + 7{\omega_d}^4))\sin(2t\omega_d)$$

$$+ 2A_2A_3A_4K_{0001}L_{0001}L_{1100}\sigma\omega_d(\sigma^4 - 6\sigma^2{\omega_d}^2 - 7{\omega_d}^4)\sin(\varphi_2 - 2t\omega_d)$$

$$+ 2A_1e^{\sigma t}K_{0001}\omega_d(\sigma^2 + {\omega_d}^2)^2(2{A_4}^2L_{0001}L_{1100}\sigma + {A_3}^2K_{0001}K_{1100}(2\sigma + \sigma^2t$$

$$+ 9t{\omega_d}^2))\sin(\varphi_1 - t\omega_d) + 2A_1{A_4}^2e^{\sigma t}K_{0001}L_{0001}L_{1100}t\omega_d(\sigma^2 + {\omega_d}^2)^2(\sigma^2$$

$$+ 9{\omega_d}^2)\sin(\varphi_1 - 2\varphi_2 - t\omega_d)$$

$$+ 2A_1A_3A_4e^{\sigma t}K_{0001}\omega_d(\sigma^2 + {\omega_d}^2)^2(K_{1100}L_{0001}t(\sigma^2 + 9{\omega_d}^2) + K_{0001}L_{1100}(2\sigma$$

$$+ \sigma^2t + 9t{\omega_d}^2))\sin(\varphi_1 - \varphi_2 - t\omega_d) + 2A_2A_3A_4e^{\sigma t}L_{0001}\omega_d(\sigma^2$$

$$+ {\omega_d}^2)(2K_{0001}L_{1100}\sigma(\sigma^2 + 9{\omega_d}^2) + K_{1100}L_{0001}(\sigma^2 + {\omega_d}^2)(2\sigma + \sigma^2t$$

$$+ 9t{\omega_d}^2))\sin(\varphi_2 - t\omega_d)$$

$$+ 4A_1A_3A_4e^{\sigma t}K_{0001}K_{1100}L_{0001}\sigma\omega_d(\sigma^2 + {\omega_d}^2)^2\sin(\varphi_1 + \varphi_2 - t\omega_d)$$

$$- 2A_1e^{\sigma t}K_{0001}\omega_d(\sigma^4 + 10\sigma^2{\omega_d}^2 + 9{\omega_d}^4)({A_4}^2L_{0001}L_{1100}(2\sigma + \sigma^2t + t{\omega_d}^2)$$

$$+ {A_3}^2 K_{0001} K_{1100} (2\sigma - \sigma^2 t + 3t{\omega_d}^2)) \sin(\varphi_1 + t\omega_d)$$

$$- 2A_1 A_3 A_4 e^{\sigma t} K_{0001} \omega_d (\sigma^4 + 10\sigma^2 {\omega_d}^2 + 9{\omega_d}^4)(2K_{0001} L_{1100} \sigma$$

$$+ K_{1100} L_{0001} t(\sigma^2 + {\omega_d}^2)) \sin(\varphi_1 - \varphi_2 + t\omega_d) - 2A_2 A_3 A_4 e^{\sigma t} L_{0001} \omega_d (\sigma^2$$

$$+ {\omega_d}^2)(-K_{1100} L_{0001} (\sigma^2 + 9{\omega_d}^2)(-2\sigma + \sigma^2 t - 3t{\omega_d}^2) + 2K_{0001} L_{1100} (\sigma^3$$

$$+ \sigma {\omega_d}^2 + 2\sigma^2 t{\omega_d}^2 + 18t{\omega_d}^4)) \sin(\varphi_2 + t\omega_d) + 2A_2 {A_4}^2 {L_{0001}}^2 L_{1100} \sigma \omega_d (\sigma^4$$

$$+ 6\sigma^2 {\omega_d}^2 - 27{\omega_d}^4) \sin(2(\varphi_2 + t\omega_d)) - 2A_1 A_3 A_4 e^{\sigma t} K_{0001} \omega_d (\sigma^4 + 10\sigma^2 {\omega_d}^2$$

$$+ 9{\omega_d}^4)(-K_{0001} L_{1100} t(\sigma^2 - 3{\omega_d}^2) + 2K_{1100} L_{0001} (\sigma - \sigma^2 t + t{\omega_d}^2)) \sin(\varphi_1$$

$$+ \varphi_2 + t\omega_d) + 2A_2 {A_4}^2 e^{\sigma t} {L_{0001}}^2 L_{1100} t\omega_d (\sigma^6 + 7\sigma^4 {\omega_d}^2 - 21\sigma^2 {\omega_d}^4$$

$$- 27{\omega_d}^6) \sin(2\varphi_2 + t\omega_d) + 4A_1 {A_4}^2 e^{\sigma t} K_{0001} L_{0001} L_{1100} t\omega_d (\sigma^6 + 9\sigma^4 {\omega_d}^2$$

$$- \sigma^2 {\omega_d}^4 - 9{\omega_d}^6) \sin(\varphi_1 + 2\varphi_2 + t\omega_d)$$

$$+ 2A_1 K_{0001} \sigma \omega_d (4{A_3}^2 K_{0001} K_{1100} {\omega_d}^2 (3\sigma^2 - 5{\omega_d}^2) + {A_4}^2 L_{0001} L_{1100} (-\sigma^4$$

$$+ 6\sigma^2 {\omega_d}^2 + 7{\omega_d}^4)) \sin(\varphi_1 + 2t\omega_d) - 2A_1 A_3 A_4 {K_{0001}}^2 L_{1100} \sigma \omega_d (\sigma^4$$

$$- 6\sigma^2 {\omega_d}^2 - 7{\omega_d}^4) \sin(\varphi_1 - \varphi_2 + 2t\omega_d)$$

$$+ 2A_2 A_3 A_4 L_{0001} \sigma \omega_d (4K_{1100} L_{0001} {\omega_d}^2 (3\sigma^2 - 5{\omega_d}^2) + K_{0001} L_{1100} (\sigma^4$$

$$+ 6\sigma^2 {\omega_d}^2 - 27{\omega_d}^4)) \sin(\varphi_2 + 2t\omega_d)$$

$$+ 2A_1 A_3 A_4 K_{0001} \sigma \omega_d (4K_{1100} L_{0001} {\omega_d}^2 (3\sigma^2 - 5{\omega_d}^2) + K_{0001} L_{1100} (\sigma^4$$

$$+ 6\sigma^2 {\omega_d}^2 - 27{\omega_d}^4)) \sin(\varphi_1 + \varphi_2 + 2t\omega_d) + 2A_1 {A_4}^2 K_{0001} L_{0001} L_{1100} \sigma \omega_d (\sigma^4$$

$$+ 6\sigma^2 {\omega_d}^2 - 27{\omega_d}^4) \sin(\varphi_1 + 2(\varphi_2 + t\omega_d))]\}$$

(C. 45)

$$x_{2,2}^{qs2}(t) = x_{2,2}^{qs2,u_1,u_1}(t) \times u_1u_1 + x_{2,2}^{qs2,u_1,u_2}(t) \times u_1u_2 + x_{2,2}^{qs2,u_2,u_1}(t) \times u_2u_1 + x_{2,2}^{qs2,u_2,u_2}(t) \times u_2u_2 \tag{C.46}$$

$$x_{2,2}^{qs2,u_1,u_1}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{\omega_d{}^2(\sigma^2+9\omega_d{}^2)(2A_3A_4{}^2L_{0010}((1$$

$$+ e^{2\sigma t})K_{0200}L_{0010} + (1+2e^{2\sigma t})K_{0010}L_{0200})(\sigma^2+\omega_d{}^2)$$

$$+ 2A_3{}^3K_{0010}{}^2K_{0200}(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2) + A_4{}^3(2+3e^{2\sigma t})L_{0010}{}^2L_{0200}(\sigma^2$$

$$+ \omega_d{}^2)\cos(\varphi_2) + 2A_3{}^2A_4K_{0010}(2K_{0200}L_{0010}+K_{0010}L_{0200})(\sigma^2+(1$$

$$+ 2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_2)) + 2A_3A_4{}^2L_{0010}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(-e^{2\sigma t}(K_{0200}L_{0010}$$

$$+ 2K_{0010}L_{0200})(\sigma^2-\omega_d{}^2) + K_{0010}L_{0200}(\sigma^2+\omega_d{}^2))\cos(2\varphi_2)$$

$$+ A_4{}^3e^{2\sigma t}L_{0010}{}^2L_{0200}\omega_d{}^2(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(3\varphi_2)$$

$$+ 4A_3e^{\sigma t}\omega_d{}^2(\sigma^2+\omega_d{}^2)(A_3{}^2K_{0010}{}^2K_{0200}(\sigma^3t-12\omega_d{}^2+9\sigma t\omega_d{}^2)$$

$$- A_4{}^2L_{0010}(-K_{0010}L_{0200}(\sigma^2-15\omega_d{}^2)+K_{0200}L_{0010}(\sigma^2+9\omega_d{}^2)))\cos(t\omega_d)$$

$$- 2A_3K_{0010}\omega_d{}^2(A_4{}^2L_{0010}L_{0200}(5\sigma^4+2\sigma^2\omega_d{}^2-3\omega_d{}^4)+A_3{}^2K_{0010}K_{0200}(\sigma^4$$

$$- 12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(2t\omega_d) + A_3{}^2A_4K_{0010}{}^2L_{0200}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(\varphi_2-2t\omega_d) - 2A_4e^{\sigma t}(\sigma^2+\omega_d{}^2)(A_4{}^2L_{0010}{}^2L_{0200}(\sigma^4+7\sigma^2\omega_d{}^2$$

$$+ 6\omega_d{}^4) + A_3{}^2K_{0010}(K_{0010}L_{0200}\omega_d{}^2(\sigma^2+9\omega_d{}^2)+K_{0200}L_{0010}(\sigma^4+6\sigma^2\omega_d{}^2$$

$$+ 21\omega_d{}^4)))\cos(\varphi_2-t\omega_d) - A_3A_4{}^2e^{\sigma t}L_{0010}(\sigma^2+\omega_d{}^2)^2(K_{0200}L_{0010}(\sigma^2$$

$$+ 3\omega_d{}^2) + K_{0010}L_{0200}(\sigma^2+9\omega_d{}^2))\cos(2\varphi_2-t\omega_d) + 2A_4e^{\sigma t}(\sigma^2$$

$$+ \omega_d{}^2)(A_4{}^2L_{0010}{}^2L_{0200}(\sigma^4+7\sigma^2\omega_d{}^2-18\omega_d{}^4) + A_3{}^2K_{0010}(K_{0200}L_{0010}(\sigma^2$$

$$+ 9\omega_d{}^2)(\sigma^2-3\omega_d{}^2+4\sigma t\omega_d{}^2) + K_{0010}L_{0200}\omega_d{}^2(\sigma^2+2\sigma^3t-15\omega_d{}^2$$

$$+ 18\sigma t\omega_d{}^2)))\cos(\varphi_2+t\omega_d) - 2A_3A_4{}^2L_{0010}\omega_d{}^2(K_{0200}L_{0010}(\sigma^4-12\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4) + K_{0010}L_{0200}(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4))\cos(2(\varphi_2+t\omega_d))$$

$$+ A_3A_4{}^2e^{\sigma t}L_{0010}(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)(K_{0200}L_{0010}(\sigma^2-\omega_d{}^2+4\sigma t\omega_d{}^2)$$

$$+ K_{0010}L_{0200}(\sigma^2-3\omega_d{}^2+8\sigma t\omega_d{}^2))\cos(2\varphi_2+t\omega_d)$$

$$+ 4A_4{}^3e^{\sigma t}L_{0010}{}^2L_{0200}\sigma t\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(3\varphi_2+t\omega_d)$$

$$+ A_4\omega_d{}^2(A_4{}^2L_{0010}{}^2L_{0200}(-5\sigma^4 - 2\sigma^2\omega_d{}^2 + 3\omega_d{}^4)$$

$$+ A_3{}^2K_{0010}(K_{0010}L_{0200}(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4) - 4K_{0200}L_{0010}(\sigma^4$$

$$- 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4)))\cos(\varphi_2 + 2t\omega_d) + A_4{}^3L_{0010}{}^2L_{0200}\omega_d{}^2(3\sigma^4 + 26\sigma^2\omega_d{}^2$$

$$- 9\omega_d{}^4)\cos(3\varphi_2 + 2t\omega_d) + A_4\sigma\omega_d(\sigma^2 + 9\omega_d{}^2)(A_4{}^2(-2$$

$$+ 3e^{2\sigma t})L_{0010}{}^2L_{0200}(\sigma^2 + \omega_d{}^2) - 2A_3{}^2K_{0010}(-4e^{2\sigma t}K_{0200}L_{0010}\omega_d{}^2$$

$$+ K_{0010}L_{0200}(\sigma^2 + \omega_d{}^2 - 2e^{2\sigma t}\omega_d{}^2)))\sin(\varphi_2) - 2A_3A_4{}^2L_{0010}\omega_d(\sigma^3$$

$$+ 9\sigma\omega_d{}^2)(-2e^{2\sigma t}K_{0200}L_{0010}\omega_d{}^2 + K_{0010}L_{0200}(\sigma^2 + (1$$

$$- 4e^{2\sigma t})\omega_d{}^2))\sin(2\varphi_2) - A_4{}^3e^{2\sigma t}L_{0010}{}^2L_{0200}\omega_d(\sigma^5 + 6\sigma^3\omega_d{}^2$$

$$- 27\sigma\omega_d{}^4)\sin(3\varphi_2) - 2A_3e^{\sigma t}\omega_d(\sigma^2 + \omega_d{}^2)(2A_3{}^2K_{0010}{}^2K_{0200}(\sigma^3 + 5\sigma\omega_d{}^2$$

$$+ \sigma^2t\omega_d{}^2 + 9t\omega_d{}^4) + A_4{}^2L_{0010}(K_{0200}L_{0010}t(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)$$

$$+ K_{0010}L_{0200}(4\sigma^3 + \sigma^4t + 20\sigma\omega_d{}^2 + 10\sigma^2t\omega_d{}^2 + 9t\omega_d{}^4)))\sin(t\omega_d)$$

$$+ 2A_3K_{0010}\sigma\omega_d(2A_3{}^2K_{0010}K_{0200}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + A_4{}^2L_{0010}L_{0200}(-\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2 + 7\omega_d{}^4))\sin(2t\omega_d) + A_3{}^2A_4K_{0010}{}^2L_{0200}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2$$

$$- 7\omega_d{}^4)\sin(\varphi_2 - 2t\omega_d) + 2A_4e^{\sigma t}\omega_d(\sigma^2 + \omega_d{}^2)(A_4{}^2L_{0010}{}^2L_{0200}\sigma(\sigma^2 + \omega_d{}^2)$$

$$+ A_3{}^2K_{0010}(K_{0010}L_{0200}\sigma(\sigma^2 + 9\omega_d{}^2) + K_{0200}L_{0010}(\sigma^2 + \omega_d{}^2)(2\sigma + \sigma^2t$$

$$+ 9t\omega_d{}^2)))\sin(\varphi_2 - t\omega_d) + 2A_3A_4{}^2e^{\sigma t}K_{0200}L_{0010}{}^2\sigma\omega_d(\sigma^2 + \omega_d{}^2)^2\sin(2\varphi_2$$

$$- t\omega_d) - 2A_4e^{\sigma t}\omega_d(\sigma^2 + \omega_d{}^2)(A_4{}^2L_{0010}{}^2L_{0200}(\sigma^2 + 9\omega_d{}^2)(\sigma + \sigma^2t + t\omega_d{}^2)$$

$$+ A_3{}^2K_{0010}(-K_{0200}L_{0010}(\sigma^2 + 9\omega_d{}^2)(-2\sigma + \sigma^2t - 3t\omega_d{}^2) + K_{0010}L_{0200}(\sigma^3$$

$$+ \sigma\omega_d{}^2 + 2\sigma^2t\omega_d{}^2 + 18t\omega_d{}^4)))\sin(\varphi_2 + t\omega_d)$$

$$+ 2A_3A_4{}^2L_{0010}\sigma\omega_d(2K_{0200}L_{0010}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + K_{0010}L_{0200}(\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4))\sin(2(\varphi_2 + t\omega_d)) - 2A_3A_4{}^2e^{\sigma t}L_{0010}\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)(-K_{0010}L_{0200}t(\sigma^2 - 3\omega_d{}^2) + K_{0200}L_{0010}(\sigma - \sigma^2t + t\omega_d{}^2))\sin(2\varphi_2$$

$$+ t\omega_d) + 2A_4{}^3 e^{\sigma t} L_{0010}{}^2 L_{0200} t\omega_d(\sigma^6 + 9\sigma^4\omega_d{}^2 - \sigma^2\omega_d{}^4 - 9\omega_d{}^6)\sin(3\varphi_2$$

$$+ t\omega_d) + A_4\sigma\omega_d(A_4{}^2 L_{0010}{}^2 L_{0200}(-\sigma^4 + 6\sigma^2\omega_d{}^2 + 7\omega_d{}^4)$$

$$+ A_3{}^2 K_{0010}(8K_{0200}L_{0010}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + K_{0010}L_{0200}(\sigma^4 + 6\sigma^2\omega_d{}^2$$

$$- 27\omega_d{}^4)))\sin(\varphi_2 + 2t\omega_d) + A_4{}^3 L_{0010}{}^2 L_{0200}\sigma\omega_d(\sigma^4 + 6\sigma^2\omega_d{}^2$$

$$- 27\omega_d{}^4)\sin(3\varphi_2 + 2t\omega_d)\}$$

(C. 47)

$$
\begin{aligned}
\boldsymbol{x_{2,2}^{qs2,u_1,u_2}}(\boldsymbol{t}) = {} & \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{\omega_d(A_3\omega_d(\sigma^2+9\omega_d{}^2)(A_4{}^2(2(1 \\
& + e^{2\sigma t})K_{0200}L_{0001}L_{0010} + (1+2e^{2\sigma t})(K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200})(\sigma^2 \\
& + \omega_d{}^2) + 2A_3{}^2K_{0001}K_{0010}K_{0200}(\sigma^2 + (1+2e^{2\sigma t})\omega_d{}^2)) + A_4\omega_d(\sigma^2 \\
& + 9\omega_d{}^2)(A_4{}^2(2+3e^{2\sigma t})L_{0001}L_{0010}L_{0200}(\sigma^2+\omega_d{}^2) + 2A_3{}^2(K_{0010}K_{0200}L_{0001} \\
& + K_{0001}K_{0200}L_{0010} + K_{0001}K_{0010}L_{0200})(\sigma^2+\omega_d{}^2+2e^{2\sigma t}\omega_d{}^2))\cos(\varphi_2) \\
& + A_3A_4{}^2\omega_d(\sigma^2+9\omega_d{}^2)(-2e^{2\sigma t}(K_{0200}L_{0001}L_{0010} + K_{0010}L_{0001}L_{0200} \\
& + K_{0001}L_{0010}L_{0200})(\sigma-\omega_d)(\sigma+\omega_d) + (K_{0010}L_{0001}+K_{0001}L_{0010})L_{0200}(\sigma^2 \\
& + \omega_d{}^2))\cos(2\varphi_2) + A_4{}^3e^{2\sigma t}L_{0001}L_{0010}L_{0200}\omega_d(-3\sigma^2+\omega_d{}^2)(\sigma^2 \\
& + 9\omega_d{}^2)\cos(3\varphi_2) + A_4\sigma(\sigma^2+9\omega_d{}^2)(A_4{}^2(-2+3e^{2\sigma t})L_{0001}L_{0010}L_{0200}(\sigma^2 \\
& + \omega_d{}^2) + 2A_3{}^2(2e^{2\sigma t}(K_{0010}K_{0200}L_{0001} + K_{0001}K_{0200}L_{0010} \\
& + K_{0001}K_{0010}L_{0200})\omega_d{}^2 - K_{0001}K_{0010}L_{0200}(\sigma^2+\omega_d{}^2)))\sin(\varphi_2) + A_3A_4{}^2\sigma(\sigma^2 \\
& + 9\omega_d{}^2)(4e^{2\sigma t}(K_{0200}L_{0001}L_{0010} + K_{0010}L_{0001}L_{0200} + K_{0001}L_{0010}L_{0200})\omega_d{}^2 \\
& - (K_{0010}L_{0001}+K_{0001}L_{0010})L_{0200}(\sigma^2+\omega_d{}^2))\sin(2\varphi_2) \\
& - A_4{}^3e^{2\sigma t}L_{0001}L_{0010}L_{0200}\sigma(\sigma^4+6\sigma^2\omega_d{}^2-27\omega_d{}^4)\sin(3\varphi_2)) \\
& + \omega_d\cos(2t\omega_d)(A_3\omega_d(-A_4{}^2(K_{0010}L_{0001}+K_{0001}L_{0010})L_{0200}(5\sigma^2-3\omega_d{}^2)(\sigma^2 \\
& + \omega_d{}^2) - 2A_3{}^2K_{0001}K_{0010}K_{0200}(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4)) \\
& + A_4(-\omega_d(A_4{}^2L_{0001}L_{0010}L_{0200}(5\sigma^2-3\omega_d{}^2)(\sigma^2+\omega_d{}^2) \\
& + 2A_3{}^2(K_{0010}K_{0200}L_{0001} + K_{0001}K_{0200}L_{0010} + K_{0001}K_{0010}L_{0200})(\sigma^4 \\
& - 12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(\varphi_2) + A_3A_4\omega_d((K_{0010}L_{0001}+K_{0001}L_{0010})L_{0200}(3\sigma^4 \\
& + 26\sigma^2\omega_d{}^2-9\omega_d{}^4) - 2K_{0200}L_{0001}L_{0010}(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(2\varphi_2) \\
& + A_4{}^2L_{0001}L_{0010}L_{0200}\omega_d(3\sigma^4+26\sigma^2\omega_d{}^2-9\omega_d{}^4)\cos(3\varphi_2) \\
& + \sigma(-A_4{}^2L_{0001}L_{0010}L_{0200}(\sigma^2-7\omega_d{}^2)(\sigma^2+\omega_d{}^2) + 2A_3{}^2(K_{0001}K_{0010}L_{0200}\sigma^4
\end{aligned}
$$

$+ 6K_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma^2{\omega_d}^2 - (10K_{0200}(K_{0010}L_{0001}$

$+ K_{0001}L_{0010}) + 17K_{0001}K_{0010}L_{0200}){\omega_d}^4))\sin(\varphi_2) + A_3A_4\sigma((K_{0010}L_{0001}$

$+ K_{0001}L_{0010})L_{0200}\sigma^4 + 6(2K_{0200}L_{0001}L_{0010} + K_{0010}L_{0001}L_{0200}$

$+ K_{0001}L_{0010}L_{0200})\sigma^2{\omega_d}^2 - (20K_{0200}L_{0001}L_{0010} + 27(K_{0010}L_{0001}$

$+ K_{0001}L_{0010})L_{0200}){\omega_d}^4)\sin(2\varphi_2) + {A_4}^2L_{0001}L_{0010}L_{0200}\sigma(\sigma^4 + 6\sigma^2{\omega_d}^2$

$- 27{\omega_d}^4)\sin(3\varphi_2))) + e^{\sigma t}\omega_d(\sigma^2$

$+ {\omega_d}^2)\cos(t\omega_d)(4{A_3}^3K_{0001}K_{0010}K_{0200}\omega_d(\sigma^3 t + 3(-4 + 3\sigma t){\omega_d}^2)$

$+ 2A_3{A_4}^2\omega_d((K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}(\sigma^2 - 15{\omega_d}^2)$

$- 2K_{0200}L_{0001}L_{0010}(\sigma^2 + 9{\omega_d}^2)) + A_4(4\omega_d(-12{A_4}^2L_{0001}L_{0010}L_{0200}{\omega_d}^2$

$+ {A_3}^2(K_{0010}K_{0200}L_{0001} + K_{0001}K_{0200}L_{0010} + K_{0001}K_{0010}L_{0200})(\sigma^3 t + 3(-4$

$+ 3\sigma t){\omega_d}^2))\cos(\varphi_2) + 2A_3A_4\omega_d((K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}(-1$

$+ 2\sigma t)(\sigma^2 + 9{\omega_d}^2) + 2K_{0200}L_{0001}L_{0010}(\sigma^2 + \sigma^3 t - 3{\omega_d}^2$

$+ 9\sigma t{\omega_d}^2))\cos(2\varphi_2) + 4{A_4}^2L_{0001}L_{0010}L_{0200}\sigma t\omega_d(\sigma^2 + 9{\omega_d}^2)\cos(3\varphi_2)$

$+ 2(-{A_4}^2L_{0001}L_{0010}L_{0200}(\sigma^4 t + 2\sigma(4 + 5\sigma t){\omega_d}^2 + 9t{\omega_d}^4)$

$+ {A_3}^2(K_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma^4 t + 2\sigma(K_{0001}K_{0010}L_{0200}(4 - \sigma t)$

$+ 4K_{0010}K_{0200}L_{0001}(-1 + \sigma t) + 4K_{0001}K_{0200}L_{0010}(-1 + \sigma t)){\omega_d}^2$

$- 9(K_{0010}K_{0200}L_{0001} + K_{0001}K_{0200}L_{0010} + 2K_{0001}K_{0010}L_{0200})t{\omega_d}^4))\sin(\varphi_2)$

$+ A_3A_4((K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}t(\sigma^4 + 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4)$

$+ 2K_{0200}L_{0001}L_{0010}(\sigma^4 t + 8\sigma(-1 + \sigma t){\omega_d}^2 - 9t{\omega_d}^4))\sin(2\varphi_2)$

$+ 2{A_4}^2L_{0001}L_{0010}L_{0200}t(\sigma^4 + 8\sigma^2{\omega_d}^2 - 9{\omega_d}^4)\sin(3\varphi_2))) + e^{\sigma t}(\sigma^2$

$+ {\omega_d}^2)(-A_3\omega_d(4{A_3}^2K_{0001}K_{0010}K_{0200}(\sigma^3 + \sigma(5 + \sigma t){\omega_d}^2 + 9t{\omega_d}^4)$

$+ {A_4}^2(\sigma^3(2K_{0200}L_{0001}L_{0010}\sigma t + K_{0010}L_{0001}L_{0200}(4 + \sigma t)$

$+ K_{0001}L_{0010}L_{0200}(4 + \sigma t)) + 10\sigma(2K_{0200}L_{0001}L_{0010}\sigma t + K_{0010}L_{0001}L_{0200}(2$

$$+ \sigma t) + K_{0001}L_{0010}L_{0200}(2 + \sigma t))\omega_d{}^2 + 9(2K_{0200}L_{0001}L_{0010}$$

$$+ K_{0010}L_{0001}L_{0200} + K_{0001}L_{0010}L_{0200})t\omega_d{}^4))$$

$$- 2A_4\omega_d(A_4{}^2L_{0001}L_{0010}L_{0200}(\sigma^3(2 + \sigma t) + 10\sigma(1 + \sigma t)\omega_d{}^2 + 9t\omega_d{}^4)$$

$$+ 2A_3{}^2(K_{0010}K_{0200}L_{0001} + K_{0001}K_{0200}L_{0010} + K_{0001}K_{0010}L_{0200})(\sigma^3 + \sigma(5$$

$$+ \sigma t)\omega_d{}^2 + 9t\omega_d{}^4))\cos(\varphi_2) + A_3A_4{}^2\omega_d((K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}t(\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4) + 2K_{0200}L_{0001}L_{0010}(\sigma^3(-2 + \sigma t) + 2\sigma(-5 + 4\sigma t)\omega_d{}^2$$

$$- 9t\omega_d{}^4))\cos(2\varphi_2) + 2A_4{}^3L_{0001}L_{0010}L_{0200}t\omega_d(\sigma^4 + 8\sigma^2\omega_d{}^2$$

$$- 9\omega_d{}^4)\cos(3\varphi_2) - 2A_4(2A_4{}^2L_{0001}L_{0010}L_{0200}(\sigma^4 + 7\sigma^2\omega_d{}^2 - 6\omega_d{}^4)$$

$$+ A_3{}^2(K_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma^4 + 2\sigma^2(K_{0001}K_{0010}L_{0200}(1 + \sigma t)$$

$$+ K_{0010}K_{0200}L_{0001}(3 + \sigma t) + K_{0001}K_{0200}L_{0010}(3 + \sigma t))\omega_d{}^2$$

$$+ 3(2K_{0001}K_{0010}L_{0200}(-1 + 3\sigma t) + K_{0010}K_{0200}L_{0001}(-1 + 6\sigma t)$$

$$+ K_{0001}K_{0200}L_{0010}(-1 + 6\sigma t))\omega_d{}^4))\sin(\varphi_2) - A_3A_4{}^2((K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200}(\sigma^2 + 9\omega_d{}^2)(\sigma^2 + (-1 + 4\sigma t)\omega_d{}^2)$$

$$+ 2K_{0200}L_{0001}L_{0010}(\sigma^4 + 2\sigma^2(3 + \sigma t)\omega_d{}^2 + 3(-1 + 6\sigma t)\omega_d{}^4))\sin(2\varphi_2)$$

$$- 4A_4{}^3L_{0001}L_{0010}L_{0200}\sigma t\omega_d{}^2(\sigma^2 + 9\omega_d{}^2)\sin(3\varphi_2))\sin(t\omega_d)$$

$$+ \omega_d(A_3\sigma(4A_3{}^2K_{0001}K_{0010}K_{0200}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) - A_4{}^2(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200}(\sigma^2 - 7\omega_d{}^2)(\sigma^2 + \omega_d{}^2)) + A_4(\sigma(4A_3{}^2(K_{0010}K_{0200}L_{0001}$$

$$+ K_{0001}K_{0200}L_{0010} + K_{0001}K_{0010}L_{0200})\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2)$$

$$- A_4{}^2L_{0001}L_{0010}L_{0200}(\sigma^2 - 7\omega_d{}^2)(\sigma^2 + \omega_d{}^2))\cos(\varphi_2) + A_3A_4\sigma((K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200}\sigma^4 + 6(2K_{0200}L_{0001}L_{0010} + K_{0010}L_{0001}L_{0200}$$

$$+ K_{0001}L_{0010}L_{0200})\sigma^2\omega_d{}^2 - (20K_{0200}L_{0001}L_{0010} + 27(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200})\omega_d{}^4)\cos(2\varphi_2) + A_4{}^2L_{0001}L_{0010}L_{0200}\sigma(\sigma^2 - 3\omega_d{}^2)(\sigma^2$$

$$+ 9{\omega_d}^2)\cos(3\varphi_2) + \omega_d({A_4}^2 L_{0001}L_{0010}L_{0200}(5\sigma^2 - 3{\omega_d}^2)(\sigma^2 + {\omega_d}^2)$$

$$+ 2{A_3}^2((K_{0010}K_{0200}L_{0001} + K_{0001}K_{0200}L_{0010} - 4K_{0001}K_{0010}L_{0200})\sigma^4$$

$$- 2(6K_{0010}K_{0200}L_{0001} + 6K_{0001}K_{0200}L_{0010} + 7K_{0001}K_{0010}L_{0200})\sigma^2{\omega_d}^2$$

$$+ 3(K_{0010}K_{0200}L_{0001} + K_{0001}K_{0200}L_{0010} + 2K_{0001}K_{0010}L_{0200}){\omega_d}^4))\sin(\varphi_2)$$

$$+ A_3A_4\omega_d(2K_{0200}L_{0001}L_{0010}(\sigma^4 - 12\sigma^2{\omega_d}^2 + 3{\omega_d}^4) + (K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200}(-3\sigma^4 - 26\sigma^2{\omega_d}^2 + 9{\omega_d}^4))\sin(2\varphi_2)$$

$$+ {A_4}^2 L_{0001}L_{0010}L_{0200}\omega_d(-3\sigma^2 + {\omega_d}^2)(\sigma^2 + 9{\omega_d}^2)\sin(3\varphi_2)))\sin(2t\omega_d)\}$$

(C. 48)

$$x_{2,2}^{qs2,u_2,u_1}(\boldsymbol{t}) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{\omega_d(A_3\omega_d(\sigma^2+9\omega_d{}^2)(A_4{}^2(2(1$$

$$+ e^{2\sigma t})K_{0200}L_{0001}L_{0010} + (1+2e^{2\sigma t})(K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200})(\sigma^2$$

$$+ \omega_d{}^2) + 2A_3{}^2K_{0001}K_{0010}K_{0200}(\sigma^2 + (1+2e^{2\sigma t})\omega_d{}^2)) + A_4\omega_d(\sigma^2$$

$$+ 9\omega_d{}^2)(A_4{}^2(2+3e^{2\sigma t})L_{0001}L_{0010}L_{0200}(\sigma^2+\omega_d{}^2) + 2A_3{}^2(K_{0010}K_{0200}L_{0001}$$

$$+ K_{0001}K_{0200}L_{0010} + K_{0001}K_{0010}L_{0200})(\sigma^2+\omega_d{}^2+2e^{2\sigma t}\omega_d{}^2))\cos(\varphi_2)$$

$$+ A_3A_4{}^2\omega_d(\sigma^2+9\omega_d{}^2)(-2e^{2\sigma t}(K_{0200}L_{0001}L_{0010} + K_{0010}L_{0001}L_{0200}$$

$$+ K_{0001}L_{0010}L_{0200})(\sigma-\omega_d)(\sigma+\omega_d) + (K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}(\sigma^2$$

$$+ \omega_d{}^2))\cos(2\varphi_2) + A_4{}^3e^{2\sigma t}L_{0001}L_{0010}L_{0200}\omega_d(-3\sigma^2+\omega_d{}^2)(\sigma^2$$

$$+ 9\omega_d{}^2)\cos(3\varphi_2) + A_4\sigma(\sigma^2+9\omega_d{}^2)(A_4{}^2(-2+3e^{2\sigma t})L_{0001}L_{0010}L_{0200}(\sigma^2$$

$$+ \omega_d{}^2) + 2A_3{}^2(2e^{2\sigma t}(K_{0010}K_{0200}L_{0001} + K_{0001}K_{0200}L_{0010}$$

$$+ K_{0001}K_{0010}L_{0200})\omega_d{}^2 - K_{0001}K_{0010}L_{0200}(\sigma^2+\omega_d{}^2)))\sin(\varphi_2) + A_3A_4{}^2\sigma(\sigma^2$$

$$+ 9\omega_d{}^2)(4e^{2\sigma t}(K_{0200}L_{0001}L_{0010} + K_{0010}L_{0001}L_{0200} + K_{0001}L_{0010}L_{0200})\omega_d{}^2$$

$$- (K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}(\sigma^2+\omega_d{}^2))\sin(2\varphi_2)$$

$$- A_4{}^3e^{2\sigma t}L_{0001}L_{0010}L_{0200}\sigma(\sigma^4+6\sigma^2\omega_d{}^2-27\omega_d{}^4)\sin(3\varphi_2))$$

$$+ \omega_d\cos(2t\omega_d)(A_3\omega_d(-A_4{}^2(K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}(5\sigma^2-3\omega_d{}^2)(\sigma^2$$

$$+ \omega_d{}^2) - 2A_3{}^2K_{0001}K_{0010}K_{0200}(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4))$$

$$+ A_4(-\omega_d(A_4{}^2L_{0001}L_{0010}L_{0200}(5\sigma^2-3\omega_d{}^2)(\sigma^2+\omega_d{}^2)$$

$$+ 2A_3{}^2(K_{0010}K_{0200}L_{0001} + K_{0001}K_{0200}L_{0010} + K_{0001}K_{0010}L_{0200})(\sigma^4$$

$$- 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4))\cos(\varphi_2) + A_3A_4\omega_d((K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}(3\sigma^4$$

$$+ 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4) - 2K_{0200}L_{0001}L_{0010}(\sigma^4-12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(2\varphi_2)$$

$$+ A_4{}^2L_{0001}L_{0010}L_{0200}\omega_d(3\sigma^4+26\sigma^2\omega_d{}^2-9\omega_d{}^4)\cos(3\varphi_2)$$

$$+ \sigma(-A_4{}^2L_{0001}L_{0010}L_{0200}(\sigma^2-7\omega_d{}^2)(\sigma^2+\omega_d{}^2) + 2A_3{}^2(K_{0001}K_{0010}L_{0200}\sigma^4$$

$$+ 6K_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma^2{\omega_d}^2 - (10K_{0200}(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010}) + 17K_{0001}K_{0010}L_{0200}){\omega_d}^4))\sin(\varphi_2) + A_3A_4\sigma((K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200}\sigma^4 + 6(2K_{0200}L_{0001}L_{0010} + K_{0010}L_{0001}L_{0200}$$

$$+ K_{0001}L_{0010}L_{0200})\sigma^2{\omega_d}^2 - (20K_{0200}L_{0001}L_{0010} + 27(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200}){\omega_d}^4)\sin(2\varphi_2) + {A_4}^2L_{0001}L_{0010}L_{0200}\sigma(\sigma^4 + 6\sigma^2{\omega_d}^2$$

$$- 27{\omega_d}^4)\sin(3\varphi_2))) + e^{\sigma t}\omega_d(\sigma^2$$

$$+ {\omega_d}^2)\cos(t\omega_d)(4{A_3}^3K_{0001}K_{0010}K_{0200}\omega_d(\sigma^3 t + 3(-4 + 3\sigma t){\omega_d}^2)$$

$$+ 2A_3{A_4}^2\omega_d((K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}(\sigma^2 - 15{\omega_d}^2)$$

$$- 2K_{0200}L_{0001}L_{0010}(\sigma^2 + 9{\omega_d}^2)) + A_4(4\omega_d(-12{A_4}^2L_{0001}L_{0010}L_{0200}{\omega_d}^2$$

$$+ {A_3}^2(K_{0010}K_{0200}L_{0001} + K_{0001}K_{0200}L_{0010} + K_{0001}K_{0010}L_{0200})(\sigma^3 t + 3(-4$$

$$+ 3\sigma t){\omega_d}^2))\cos(\varphi_2) + 2A_3A_4\omega_d((K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}(-1$$

$$+ 2\sigma t)(\sigma^2 + 9{\omega_d}^2) + 2K_{0200}L_{0001}L_{0010}(\sigma^2 + \sigma^3 t - 3{\omega_d}^2$$

$$+ 9\sigma t{\omega_d}^2))\cos(2\varphi_2) + 4{A_4}^2L_{0001}L_{0010}L_{0200}\sigma t\omega_d(\sigma^2 + 9{\omega_d}^2)\cos(3\varphi_2)$$

$$+ 2(-{A_4}^2L_{0001}L_{0010}L_{0200}(\sigma^4 t + 2\sigma(4 + 5\sigma t){\omega_d}^2 + 9t{\omega_d}^4)$$

$$+ {A_3}^2(K_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma^4 t + 2\sigma(K_{0001}K_{0010}L_{0200}(4 - \sigma t)$$

$$+ 4K_{0010}K_{0200}L_{0001}(-1 + \sigma t) + 4K_{0001}K_{0200}L_{0010}(-1 + \sigma t)){\omega_d}^2$$

$$- 9(K_{0010}K_{0200}L_{0001} + K_{0001}K_{0200}L_{0010} + 2K_{0001}K_{0010}L_{0200})t{\omega_d}^4))\sin(\varphi_2)$$

$$+ A_3A_4((K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}t(\sigma^4 + 6\sigma^2{\omega_d}^2 - 27{\omega_d}^4)$$

$$+ 2K_{0200}L_{0001}L_{0010}(\sigma^4 t + 8\sigma(-1 + \sigma t){\omega_d}^2 - 9t{\omega_d}^4))\sin(2\varphi_2)$$

$$+ 2{A_4}^2L_{0001}L_{0010}L_{0200}t(\sigma^4 + 8\sigma^2{\omega_d}^2 - 9{\omega_d}^4)\sin(3\varphi_2))) + e^{\sigma t}(\sigma^2$$

$$+ {\omega_d}^2)(-A_3\omega_d(4{A_3}^2K_{0001}K_{0010}K_{0200}(\sigma^3 + \sigma(5 + \sigma t){\omega_d}^2 + 9t{\omega_d}^4)$$

$$+ {A_4}^2(\sigma^3(2K_{0200}L_{0001}L_{0010}\sigma t + K_{0010}L_{0001}L_{0200}(4 + \sigma t)$$

$$+ K_{0001}L_{0010}L_{0200}(4 + \sigma t)) + 10\sigma(2K_{0200}L_{0001}L_{0010}\sigma t + K_{0010}L_{0001}L_{0200}(2$$

$$+ \sigma t) + K_{0001}L_{0010}L_{0200}(2 + \sigma t))\omega_d{}^2 + 9(2K_{0200}L_{0001}L_{0010}$$

$$+ K_{0010}L_{0001}L_{0200} + K_{0001}L_{0010}L_{0200})t\omega_d{}^4))$$

$$- 2A_4\omega_d(A_4{}^2L_{0001}L_{0010}L_{0200}(\sigma^3(2 + \sigma t) + 10\sigma(1 + \sigma t)\omega_d{}^2 + 9t\omega_d{}^4)$$

$$+ 2A_3{}^2(K_{0010}K_{0200}L_{0001} + K_{0001}K_{0200}L_{0010} + K_{0001}K_{0010}L_{0200})(\sigma^3 + \sigma(5$$

$$+ \sigma t)\omega_d{}^2 + 9t\omega_d{}^4))\cos(\varphi_2) + A_3A_4{}^2\omega_d((K_{0010}L_{0001} + K_{0001}L_{0010})L_{0200}t(\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4) + 2K_{0200}L_{0001}L_{0010}(\sigma^3(-2 + \sigma t) + 2\sigma(-5 + 4\sigma t)\omega_d{}^2$$

$$- 9t\omega_d{}^4))\cos(2\varphi_2) + 2A_4{}^3L_{0001}L_{0010}L_{0200}t\omega_d(\sigma^4 + 8\sigma^2\omega_d{}^2$$

$$- 9\omega_d{}^4)\cos(3\varphi_2) - 2A_4(2A_4{}^2L_{0001}L_{0010}L_{0200}(\sigma^4 + 7\sigma^2\omega_d{}^2 - 6\omega_d{}^4)$$

$$+ A_3{}^2(K_{0200}(K_{0010}L_{0001} + K_{0001}L_{0010})\sigma^4 + 2\sigma^2(K_{0001}K_{0010}L_{0200}(1 + \sigma t)$$

$$+ K_{0010}K_{0200}L_{0001}(3 + \sigma t) + K_{0001}K_{0200}L_{0010}(3 + \sigma t))\omega_d{}^2$$

$$+ 3(2K_{0001}K_{0010}L_{0200}(-1 + 3\sigma t) + K_{0010}K_{0200}L_{0001}(-1 + 6\sigma t)$$

$$+ K_{0001}K_{0200}L_{0010}(-1 + 6\sigma t))\omega_d{}^4))\sin(\varphi_2) - A_3A_4{}^2((K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200}(\sigma^2 + 9\omega_d{}^2)(\sigma^2 + (-1 + 4\sigma t)\omega_d{}^2)$$

$$+ 2K_{0200}L_{0001}L_{0010}(\sigma^4 + 2\sigma^2(3 + \sigma t)\omega_d{}^2 + 3(-1 + 6\sigma t)\omega_d{}^4))\sin(2\varphi_2)$$

$$- 4A_4{}^3L_{0001}L_{0010}L_{0200}\sigma t\omega_d{}^2(\sigma^2 + 9\omega_d{}^2)\sin(3\varphi_2))\sin(t\omega_d)$$

$$+ \omega_d(A_3\sigma(4A_3{}^2K_{0001}K_{0010}K_{0200}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) - A_4{}^2(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200}(\sigma^2 - 7\omega_d{}^2)(\sigma^2 + \omega_d{}^2)) + A_4(\sigma(4A_3{}^2(K_{0010}K_{0200}L_{0001}$$

$$+ K_{0001}K_{0200}L_{0010} + K_{0001}K_{0010}L_{0200})\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2)$$

$$- A_4{}^2L_{0001}L_{0010}L_{0200}(\sigma^2 - 7\omega_d{}^2)(\sigma^2 + \omega_d{}^2))\cos(\varphi_2) + A_3A_4\sigma((K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200}\sigma^4 + 6(2K_{0200}L_{0001}L_{0010} + K_{0010}L_{0001}L_{0200}$$

$$+ K_{0001}L_{0010}L_{0200})\sigma^2\omega_d{}^2 - (20K_{0200}L_{0001}L_{0010} + 27(K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200})\omega_d{}^4)\cos(2\varphi_2) + A_4{}^2L_{0001}L_{0010}L_{0200}\sigma(\sigma^2 - 3\omega_d{}^2)(\sigma^2$$

$$+ 9{\omega_d}^2)\cos(3\varphi_2) + \omega_d({A_4}^2 L_{0001} L_{0010} L_{0200}(5\sigma^2 - 3{\omega_d}^2)(\sigma^2 + {\omega_d}^2)$$

$$+ 2{A_3}^2((K_{0010}K_{0200}L_{0001} + K_{0001}K_{0200}L_{0010} - 4K_{0001}K_{0010}L_{0200})\sigma^4$$

$$- 2(6K_{0010}K_{0200}L_{0001} + 6K_{0001}K_{0200}L_{0010} + 7K_{0001}K_{0010}L_{0200})\sigma^2{\omega_d}^2$$

$$+ 3(K_{0010}K_{0200}L_{0001} + K_{0001}K_{0200}L_{0010} + 2K_{0001}K_{0010}L_{0200}){\omega_d}^4))\sin(\varphi_2)$$

$$+ A_3A_4\omega_d(2K_{0200}L_{0001}L_{0010}(\sigma^4 - 12\sigma^2{\omega_d}^2 + 3{\omega_d}^4) + (K_{0010}L_{0001}$$

$$+ K_{0001}L_{0010})L_{0200}(-3\sigma^4 - 26\sigma^2{\omega_d}^2 + 9{\omega_d}^4))\sin(2\varphi_2)$$

$$+ {A_4}^2L_{0001}L_{0010}L_{0200}\omega_d(-3\sigma^2 + {\omega_d}^2)(\sigma^2 + 9{\omega_d}^2)\sin(3\varphi_2)))\sin(2t\omega_d)\}$$

(C. 49)

$$x_{2,2}^{qs2,u_2,u_2}(t) = \frac{1}{4\omega_d(\sigma^2+\omega_d{}^2)^3(\sigma^2+9\omega_d{}^2)} e^{-2\sigma t}\{\omega_d{}^2(\sigma^2+9\omega_d{}^2)(2A_3A_4{}^2L_{0001}((1$$

$$+ e^{2\sigma t})K_{0200}L_{0001} + (1+2e^{2\sigma t})K_{0001}L_{0200})(\sigma^2+\omega_d{}^2)$$

$$+ 2A_3{}^3K_{0001}{}^2K_{0200}(\sigma^2+(1+2e^{2\sigma t})\omega_d{}^2) + A_4{}^3(2+3e^{2\sigma t})L_{0001}{}^2L_{0200}(\sigma^2$$

$$+ \omega_d{}^2)\cos(\varphi_2) + 2A_3{}^2A_4K_{0001}(2K_{0200}L_{0001}+K_{0001}L_{0200})(\sigma^2+(1$$

$$+ 2e^{2\sigma t})\omega_d{}^2)\cos(\varphi_2)) + 2A_3A_4{}^2L_{0001}\omega_d{}^2(\sigma^2+9\omega_d{}^2)(-e^{2\sigma t}(K_{0200}L_{0001}$$

$$+ 2K_{0001}L_{0200})(\sigma^2-\omega_d{}^2) + K_{0001}L_{0200}(\sigma^2+\omega_d{}^2))\cos(2\varphi_2)$$

$$+ A_4{}^3e^{2\sigma t}L_{0001}{}^2L_{0200}\omega_d{}^2(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(3\varphi_2)$$

$$+ 4A_3e^{\sigma t}\omega_d{}^2(\sigma^2+\omega_d{}^2)(A_3{}^2K_{0001}{}^2K_{0200}(\sigma^3t-12\omega_d{}^2+9\sigma t\omega_d{}^2)$$

$$- A_4{}^2L_{0001}(-K_{0001}L_{0200}(\sigma^2-15\omega_d{}^2)+K_{0200}L_{0001}(\sigma^2+9\omega_d{}^2)))\cos(t\omega_d)$$

$$- 2A_3K_{0001}\omega_d{}^2(A_4{}^2L_{0001}L_{0200}(5\sigma^4+2\sigma^2\omega_d{}^2-3\omega_d{}^4) + A_3{}^2K_{0001}K_{0200}(\sigma^4$$

$$- 12\sigma^2\omega_d{}^2+3\omega_d{}^4))\cos(2t\omega_d) + A_3{}^2A_4K_{0001}{}^2L_{0200}\omega_d{}^2(-5\sigma^4-2\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4)\cos(\varphi_2-2t\omega_d) - 2A_4e^{\sigma t}(\sigma^2+\omega_d{}^2)(A_4{}^2L_{0001}{}^2L_{0200}(\sigma^4+7\sigma^2\omega_d{}^2$$

$$+ 6\omega_d{}^4) + A_3{}^2K_{0001}(K_{0001}L_{0200}\omega_d{}^2(\sigma^2+9\omega_d{}^2)+K_{0200}L_{0001}(\sigma^4+6\sigma^2\omega_d{}^2$$

$$+ 21\omega_d{}^4)))\cos(\varphi_2-t\omega_d) - A_3A_4{}^2e^{\sigma t}L_{0001}(\sigma^2+\omega_d{}^2)^2(K_{0200}L_{0001}(\sigma^2$$

$$+ 3\omega_d{}^2) + K_{0001}L_{0200}(\sigma^2+9\omega_d{}^2))\cos(2\varphi_2-t\omega_d) + 2A_4e^{\sigma t}(\sigma^2$$

$$+ \omega_d{}^2)(A_4{}^2L_{0001}{}^2L_{0200}(\sigma^4+7\sigma^2\omega_d{}^2-18\omega_d{}^4) + A_3{}^2K_{0001}(K_{0200}L_{0001}(\sigma^2$$

$$+ 9\omega_d{}^2)(\sigma^2-3\omega_d{}^2+4\sigma t\omega_d{}^2) + K_{0001}L_{0200}\omega_d{}^2(\sigma^2+2\sigma^3t-15\omega_d{}^2$$

$$+ 18\sigma t\omega_d{}^2)))\cos(\varphi_2+t\omega_d) - 2A_3A_4{}^2L_{0001}\omega_d{}^2(K_{0200}L_{0001}(\sigma^4-12\sigma^2\omega_d{}^2$$

$$+ 3\omega_d{}^4) + K_{0001}L_{0200}(-3\sigma^4-26\sigma^2\omega_d{}^2+9\omega_d{}^4))\cos(2(\varphi_2+t\omega_d))$$

$$+ A_3A_4{}^2e^{\sigma t}L_{0001}(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)(K_{0200}L_{0001}(\sigma^2-\omega_d{}^2+4\sigma t\omega_d{}^2)$$

$$+ K_{0001}L_{0200}(\sigma^2-3\omega_d{}^2+8\sigma t\omega_d{}^2))\cos(2\varphi_2+t\omega_d)$$

$$+ 4A_4{}^3e^{\sigma t}L_{0001}{}^2L_{0200}\sigma t\omega_d{}^2(\sigma^4+10\sigma^2\omega_d{}^2+9\omega_d{}^4)\cos(3\varphi_2+t\omega_d)$$

$$+ A_4\omega_d{}^2(A_4{}^2L_{0001}{}^2L_{0200}(-5\sigma^4 - 2\sigma^2\omega_d{}^2 + 3\omega_d{}^4)$$

$$+ A_3{}^2K_{0001}(K_{0001}L_{0200}(3\sigma^4 + 26\sigma^2\omega_d{}^2 - 9\omega_d{}^4) - 4K_{0200}L_{0001}(\sigma^4$$

$$- 12\sigma^2\omega_d{}^2 + 3\omega_d{}^4)))\cos(\varphi_2 + 2t\omega_d) + A_4{}^3L_{0001}{}^2L_{0200}\omega_d{}^2(3\sigma^4 + 26\sigma^2\omega_d{}^2$$

$$- 9\omega_d{}^4)\cos(3\varphi_2 + 2t\omega_d) + A_4\sigma\omega_d(\sigma^2 + 9\omega_d{}^2)(A_4{}^2(-2$$

$$+ 3e^{2\sigma t})L_{0001}{}^2L_{0200}(\sigma^2 + \omega_d{}^2) - 2A_3{}^2K_{0001}(-4e^{2\sigma t}K_{0200}L_{0001}\omega_d{}^2$$

$$+ K_{0001}L_{0200}(\sigma^2 + \omega_d{}^2 - 2e^{2\sigma t}\omega_d{}^2)))\sin(\varphi_2) - 2A_3A_4{}^2L_{0001}\omega_d(\sigma^3$$

$$+ 9\sigma\omega_d{}^2)(-2e^{2\sigma t}K_{0200}L_{0001}\omega_d{}^2 + K_{0001}L_{0200}(\sigma^2 + (1$$

$$- 4e^{2\sigma t})\omega_d{}^2))\sin(2\varphi_2) - A_4{}^3e^{2\sigma t}L_{0001}{}^2L_{0200}\omega_d(\sigma^5 + 6\sigma^3\omega_d{}^2$$

$$- 27\sigma\omega_d{}^4)\sin(3\varphi_2) - 2A_3e^{\sigma t}\omega_d(\sigma^2 + \omega_d{}^2)(2A_3{}^2K_{0001}{}^2K_{0200}(\sigma^3 + 5\sigma\omega_d{}^2$$

$$+ \sigma^2t\omega_d{}^2 + 9t\omega_d{}^4) + A_4{}^2L_{0001}(K_{0200}L_{0001}t(\sigma^4 + 10\sigma^2\omega_d{}^2 + 9\omega_d{}^4)$$

$$+ K_{0001}L_{0200}(4\sigma^3 + \sigma^4t + 20\sigma\omega_d{}^2 + 10\sigma^2t\omega_d{}^2 + 9t\omega_d{}^4)))\sin(t\omega_d)$$

$$+ 2A_3K_{0001}\sigma\omega_d(2A_3{}^2K_{0001}K_{0200}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + A_4{}^2L_{0001}L_{0200}(-\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2 + 7\omega_d{}^4))\sin(2t\omega_d) + A_3{}^2A_4K_{0001}{}^2L_{0200}\sigma\omega_d(\sigma^4 - 6\sigma^2\omega_d{}^2$$

$$- 7\omega_d{}^4)\sin(\varphi_2 - 2t\omega_d) + 2A_4e^{\sigma t}\omega_d(\sigma^2 + \omega_d{}^2)(A_4{}^2L_{0001}{}^2L_{0200}\sigma(\sigma^2 + \omega_d{}^2)$$

$$+ A_3{}^2K_{0001}(K_{0001}L_{0200}\sigma(\sigma^2 + 9\omega_d{}^2) + K_{0200}L_{0001}(\sigma^2 + \omega_d{}^2)(2\sigma + \sigma^2t$$

$$+ 9t\omega_d{}^2)))\sin(\varphi_2 - t\omega_d) + 2A_3A_4{}^2e^{\sigma t}K_{0200}L_{0001}{}^2\sigma\omega_d(\sigma^2 + \omega_d{}^2)^2\sin(2\varphi_2$$

$$- t\omega_d) - 2A_4e^{\sigma t}\omega_d(\sigma^2 + \omega_d{}^2)(A_4{}^2L_{0001}{}^2L_{0200}(\sigma^2 + 9\omega_d{}^2)(\sigma + \sigma^2t + t\omega_d{}^2)$$

$$+ A_3{}^2K_{0001}(-K_{0200}L_{0001}(\sigma^2 + 9\omega_d{}^2)(-2\sigma + \sigma^2t - 3t\omega_d{}^2) + K_{0001}L_{0200}(\sigma^3$$

$$+ \sigma\omega_d{}^2 + 2\sigma^2t\omega_d{}^2 + 18t\omega_d{}^4)))\sin(\varphi_2 + t\omega_d)$$

$$+ 2A_3A_4{}^2L_{0001}\sigma\omega_d(2K_{0200}L_{0001}\omega_d{}^2(3\sigma^2 - 5\omega_d{}^2) + K_{0001}L_{0200}(\sigma^4$$

$$+ 6\sigma^2\omega_d{}^2 - 27\omega_d{}^4))\sin(2(\varphi_2 + t\omega_d)) - 2A_3A_4{}^2e^{\sigma t}L_{0001}\omega_d(\sigma^4 + 10\sigma^2\omega_d{}^2$$

$$+ 9\omega_d{}^4)(-K_{0001}L_{0200}t(\sigma^2 - 3\omega_d{}^2) + K_{0200}L_{0001}(\sigma - \sigma^2t + t\omega_d{}^2))\sin(2\varphi_2$$

$$+ t\omega_d) + 2A_4{}^3 e^{\sigma t} L_{0001}{}^2 L_{0200} t\omega_d (\sigma^6 + 9\sigma^4 \omega_d{}^2 - \sigma^2 \omega_d{}^4 - 9\omega_d{}^6) \sin(3\varphi_2$$

$$+ t\omega_d) + A_4 \sigma \omega_d (A_4{}^2 L_{0001}{}^2 L_{0200} (-\sigma^4 + 6\sigma^2 \omega_d{}^2 + 7\omega_d{}^4)$$

$$+ A_3{}^2 K_{0001} (8K_{0200} L_{0001} \omega_d{}^2 (3\sigma^2 - 5\omega_d{}^2) + K_{0001} L_{0200} (\sigma^4 + 6\sigma^2 \omega_d{}^2$$

$$- 27\omega_d{}^4))) \sin(\varphi_2 + 2t\omega_d) + A_4{}^3 L_{0001}{}^2 L_{0200} \sigma \omega_d (\sigma^4 + 6\sigma^2 \omega_d{}^2$$

$$- 27\omega_d{}^4) \sin(3\varphi_2 + 2t\omega_d)\} \tag{C.50}$$

$$x_{2,2}^{bs1i1}(t) = x_{2,2}^{bs1i1,u_1,u_1}(t) \times u_1u_1 + x_{2,2}^{bs1i1,u_2,u_1}(t) \times u_2u_1 \qquad (C.51)$$

$$x_{2,2}^{bs1i1,u_1,u_1}(t) = \frac{1}{4\omega_d(\sigma^2+{\omega_d}^2)^2}\{4A_3K_{1010}{\omega_d}^3(A_2L_{0010}+A_1K_{0010}\cos(\varphi_1))$$
$$+2A_1A_4K_{0010}L_{1010}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1-\varphi_2)$$
$$+4A_2A_4L_{0010}L_{1010}{\omega_d}^3\cos(\varphi_2)+2A_1A_4K_{0010}L_{1010}\omega_d(-\sigma^2+{\omega_d}^2)\cos(\varphi_1$$
$$+\varphi_2)+2A_2A_3e^{-\sigma t}K_{1010}L_{0010}\omega_d(\sigma^3t-2{\omega_d}^2+\sigma t{\omega_d}^2)\cos(t\omega_d)$$
$$-A_1A_3e^{-\sigma t}K_{0010}K_{1010}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1-t\omega_d)$$
$$-A_1A_4e^{-\sigma t}K_{0010}L_{1010}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1-\varphi_2-t\omega_d)$$
$$-A_2A_4e^{-\sigma t}L_{0010}L_{1010}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_2-t\omega_d)$$
$$+A_1A_3e^{-\sigma t}K_{0010}K_{1010}\omega_d(\sigma^2+2\sigma^3t-3{\omega_d}^2+2\sigma t{\omega_d}^2)\cos(\varphi_1+t\omega_d)$$
$$-A_1A_4e^{-\sigma t}K_{0010}L_{1010}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1-\varphi_2+t\omega_d)$$
$$+A_2A_4e^{-\sigma t}L_{0010}L_{1010}\omega_d(\sigma^2+2\sigma^3t-3{\omega_d}^2+2\sigma t{\omega_d}^2)\cos(\varphi_2+t\omega_d)$$
$$+2A_1A_4e^{-\sigma t}K_{0010}L_{1010}\omega_d(\sigma^2+\sigma^3t-{\omega_d}^2+\sigma t{\omega_d}^2)\cos(\varphi_1+\varphi_2+t\omega_d)$$
$$+4A_1A_3K_{0010}K_{1010}\sigma{\omega_d}^2\sin(\varphi_1)+4A_2A_4L_{0010}L_{1010}\sigma{\omega_d}^2\sin(\varphi_2)$$
$$+4A_1A_4K_{0010}L_{1010}\sigma{\omega_d}^2\sin(\varphi_1+\varphi_2)-2A_2A_3e^{-\sigma t}K_{1010}L_{0010}(\sigma^3+3\sigma{\omega_d}^2$$
$$+\sigma^2t{\omega_d}^2+t{\omega_d}^4)\sin(t\omega_d)+A_1A_3e^{-\sigma t}K_{0010}K_{1010}\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_1-t\omega_d)$$
$$+A_1A_4e^{-\sigma t}K_{0010}L_{1010}\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_1-\varphi_2-t\omega_d)$$
$$+A_2A_4e^{-\sigma t}L_{0010}L_{1010}\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_2-t\omega_d)-A_1A_3e^{-\sigma t}K_{0010}K_{1010}(\sigma^3$$
$$+5\sigma{\omega_d}^2+2\sigma^2t{\omega_d}^2+2t{\omega_d}^4)\sin(\varphi_1+t\omega_d)-A_1A_4e^{-\sigma t}K_{0010}L_{1010}\sigma(\sigma^2$$
$$+{\omega_d}^2)\sin(\varphi_1-\varphi_2+t\omega_d)-A_2A_4e^{-\sigma t}L_{0010}L_{1010}(\sigma^3+5\sigma{\omega_d}^2+2\sigma^2t{\omega_d}^2$$
$$+2t{\omega_d}^4)\sin(\varphi_2+t\omega_d)-2A_1A_4e^{-\sigma t}K_{0010}L_{1010}{\omega_d}^2(2\sigma+\sigma^2t+t{\omega_d}^2)\sin(\varphi_1$$
$$+\varphi_2+t\omega_d)\}$$

(C. 52)

$$x_{2,2}^{bs1i1,u_2,u_1}(t) = \frac{1}{4\omega_d(\sigma^2+{\omega_d}^2)^2}\{4A_3K_{1010}{\omega_d}^3(A_2L_{0001}+A_1K_{0001}\cos(\varphi_1))$$

$$+ 2A_1A_4K_{0001}L_{1010}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1-\varphi_2)$$

$$+ 4A_2A_4L_{0001}L_{1010}{\omega_d}^3\cos(\varphi_2) + 2A_1A_4K_{0001}L_{1010}\omega_d(-\sigma^2+{\omega_d}^2)\cos(\varphi_1$$

$$+ \varphi_2) + 2A_2A_3e^{-\sigma t}K_{1010}L_{0001}\omega_d(\sigma^3 t - 2{\omega_d}^2 + \sigma t{\omega_d}^2)\cos(t\omega_d)$$

$$- A_1A_3e^{-\sigma t}K_{0001}K_{1010}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1 - t\omega_d)$$

$$- A_1A_4e^{-\sigma t}K_{0001}L_{1010}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1-\varphi_2-t\omega_d)$$

$$- A_2A_4e^{-\sigma t}L_{0001}L_{1010}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_2-t\omega_d)$$

$$+ A_1A_3e^{-\sigma t}K_{0001}K_{1010}\omega_d(\sigma^2+2\sigma^3 t-3{\omega_d}^2+2\sigma t{\omega_d}^2)\cos(\varphi_1+t\omega_d)$$

$$- A_1A_4e^{-\sigma t}K_{0001}L_{1010}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1-\varphi_2+t\omega_d)$$

$$+ A_2A_4e^{-\sigma t}L_{0001}L_{1010}\omega_d(\sigma^2+2\sigma^3 t-3{\omega_d}^2+2\sigma t{\omega_d}^2)\cos(\varphi_2+t\omega_d)$$

$$+ 2A_1A_4e^{-\sigma t}K_{0001}L_{1010}\omega_d(\sigma^2+\sigma^3 t-{\omega_d}^2+\sigma t{\omega_d}^2)\cos(\varphi_1+\varphi_2+t\omega_d)$$

$$+ 4A_1A_3K_{0001}K_{1010}\sigma{\omega_d}^2\sin(\varphi_1) + 4A_2A_4L_{0001}L_{1010}\sigma{\omega_d}^2\sin(\varphi_2)$$

$$+ 4A_1A_4K_{0001}L_{1010}\sigma{\omega_d}^2\sin(\varphi_1+\varphi_2) - 2A_2A_3e^{-\sigma t}K_{1010}L_{0001}(\sigma^3+3\sigma{\omega_d}^2$$

$$+ \sigma^2 t{\omega_d}^2 + t{\omega_d}^4)\sin(t\omega_d) + A_1A_3e^{-\sigma t}K_{0001}K_{1010}\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_1-t\omega_d)$$

$$+ A_1A_4e^{-\sigma t}K_{0001}L_{1010}\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_1-\varphi_2-t\omega_d)$$

$$+ A_2A_4e^{-\sigma t}L_{0001}L_{1010}\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_2-t\omega_d) - A_1A_3e^{-\sigma t}K_{0001}K_{1010}(\sigma^3$$

$$+ 5\sigma{\omega_d}^2 + 2\sigma^2 t{\omega_d}^2 + 2t{\omega_d}^4)\sin(\varphi_1+t\omega_d) - A_1A_4e^{-\sigma t}K_{0001}L_{1010}\sigma(\sigma^2$$

$$+ {\omega_d}^2)\sin(\varphi_1-\varphi_2+t\omega_d) - A_2A_4e^{-\sigma t}L_{0001}L_{1010}(\sigma^3+5\sigma{\omega_d}^2+2\sigma^2 t{\omega_d}^2$$

$$+ 2t{\omega_d}^4)\sin(\varphi_2+t\omega_d) - 2A_1A_4e^{-\sigma t}K_{0001}L_{1010}{\omega_d}^2(2\sigma+\sigma^2 t+t{\omega_d}^2)\sin(\varphi_1$$

$$+ \varphi_2 + t\omega_d)\}$$

(C. 53)

$$x_{2,2}^{bs2i1}(t) = x_{2,2}^{bs2i1,u_1,u_1}(t) \times u_1 u_1 + x_{2,2}^{bs2i1,u_2,u_1}(t) \times u_2 u_1 \qquad (C.54)$$

$$x_{2,2}^{bs2i1,u_1,u_1}(t) = \frac{1}{4(\sigma^2+{\omega_d}^2)^2}\{4{A_3}^2K_{0010}K_{0110}{\omega_d}^2 + 2{A_4}^2L_{0010}L_{0110}(\sigma^2+{\omega_d}^2)$$
$$+ 4A_3A_4(K_{0110}L_{0010} + K_{0010}L_{0110}){\omega_d}^2\cos(\varphi_2) + 2{A_4}^2L_{0010}L_{0110}(-\sigma^2$$
$$+ {\omega_d}^2)\cos(2\varphi_2) + 2e^{-\sigma t}(-{A_4}^2L_{0010}L_{0110}(\sigma^2+{\omega_d}^2) + {A_3}^2K_{0010}K_{0110}(\sigma^3 t$$
$$- 2{\omega_d}^2 + \sigma t{\omega_d}^2))\cos(t\omega_d) - A_3A_4e^{-\sigma t}(K_{0110}L_{0010} + K_{0010}L_{0110})(\sigma^2$$
$$+ {\omega_d}^2)\cos(\varphi_2 - t\omega_d) + A_3A_4e^{-\sigma t}(K_{0110}L_{0010} + K_{0010}L_{0110})(\sigma^2 + 2\sigma^3 t$$
$$- 3{\omega_d}^2 + 2\sigma t{\omega_d}^2)\cos(\varphi_2 + t\omega_d) + 2{A_4}^2e^{-\sigma t}L_{0010}L_{0110}(\sigma^2 + \sigma^3 t - {\omega_d}^2$$
$$+ \sigma t{\omega_d}^2)\cos(2\varphi_2 + t\omega_d) + 4A_3A_4(K_{0110}L_{0010} + K_{0010}L_{0110})\sigma\omega_d\sin(\varphi_2)$$
$$+ 4{A_4}^2L_{0010}L_{0110}\sigma\omega_d\sin(2\varphi_2) - \frac{1}{\omega_d}2e^{-\sigma t}({A_4}^2L_{0010}L_{0110}\sigma(\sigma^2+{\omega_d}^2)$$
$$+ {A_3}^2K_{0010}K_{0110}(\sigma^3 + 3\sigma{\omega_d}^2 + \sigma^2 t{\omega_d}^2 + t{\omega_d}^4))\sin(t\omega_d)$$
$$+ \frac{1}{\omega_d}A_3A_4e^{-\sigma t}(K_{0110}L_{0010} + K_{0010}L_{0110})\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_2 - t\omega_d)$$
$$- \frac{1}{\omega_d}A_3A_4e^{-\sigma t}(K_{0110}L_{0010} + K_{0010}L_{0110})(\sigma^3 + 5\sigma{\omega_d}^2 + 2\sigma^2 t{\omega_d}^2$$
$$+ 2t{\omega_d}^4)\sin(\varphi_2 + t\omega_d) - 2{A_4}^2e^{-\sigma t}L_{0010}L_{0110}\omega_d(2\sigma + \sigma^2 t + t{\omega_d}^2)\sin(2\varphi_2$$
$$+ t\omega_d)\}$$

(C. 55)

$$x_{2,2}^{bs2i1,u_2,u_1}(t) = \frac{1}{4(\sigma^2+{\omega_d}^2)^2}\{4{A_3}^2K_{0001}K_{0110}{\omega_d}^2 + 2{A_4}^2L_{0001}L_{0110}(\sigma^2+{\omega_d}^2)$$

$$+ 4A_3A_4(K_{0110}L_{0001}+K_{0001}L_{0110}){\omega_d}^2\cos(\varphi_2) + 2{A_4}^2L_{0001}L_{0110}(-\sigma^2$$

$$+ {\omega_d}^2)\cos(2\varphi_2) + 2e^{-\sigma t}(-{A_4}^2L_{0001}L_{0110}(\sigma^2+{\omega_d}^2) + {A_3}^2K_{0001}K_{0110}(\sigma^3 t$$

$$- 2{\omega_d}^2 + \sigma t{\omega_d}^2))\cos(t\omega_d) - A_3A_4e^{-\sigma t}(K_{0110}L_{0001}+K_{0001}L_{0110})(\sigma^2$$

$$+ {\omega_d}^2)\cos(\varphi_2 - t\omega_d) + A_3A_4e^{-\sigma t}(K_{0110}L_{0001}+K_{0001}L_{0110})(\sigma^2 + 2\sigma^3 t$$

$$- 3{\omega_d}^2 + 2\sigma t{\omega_d}^2)\cos(\varphi_2 + t\omega_d) + 2{A_4}^2e^{-\sigma t}L_{0001}L_{0110}(\sigma^2 + \sigma^3 t - {\omega_d}^2$$

$$+ \sigma t{\omega_d}^2)\cos(2\varphi_2 + t\omega_d) + 4A_3A_4(K_{0110}L_{0001}+K_{0001}L_{0110})\sigma\omega_d\sin(\varphi_2)$$

$$+ 4{A_4}^2L_{0001}L_{0110}\sigma\omega_d\sin(2\varphi_2) - \frac{1}{\omega_d}2e^{-\sigma t}({A_4}^2L_{0001}L_{0110}\sigma(\sigma^2+{\omega_d}^2)$$

$$+ {A_3}^2K_{0001}K_{0110}(\sigma^3 + 3\sigma{\omega_d}^2 + \sigma^2 t{\omega_d}^2 + t{\omega_d}^4))\sin(t\omega_d)$$

$$+ \frac{1}{\omega_d}A_3A_4e^{-\sigma t}(K_{0110}L_{0001}+K_{0001}L_{0110})\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_2 - t\omega_d)$$

$$- \frac{1}{\omega_d}A_3A_4e^{-\sigma t}(K_{0110}L_{0001}+K_{0001}L_{0110})(\sigma^3 + 5\sigma{\omega_d}^2 + 2\sigma^2 t{\omega_d}^2$$

$$+ 2t{\omega_d}^4)\sin(\varphi_2 + t\omega_d) - 2{A_4}^2e^{-\sigma t}L_{0001}L_{0110}\omega_d(2\sigma + \sigma^2 t + t{\omega_d}^2)\sin(2\varphi_2$$

$$+ t\omega_d)\}$$

(C. 56)

$$x_{2,2}^{bs1i2}(t) = x_{2,2}^{bs1i2,u_1,u_2}(t) \times u_1 u_2 + x_{2,2}^{bs1i2,u_2,u_2}(t) \times u_2 u_2 \qquad (C.57)$$

$$
\begin{aligned}
\boldsymbol{x_{2,2}^{bs1i2,u_1,u_2}(t)} = {} & \frac{1}{4\omega_d(\sigma^2+{\omega_d}^2)^2}\{4A_3K_{1001}{\omega_d}^3(A_2L_{0010}+A_1K_{0010}\cos(\varphi_1)) \\
& + 2A_1A_4K_{0010}L_{1001}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1-\varphi_2) \\
& + 4A_2A_4L_{0010}L_{1001}{\omega_d}^3\cos(\varphi_2) + 2A_1A_4K_{0010}L_{1001}\omega_d(-\sigma^2+{\omega_d}^2)\cos(\varphi_1 \\
& + \varphi_2) + 2A_2A_3e^{-\sigma t}K_{1001}L_{0010}\omega_d(\sigma^3 t - 2{\omega_d}^2 + \sigma t{\omega_d}^2)\cos(t\omega_d) \\
& - A_1A_3e^{-\sigma t}K_{0010}K_{1001}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1 - t\omega_d) \\
& - A_1A_4e^{-\sigma t}K_{0010}L_{1001}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1-\varphi_2-t\omega_d) \\
& - A_2A_4e^{-\sigma t}L_{0010}L_{1001}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_2-t\omega_d) \\
& + A_1A_3e^{-\sigma t}K_{0010}K_{1001}\omega_d(\sigma^2+2\sigma^3 t-3{\omega_d}^2+2\sigma t{\omega_d}^2)\cos(\varphi_1+t\omega_d) \\
& - A_1A_4e^{-\sigma t}K_{0010}L_{1001}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1-\varphi_2+t\omega_d) \\
& + A_2A_4e^{-\sigma t}L_{0010}L_{1001}\omega_d(\sigma^2+2\sigma^3 t-3{\omega_d}^2+2\sigma t{\omega_d}^2)\cos(\varphi_2+t\omega_d) \\
& + 2A_1A_4e^{-\sigma t}K_{0010}L_{1001}\omega_d(\sigma^2+\sigma^3 t-{\omega_d}^2+\sigma t{\omega_d}^2)\cos(\varphi_1+\varphi_2+t\omega_d) \\
& + 4A_1A_3K_{0010}K_{1001}\sigma{\omega_d}^2\sin(\varphi_1) + 4A_2A_4L_{0010}L_{1001}\sigma{\omega_d}^2\sin(\varphi_2) \\
& + 4A_1A_4K_{0010}L_{1001}\sigma{\omega_d}^2\sin(\varphi_1+\varphi_2) - 2A_2A_3e^{-\sigma t}K_{1001}L_{0010}(\sigma^3+3\sigma{\omega_d}^2 \\
& + \sigma^2 t{\omega_d}^2 + t{\omega_d}^4)\sin(t\omega_d) + A_1A_3e^{-\sigma t}K_{0010}K_{1001}\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_1-t\omega_d) \\
& + A_1A_4e^{-\sigma t}K_{0010}L_{1001}\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_1-\varphi_2-t\omega_d) \\
& + A_2A_4e^{-\sigma t}L_{0010}L_{1001}\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_2-t\omega_d) - A_1A_3e^{-\sigma t}K_{0010}K_{1001}(\sigma^3 \\
& + 5\sigma{\omega_d}^2 + 2\sigma^2 t{\omega_d}^2 + 2t{\omega_d}^4)\sin(\varphi_1+t\omega_d) - A_1A_4e^{-\sigma t}K_{0010}L_{1001}\sigma(\sigma^2 \\
& + {\omega_d}^2)\sin(\varphi_1-\varphi_2+t\omega_d) - A_2A_4e^{-\sigma t}L_{0010}L_{1001}(\sigma^3+5\sigma{\omega_d}^2+2\sigma^2 t{\omega_d}^2 \\
& + 2t{\omega_d}^4)\sin(\varphi_2+t\omega_d) - 2A_1A_4e^{-\sigma t}K_{0010}L_{1001}{\omega_d}^2(2\sigma+\sigma^2 t+t{\omega_d}^2)\sin(\varphi_1 \\
& + \varphi_2 + t\omega_d)\}
\end{aligned}
\tag{C.58}
$$

$$\boldsymbol{x_{2,2}^{bs1i2,u_2,u_2}(t)} = \frac{1}{4\omega_d(\sigma^2+{\omega_d}^2)^2}\{4A_3K_{1001}{\omega_d}^3(A_2L_{0001}+A_1K_{0001}\cos(\varphi_1))$$
$$+2A_1A_4K_{0001}L_{1001}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1-\varphi_2)$$
$$+4A_2A_4L_{0001}L_{1001}{\omega_d}^3\cos(\varphi_2)+2A_1A_4K_{0001}L_{1001}\omega_d(-\sigma^2+{\omega_d}^2)\cos(\varphi_1$$
$$+\varphi_2)+2A_2A_3e^{-\sigma t}K_{1001}L_{0001}\omega_d(\sigma^3t-2{\omega_d}^2+\sigma t{\omega_d}^2)\cos(t\omega_d)$$
$$-A_1A_3e^{-\sigma t}K_{0001}K_{1001}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1-t\omega_d)$$
$$-A_1A_4e^{-\sigma t}K_{0001}L_{1001}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1-\varphi_2-t\omega_d)$$
$$-A_2A_4e^{-\sigma t}L_{0001}L_{1001}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_2-t\omega_d)$$
$$+A_1A_3e^{-\sigma t}K_{0001}K_{1001}\omega_d(\sigma^2+2\sigma^3t-3{\omega_d}^2+2\sigma t{\omega_d}^2)\cos(\varphi_1+t\omega_d)$$
$$-A_1A_4e^{-\sigma t}K_{0001}L_{1001}\omega_d(\sigma^2+{\omega_d}^2)\cos(\varphi_1-\varphi_2+t\omega_d)$$
$$+A_2A_4e^{-\sigma t}L_{0001}L_{1001}\omega_d(\sigma^2+2\sigma^3t-3{\omega_d}^2+2\sigma t{\omega_d}^2)\cos(\varphi_2+t\omega_d)$$
$$+2A_1A_4e^{-\sigma t}K_{0001}L_{1001}\omega_d(\sigma^2+\sigma^3t-{\omega_d}^2+\sigma t{\omega_d}^2)\cos(\varphi_1+\varphi_2+t\omega_d)$$
$$+4A_1A_3K_{0001}K_{1001}\sigma{\omega_d}^2\sin(\varphi_1)+4A_2A_4L_{0001}L_{1001}\sigma{\omega_d}^2\sin(\varphi_2)$$
$$+4A_1A_4K_{0001}L_{1001}\sigma{\omega_d}^2\sin(\varphi_1+\varphi_2)-2A_2A_3e^{-\sigma t}K_{1001}L_{0001}(\sigma^3+3\sigma{\omega_d}^2$$
$$+\sigma^2t{\omega_d}^2+t{\omega_d}^4)\sin(t\omega_d)+A_1A_3e^{-\sigma t}K_{0001}K_{1001}\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_1-t\omega_d)$$
$$+A_1A_4e^{-\sigma t}K_{0001}L_{1001}\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_1-\varphi_2-t\omega_d)$$
$$+A_2A_4e^{-\sigma t}L_{0001}L_{1001}\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_2-t\omega_d)-A_1A_3e^{-\sigma t}K_{0001}K_{1001}(\sigma^3$$
$$+5\sigma{\omega_d}^2+2\sigma^2t{\omega_d}^2+2t{\omega_d}^4)\sin(\varphi_1+t\omega_d)-A_1A_4e^{-\sigma t}K_{0001}L_{1001}\sigma(\sigma^2$$
$$+{\omega_d}^2)\sin(\varphi_1-\varphi_2+t\omega_d)-A_2A_4e^{-\sigma t}L_{0001}L_{1001}(\sigma^3+5\sigma{\omega_d}^2+2\sigma^2t{\omega_d}^2$$
$$+2t{\omega_d}^4)\sin(\varphi_2+t\omega_d)-2A_1A_4e^{-\sigma t}K_{0001}L_{1001}{\omega_d}^2(2\sigma+\sigma^2t+t{\omega_d}^2)\sin(\varphi_1$$
$$+\varphi_2+t\omega_d)\}$$

(C. 59)

$$x_{2,2}^{bs2i2}(t) = x_{2,2}^{bs2i2,u_1,u_2}(t) \times u_1 u_2 + x_{2,2}^{bs2i2,u_2,u_2}(t) \times u_2 u_2 \qquad (C.60)$$

$$
\begin{aligned}
\boldsymbol{x}_{2,2}^{bs2i2,u_1,u_2}(\boldsymbol{t}) = \frac{1}{4(\sigma^2+{\omega_d}^2)^2}\{&4{A_3}^2K_{0010}K_{0101}{\omega_d}^2 + 2{A_4}^2L_{0010}L_{0101}(\sigma^2+{\omega_d}^2) \\
&+ 4A_3A_4(K_{0101}L_{0010}+K_{0010}L_{0101}){\omega_d}^2\cos(\varphi_2) + 2{A_4}^2L_{0010}L_{0101}(-\sigma^2 \\
&+ {\omega_d}^2)\cos(2\varphi_2) + 2e^{-\sigma t}(-{A_4}^2L_{0010}L_{0101}(\sigma^2+{\omega_d}^2) + {A_3}^2K_{0010}K_{0101}(\sigma^3 t \\
&- 2{\omega_d}^2 + \sigma t{\omega_d}^2))\cos(t\omega_d) - A_3A_4e^{-\sigma t}(K_{0101}L_{0010}+K_{0010}L_{0101})(\sigma^2 \\
&+ {\omega_d}^2)\cos(\varphi_2 - t\omega_d) + A_3A_4e^{-\sigma t}(K_{0101}L_{0010}+K_{0010}L_{0101})(\sigma^2 + 2\sigma^3 t \\
&- 3{\omega_d}^2 + 2\sigma t{\omega_d}^2)\cos(\varphi_2 + t\omega_d) + 2{A_4}^2e^{-\sigma t}L_{0010}L_{0101}(\sigma^2 + \sigma^3 t - {\omega_d}^2 \\
&+ \sigma t{\omega_d}^2)\cos(2\varphi_2 + t\omega_d) + 4A_3A_4(K_{0101}L_{0010}+K_{0010}L_{0101})\sigma\omega_d\sin(\varphi_2) \\
&+ 4{A_4}^2L_{0010}L_{0101}\sigma\omega_d\sin(2\varphi_2) - \frac{1}{\omega_d}2e^{-\sigma t}({A_4}^2L_{0010}L_{0101}\sigma(\sigma^2+{\omega_d}^2) \\
&+ {A_3}^2K_{0010}K_{0101}(\sigma^3 + 3\sigma{\omega_d}^2 + \sigma^2 t{\omega_d}^2 + t{\omega_d}^4))\sin(t\omega_d) \\
&+ \frac{1}{\omega_d}A_3A_4e^{-\sigma t}(K_{0101}L_{0010}+K_{0010}L_{0101})\sigma(\sigma^2+{\omega_d}^2)\sin(\varphi_2 - t\omega_d) \\
&- \frac{1}{\omega_d}A_3A_4e^{-\sigma t}(K_{0101}L_{0010}+K_{0010}L_{0101})(\sigma^3 + 5\sigma{\omega_d}^2 + 2\sigma^2 t{\omega_d}^2 \\
&+ 2t{\omega_d}^4)\sin(\varphi_2 + t\omega_d) - 2{A_4}^2e^{-\sigma t}L_{0010}L_{0101}\omega_d(2\sigma + \sigma^2 t + t{\omega_d}^2)\sin(2\varphi_2 \\
&+ t\omega_d)\}
\end{aligned}
$$

(C. 61)

$$x_{2,2}^{bs2i2,u_2,u_2}(t) = \frac{1}{4(\sigma^2+\omega_d{}^2)^2}\{4A_3{}^2K_{0001}K_{0101}\omega_d{}^2 + 2A_4{}^2L_{0001}L_{0101}(\sigma^2+\omega_d{}^2)$$

$$+ 4A_3A_4(K_{0101}L_{0001}+K_{0001}L_{0101})\omega_d{}^2\cos(\varphi_2) + 2A_4{}^2L_{0001}L_{0101}(-\sigma^2$$

$$+ \omega_d{}^2)\cos(2\varphi_2) + 2e^{-\sigma t}(-A_4{}^2L_{0001}L_{0101}(\sigma^2+\omega_d{}^2) + A_3{}^2K_{0001}K_{0101}(\sigma^3 t$$

$$- 2\omega_d{}^2 + \sigma t\omega_d{}^2))\cos(t\omega_d) - A_3A_4e^{-\sigma t}(K_{0101}L_{0001}+K_{0001}L_{0101})(\sigma^2$$

$$+ \omega_d{}^2)\cos(\varphi_2 - t\omega_d) + A_3A_4e^{-\sigma t}(K_{0101}L_{0001}+K_{0001}L_{0101})(\sigma^2 + 2\sigma^3 t$$

$$- 3\omega_d{}^2 + 2\sigma t\omega_d{}^2)\cos(\varphi_2 + t\omega_d) + 2A_4{}^2e^{-\sigma t}L_{0001}L_{0101}(\sigma^2 + \sigma^3 t - \omega_d{}^2$$

$$+ \sigma t\omega_d{}^2)\cos(2\varphi_2 + t\omega_d) + 4A_3A_4(K_{0101}L_{0001}+K_{0001}L_{0101})\sigma\omega_d\sin(\varphi_2)$$

$$+ 4A_4{}^2L_{0001}L_{0101}\sigma\omega_d\sin(2\varphi_2) - \frac{1}{\omega_d}2e^{-\sigma t}(A_4{}^2L_{0001}L_{0101}\sigma(\sigma^2+\omega_d{}^2)$$

$$+ A_3{}^2K_{0001}K_{0101}(\sigma^3 + 3\sigma\omega_d{}^2 + \sigma^2 t\omega_d{}^2 + t\omega_d{}^4))\sin(t\omega_d)$$

$$+ \frac{1}{\omega_d}A_3A_4e^{-\sigma t}(K_{0101}L_{0001}+K_{0001}L_{0101})\sigma(\sigma^2+\omega_d{}^2)\sin(\varphi_2 - t\omega_d)$$

$$- \frac{1}{\omega_d}A_3A_4e^{-\sigma t}(K_{0101}L_{0001}+K_{0001}L_{0101})(\sigma^3 + 5\sigma\omega_d{}^2 + 2\sigma^2 t\omega_d{}^2$$

$$+ 2t\omega_d{}^4)\sin(\varphi_2 + t\omega_d) - 2A_4{}^2e^{-\sigma t}L_{0001}L_{0101}\omega_d(2\sigma + \sigma^2 t + t\omega_d{}^2)\sin(2\varphi_2$$

$$+ t\omega_d)\}$$

(C. 62)

$$x_{2,2}^{qi1}(t) = x_{2,2}^{qi1,u_1,u_1}(t) \times u_1 u_1 \tag{C.63}$$

$$\begin{aligned}\boldsymbol{x}_{\mathbf{2,2}}^{\boldsymbol{qi1},\boldsymbol{u_1},\boldsymbol{u_1}}(\boldsymbol{t}) = \frac{1}{\sigma^2+{\omega_d}^2} e^{-\mathrm{\sigma t}}\{A_3 e^{\sigma t} K_{0020}\omega_d + A_4 e^{\sigma t} L_{0020}\omega_d \cos(\varphi_2) - A_3 K_{0020}\omega_d \cos(t\omega_d) \\ - A_4 L_{0020}\omega_d \cos(\varphi_2 + t\omega_d) + A_4 e^{\sigma t} L_{0020}\sigma \sin(\varphi_2) - A_3 K_{0020}\sigma \sin(t\omega_d) \\ - A_4 L_{0020}\sigma \sin(\varphi_2 + t\omega_d)\}\end{aligned}$$

(C. 64)

$$x_{2,2}^{bi1i2}(t) = x_{2,2}^{bi1i2,u_1,u_2}(t) \times u_1 u_2 \qquad (C.65)$$

$$\boldsymbol{x_{2,2}^{bi1i2,u_1,u_2}(t)} = \frac{1}{\sigma^2 + {\omega_d}^2} e^{-\sigma t}\{A_3 e^{\sigma t} K_{0011}\omega_d + A_4 e^{\sigma t} L_{0011}\omega_d \cos(\varphi_2)$$

$$- A_3 K_{0011}\omega_d \cos(t\omega_d) - A_4 L_{0011}\omega_d \cos(\varphi_2 + t\omega_d) + A_4 e^{\sigma t} L_{0011}\sigma \sin(\varphi_2)$$

$$- A_3 K_{0011}\sigma \sin(t\omega_d) - A_4 L_{0011}\sigma \sin(\varphi_2 + t\omega_d)\}$$

$$(C.66)$$

$$x_{2,2}^{qi2}(t) = x_{2,2}^{qi2,u_2,u_2}(t) \times u_2 u_2 \qquad (C.67)$$

$$\boldsymbol{x_{2,2}^{qi2,u_2,u_2}(t)} = \frac{1}{\sigma^2 + {\omega_d}^2} e^{-\sigma t}\{A_3 e^{\sigma t} K_{0002}\omega_d + A_4 e^{\sigma t} L_{0002}\omega_d \cos(\varphi_2) - A_3 K_{0002}\omega_d \cos(t\omega_d)$$

$$- A_4 L_{0002}\omega_d \cos(\varphi_2 + t\omega_d) + A_4 e^{\sigma t} L_{0002}\sigma \sin(\varphi_2) - A_3 K_{0002}\sigma \sin(t\omega_d)$$

$$- A_4 L_{0002}\sigma \sin(\varphi_2 + t\omega_d)\}$$

$$(C.68)$$

VITA

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Alexander J. Chen received his Bachelor of Science in Aeronautical and Astronautical Engineering from The Ohio State University in June, 2010. During his undergraduate studies, he was a member of the men's varsity swimming and diving team and earned Academic All-Big Ten Honors all four years. As a swimmer, he also qualified for Olympic Trials in 2008 and was awarded the team's Spirit of Giving award his senior year in 2010. After graduating, Alexander started employment as a contractor for the Federal Aviation Administration in Washington, D.C. and worked two years in systems engineering. Following this period, he accepted an offer to work as a civil servant for the Department of the Navy as an Aerospace Engineer in Norfolk, VA. His work consists of procurement support of aerospace systems such as the Navy's F/A-18 Super Hornet and NASA's Space Launch System. He has been working full-time and completing coursework and research as a part-time student at Old Dominion University. Alexander will complete his Master of Science in Aerospace Engineering in May 2018 with a concentration in dynamics and controls.